419(4): The Three Kepler Laws in m Theory

Kepler's first law states by empirical observation that the orbit of m around M is an ellipse. In m theory this is replaced by:

$$\frac{dH}{dt} = 0 \quad -(1)$$

and

$$\frac{dL}{dt} = 0 \quad -(2)$$

giving many different types of orbit, precession, retrograde precession, shrinking and expanding orbits and so on.

Kepler's second law is derived by considering Fig (1):

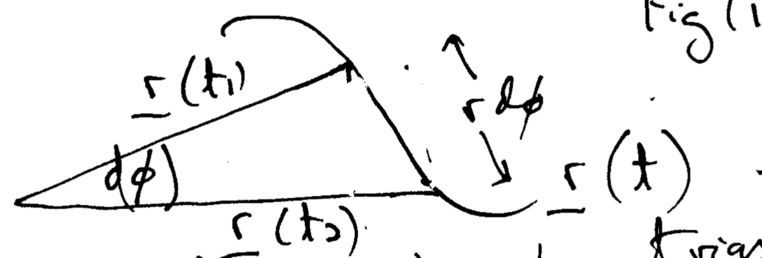

Fig (1)

So using the formula for the area of an isosceles triangle:

$$dA = \frac{1}{2} r^2 d\phi \quad -(3)$$

In Newtonian theory:

$$L = m r^2 \frac{d\phi}{dt} \quad -(4)$$

From eqs. (3) and (4):

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\phi}{dt} = \frac{L}{2m} = \text{constant} \quad -(5)$$

This is Kepler's second law of 1609, derived from data by Tycho Brahe.

In m theory, Kepler's second law is modified as follows. The angular momentum of m theory is:

$$L = \frac{\gamma m r^2 \dot{\phi}}{m(r)} \quad -(6)$$

in which:

$$\gamma = \left(m(r) - \frac{v_N^2}{m(r)c^2} \right)^{-1/2} \quad -(7)$$

Therefore

$$\frac{d\phi}{dt} = \frac{m(r)\,L}{\gamma m r^2} \quad -(8)$$

so

$$\boxed{\frac{dA}{dt} = \frac{1}{2}\,\frac{m(r)\,L}{\gamma m}} \quad -(9)$$

It follows that:

$$\frac{dA}{dt} = \frac{L}{2m}\, m(r) \left(m(r) - \frac{v_N^2}{m(r)c^2} \right)^{1/2} \quad -(10)$$

where

$$v_N^2 \sim MG\, m(r)^{3/2} \left(\frac{2\, m(r)^{1/2}}{r} - \frac{1}{a} \right) \quad -(11)$$

In m theory, dA/dt is no longer constant.

Kepler's Third Law in the standard model is obtained from eq. (5):

$$dt = \frac{2m}{L}\, dA \quad -(12)$$

so

$$T = \int_0^t dt = \frac{2m}{L} \int_0^A dA \quad -(13)$$

i.e.

$$T = \frac{2m}{L}\, A \quad -(14)$$

The time taken to complete one orbit, T, is proportional to the area of the orbit, A. This is Kepler's Third law. In Newtonian theory A is the area of an ellipse:

3)

$$A = \pi a b \quad -(15)$$

where a and b are the semi major and semi minor axes:

$$a = \frac{\alpha}{1-\epsilon^2}, \quad b = \frac{\alpha}{(1-\epsilon^2)^{1/2}} \quad -(16)$$

where α is the half right latitude and ϵ is the eccentricity.

So

$$T = \frac{2\pi m}{L} \frac{\alpha^2}{(1-\epsilon^2)^{3/2}} = \frac{2m\pi}{L} a^{3/2} \alpha^{1/2} \quad -(17)$$

It follows that:

$$T^2 = 4\pi^2 \left(\frac{m^2 \alpha}{L^2} \right) a^3 \quad -(18)$$

$$= \frac{4\pi^2}{MG} a^3$$

where we have used:

$$\alpha = \frac{L^2}{m M G} \quad -(19)$$

Eq. (18) is Kepler's third law:

$$\boxed{T^2 = \frac{4\pi^2}{MG} a^3} \quad -(20)$$

in the standard model.

In m theory Kepler's third law is modified as follows. From eq. (9):

$$dt = \frac{2m}{L} \frac{\gamma}{n(r)} dA \quad -(21)$$

so

$$\boxed{T = \frac{2m}{L} \int_0^A \frac{\gamma}{n(r)} dA} \quad -(22)$$

4) In the Newtonian limit:

$$\frac{\gamma}{n(r)} \rightarrow 1 \quad -(23)$$

and Kepler's third law is recovered.

In Eq. (22):

$$\frac{\gamma}{n(r)} = \frac{1}{n(r)}\left(n(r) - \frac{v_N^2}{n(r)c^2}\right)^{-1/2} \quad -(24)$$

in which:

$$v_N^2 \sim MG\, n(r)^{3/2}\left(\frac{2n(r)^{1/2}}{r} - \frac{1}{a}\right) \quad -(25)$$

Therefore

$$\frac{\gamma}{n(r)} = f(r) \quad -(26)$$

i.e. $\gamma / n(r)$ is a function of r. In general, assume that:

$$A = f_1(r), \quad -(27)$$

then:

$$dA = \frac{df_1(r)}{dr}\, dr \quad -(28)$$

and

$$T = \frac{2m}{L}\int_0^A \frac{\gamma}{n(r)}\frac{df_1(r)}{dr}\, dr \quad -(29)$$

$$T = \frac{2m}{L}\int_0^{f_1(r)} \frac{\gamma}{n(r)}\frac{df_1(r)}{dr}\, dr \quad -(30)$$

If it is assumed that A is not a function of r, as for example in an elliptical orbit, where:

$$A = \pi a b \quad -(31)$$

then A is a constant of motion, independent of r.

Therefore assume that:

$$T \sim \frac{\gamma}{n(r)} \cdot \frac{2mA}{L} \qquad -(32)$$

and

$$\boxed{T^2 = \left(\frac{\gamma}{n(r)}\right)^2 \left(\frac{4\pi^2}{MG}\, a^3\right)} \qquad -(33)$$

This is Kepler's third law in m theory.

From eqs. (24), (25) and (33) the time T can be evaluated in terms of $n(r)$ for a given M and a.

In the previous note it was found that the Newtonian limit

$$\frac{\gamma}{n(r)} \rightarrow 1 \qquad -(34)$$

is wildly incorrect for the S2 star. This means that the standard model mass for the "black hole" is completely incorrect. The effective mass in m theory is:

$$M(\text{effective}) = \left(\frac{n(r)}{\gamma}\right)^2 M_N \qquad -(35)$$

where M_N is the Newtonian mass:

$$M_N = \frac{4\pi^2 a^3}{T^2 G} \qquad -(36)$$