434(5): The Time Dependent Schroedinger Equation in General Relativity and m Theory

The conventional time dependent Schroedinger equation is non relativistic and is obtained from:

$$i\hbar \frac{\partial \psi}{\partial t} = E\psi = H\psi \quad -(1)$$

So the time dependent Schroedinger equation is:

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi \quad -(2)$$

In m theory and general relativity this becomes:

$$\boxed{i\hbar \frac{\partial \psi}{\partial t_1} = H_1 \psi} \quad -(3)$$

Eq. (3) can be developed as:

$$\frac{i\hbar}{m(r)^{1/2}} \frac{\partial \psi}{\partial t} = H_1 \psi \quad -(4)$$

where:

$$H_1 = \gamma_1 m(r) mc^2 + U_1 \quad -(5)$$

where

$$\gamma_1 = \left(m(r_1) - \frac{v_1^2}{m(r_1)c^2} \right)^{-1/2} \quad -(6)$$

and U_1 is the potential energy in m space. For example, for Coulombic interaction:

$$U_1 = -\frac{e^2 m(r)^{1/2}}{4\pi\epsilon_0 r} \quad -(7)$$

Using the assumption:

$$\psi(r,t) = e^{-i\omega t}\phi(r) \quad -(8)$$

in Eq. (4) it follows that:

$$\langle H_1 \rangle = \int \psi^* H_1 \psi \, d\tau = \hbar\omega \int \phi^*(r) \frac{1}{n(r)^{1/2}} \phi(r) d\tau \quad -(9)$$

In general:

$$\psi = \psi(r, \theta, \phi, t) \quad -(10)$$

$$= e^{-i\omega t} \phi(r, \theta, \phi) \quad -(11)$$

As shown in previous UFT papers eq. (9) produces the Lamb shift from m theory.

In the non-relativistic limit, Eq. (2) is the time dependent Schroedinger equation of the H atom, with:

$$\psi(r, \theta, \phi, t) = e^{-i\omega t} \phi_H(r, \theta, \phi) \quad -(12)$$

where ϕ_H are the well known hydrogenic wave functions. In the non relativistic limit:

$$n(r) = 1 \quad -(13)$$

and

$$|E| = \left| \int \psi^* H \psi \, d\tau \right| = \hbar\omega \quad -(14)$$

$$= \left(\frac{\mu e^4}{32\pi^2 \epsilon_0^2 \hbar^2} \right) \frac{1}{n^2}$$

The energy levels of the H atom are negative as is well known:

$$E = \frac{-\mu e^4}{32\pi^2 \epsilon_0^2 \hbar^2} \frac{1}{n^2} \quad -(15)$$

because they are bound states. The modulus is used in Eq. (14) because $\hbar\omega$ must be positive. Here n is the principal quantum number as is well known. Eq. (14) is an expression of the wave particle dualism of the hydrogen atom, which is both a particle and a wave.

In the n theory of Eq. (9) more energy levels of the H atom appear, and this manifests itself as the Lamb shift. Therefore the n theory of the H atom – the Lamb shift is the effect of a vacuum on the H atom.

Numerical Work

It would be interesting to compute the energy levels:

$$E = \hbar\omega \int \phi_H^* \frac{1}{n(r)^{1/2}} \phi_H \, d\tau \quad -(16)$$

from the hydrogenic wavefunctions for any $n(r)$, to illustrate the effect of $n(r)$ in producing more energy levels and shifts.