434(2): The Quantization of Linear Momentum in m Space

The Schroedinger quantization of linear momentum $\underline{p}$ takes place in a space with:

$$m(r) = 1 \quad -(1)$$

using:

$$\underline{p}\,\phi = -i\hbar \frac{\partial \phi}{\partial r} \underline{e}_r \quad -(2)$$

assuming that

$$\underline{\nabla}\,\phi = \frac{\partial \phi}{\partial r} \underline{e}_r \quad -(3)$$

so

$$\langle \underline{p} \rangle = -i\hbar \int \phi^* \frac{\partial \phi}{\partial r} d\tau\, \underline{e}_r \quad -(4)$$

and

$$\langle p \rangle = -i\hbar \int \phi^* \frac{\partial \phi}{\partial r} d\tau \quad -(5)$$

Assuming:

$$\phi = e^{i\kappa r} \quad -(6)$$

then

$$\frac{\partial \phi}{\partial r} = i\kappa\phi \quad -(7)$$

and

$$p = \langle p \rangle = \hbar\kappa \int \phi^* \phi \, d\tau \quad -(8)$$

i.e.

$$p = \hbar\kappa \quad -(9)$$

This is the fundamental equation of the wave particle dualism of Louis de Broglie.

In m space the de Broglie wave

particle dualism is changed fundamentally into a much richer structure. The Schroedinger quantization in n space is:

$$\boxed{p_1 \phi = -i\hbar \frac{\partial \phi}{\partial r_1}} \quad -(10)$$

In Eq. (10):

$$\frac{\partial \phi}{\partial r_1} = \frac{\partial \phi}{\partial r} \frac{\partial r}{\partial r_1} \quad -(11)$$

So:

$$\langle p_1 \rangle = -i\hbar \int \phi^* \frac{\partial r}{\partial r_1} \frac{\partial \phi}{\partial r} \, d\tau \quad -(12)$$

By definition:

$$r_1 = \frac{r}{n(r)^{1/2}} \quad -(13)$$

so

$$\frac{\partial r_1}{\partial r} = \left(n(r)^{1/2} - \frac{d}{dr} \left(n(r)^{1/2} \right) \right) \Big/ n(r) \quad -(14)$$

Define

$$f = n(r) \quad -(15)$$

and consider:

$$y = f^{1/2} = n^{1/2}(r) \quad -(16)$$

then

$$\frac{dy}{dr} = \frac{dy}{df} \frac{df}{dr} = \frac{1}{2} f^{-1/2} \frac{df}{dr} \quad -(17)$$

so:

$$\frac{d}{dr} n(r)^{1/2} = \frac{1}{2n(r)^{1/2}} \frac{dn(r)}{dr} \quad -(18)$$

It follows that:

$$\frac{dr_1}{dr} = \frac{1}{m(r)}\left(m(r)^{1/2} - \frac{1}{2m(r)^{1/2}}\frac{dm(r)}{dr}\right) \quad -(19)$$

$$= \frac{1}{2m(r)^{3/2}}\left(2m(r) - \frac{dm(r)}{dr}\right) \quad -(19a)$$

So

$$\frac{dr}{dr_1} = \frac{2m(r)^{3/2}}{2m(r) - \frac{dm(r)}{dr}} \quad -(20)$$

Therefore the expectation values of linear momentum in m space are: $-(21)$

$$p_1 = \langle p_1 \rangle = -i\hbar \int \phi^* \frac{2m(r)^{3/2}}{2m(r) - \frac{dm(r)}{dr}} \frac{d\phi}{dr} \cdot d\tau$$

and

$$p_1 \phi = -i\hbar \left(\frac{2m(r)^{3/2}}{2m(r) - \frac{dm(r)}{dr}}\right) \frac{d\phi}{dr} \quad -(22)$$

Therefore the de Broglie wave particle dualism is changed fundamentally in m space, creating structure and quantized particle momenta.

In the first approximation, assume that the m space is similar to the flat space, then

$$\phi \sim e^{ikr} \quad -(23)$$

It follows ~~that~~:

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$$P_1 = \langle P_1 \rangle = \hbar \kappa \int \phi^* \frac{\partial n(r)^{3/2}}{\partial n(r) - \frac{dn(r)}{dr}} \phi \, d\tau$$

So depending on the choice of $n(r)$, there are several values of P_1. — (24)

Specifically:

$$\boxed{\langle P_1 \rangle = \hbar \kappa \int e^{-i\kappa r} \frac{\partial n(r)^{3/2}}{\partial n(r) - \frac{dn(r)}{dr}} e^{i\kappa r} d\tau} \quad -(25)$$

These are momenta levels of entities which can be defined as elementary particles of a carrier field.

1) Under the condition:

$$\partial n(r) = \frac{dn(r)}{dr} \quad -(26)$$

the momenta become infinite

2) Under the condition: $n(r) \longrightarrow 0$ — (27)

the momenta disappear, and we are left with the rest energies of the elementary particles.

3) Under the condition:

$$n(r) = 1 \quad -(28)$$

$$\langle P_1 \rangle = \langle p \rangle = \hbar \kappa, \quad -(29)$$

which is the de Broglie wave particle dualism,

Q.E.D.