419(5): Velocity Curve of a Whirlpool Galaxy in m Theory

Consider the infinitesimal line element of m theory:

$$ds^2 = c^2 d\tau^2 = m(r) c^2 dt^2 - \frac{dr^2}{m(r)} - r^2 d\phi^2 \quad -(1)$$

where $m(r)$ is any function of r. It follows that:

$$d\underline{r} \cdot d\underline{r} = \frac{dr^2}{m(r)} + r^2 d\phi^2 \quad -(2)$$

where:

$$d\underline{r} = \frac{\partial \underline{r}}{\partial r} dr + \frac{\partial \underline{r}}{\partial \phi} d\phi \quad -(3)$$

Therefore:

$$d\underline{r} \cdot d\underline{r} = \left(\frac{\partial \underline{r}}{\partial r} dr + \frac{\partial \underline{r}}{\partial \phi} d\phi \right) \cdot \left(\frac{\partial \underline{r}}{\partial r} dr + \frac{\partial \underline{r}}{\partial \phi} d\phi \right)$$

$$= \frac{dr^2}{m(r)} + r^2 d\phi^2 \quad -(4)$$

A possible solution is:

$$\left(\frac{\partial \underline{r}}{\partial r} \right)^2 dr^2 = \frac{1}{m(r)} dr^2 \quad -(5)$$

$$\left(\frac{\partial \underline{r}}{\partial \phi} \right)^2 d\phi^2 = r^2 d\phi^2 \quad -(6)$$

$$\frac{\partial \underline{r}}{\partial \phi} \cdot \frac{\partial \underline{r}}{\partial r} = 0 \quad -(7)$$

It follows that:

$$\frac{\partial \underline{r}}{\partial r} = \frac{1}{m(r)} \underline{e}_r \quad -(8)$$

$$\frac{\partial \underline{r}}{\partial \phi} = r \underline{e}_\phi \quad -(9)$$

So

$$\underline{r} = \frac{r}{m(r)^{1/2}} \underline{e}_r \quad -(10)$$

and

$$r^2 \longrightarrow \frac{r^2}{m(r)} \quad -(11)$$

<u>in m space</u>.

The velocity in m space is:

$$v^2 = \frac{1}{m(r)} \left(\dot{r}^2 + r^2 \dot{\phi}^2 \right) \quad -(12)$$

Now use:

$$\frac{dr}{dt} = \frac{dr}{d\phi} \frac{d\phi}{dt} = \frac{dr}{d\phi} \dot{\phi} \quad -(13)$$

so

$$v^2 = \frac{\dot{\phi}^2}{m(r)} \left(r^2 + \left(\frac{dr}{d\phi} \right)^2 \right) \quad -(14)$$

In m theory the constant angular momentum is:

$$L = \frac{\gamma m r^2 \dot{\phi}}{m(r)} \quad -(15)$$

where γ is the generalized Lorentz factor:

$$\gamma = \left(m(r) - \frac{v^2}{m(r) c^2} \right)^{-1/2} \quad -(16)$$

Therefore:

$$\dot{\phi}^2 = \frac{m(r)^2 L^2}{\gamma^2 m^2 r^4} \quad -(17)$$

and

$$v^2 = \frac{m(r) L^2}{\gamma^2 m^2 r^4} \left(r^2 + \left(\frac{dr}{d\phi} \right)^2 \right) \quad -(18)$$

$$= \frac{L^2 m(r)}{\gamma^2 m^2} \left(\frac{1}{r^2} + \frac{1}{r^4} \left(\frac{dr}{d\phi} \right)^2 \right)$$

) In a whirlpool galaxy the spiral arms can be modelled by:

$$\frac{1}{r} = \frac{\phi}{r_0} \quad -(19)$$

So

$$\frac{dr}{d\phi} = -\frac{r_0}{\phi^2} \quad -(20)$$

and

$$\left(\frac{dr}{d\phi}\right)^2 = \frac{r_0^2}{\phi^4} = \frac{r^4}{r_0^2} \quad -(21)$$

It follows from eqs. (18) and (21) that:

$$v^2 = \frac{L^2 m(r)}{\gamma^2 m^2}\left(\frac{1}{r^2} + \frac{1}{r_0^2}\right) \quad -(22)$$

and

$$\boxed{v^2 \xrightarrow[r \to \infty]{} \frac{L^2 m(r)}{\gamma^2 m^2 r_0^2}} \quad -(23)$$

i.e:

$$v^2 \rightarrow \frac{L^2}{m^2 r_0^2} \cdot \frac{m(r)}{\gamma^2} \quad -(24)$$

in which $(L/(m r_0))^2$ is a constant. It follows that as r becomes infinite:

$$\frac{m(r)}{\gamma^2} = \text{constant} := A \quad -(25)$$

and

$$v^2 \xrightarrow[r \to \infty]{} \left(\frac{L}{m r_0}\right)^2 A \quad -(26)$$

where

$$\gamma \rightarrow \left(m(r) - \frac{L^2 A}{m^2 r_0^2 m(r) c^2}\right)^{-1/2} \quad -(27)$$

From eqs. (25) and (27):

$$n(r) \xrightarrow[r \to \infty]{} \frac{A}{m(r) - \frac{B}{m(r)}} \quad -(28)$$

where

$$B := \frac{L^2 A}{m^2 r_0^2 c^2} \quad -(29)$$

So:

$$n(r) \xrightarrow[r \to \infty]{} \frac{m(r) A}{m(r)^2 - B} \quad -(30)$$

i.e.

$$m(r)^2 = A + B$$
$$= A\left(1 + \frac{L^2}{m^2 r_0^2 c^2}\right) \quad -(31)$$

This means that $n(r)$ evolves to a constant value.

In general, the velocity curve in m theory is given by eq. (18):

$$v^2 = \frac{L^2 n(r)}{\gamma^2 m^2}\left(\frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\phi}\right)^2\right) \quad -(32)$$

where

$$\frac{1}{\gamma^2} = m(r) - \frac{v^2}{m(r) c^2} \quad -(33)$$

Eqs. (32) and (33) can be solved using computer algebra to find v in terms of any $n(r)$ and any $dr/d\phi$.

For example if it is assumed that:

$$\frac{1}{r} = \frac{\phi}{r_0} \quad -(34)$$

then:

$$v^2 = \frac{L^2 m(r)}{m^2}\left(m(r) - \frac{v^2}{m(r)c^2}\right)\left(\frac{1}{r^2}+\frac{1}{r_0^2}\right) \quad -(35)$$

and

$$v^2 = \frac{\frac{L^2}{m^2} m^2(r)\left(\frac{1}{r^2}+\frac{1}{r_0^2}\right)}{1+\frac{L^2}{m^2c^2}\left(\frac{1}{r^2}+\frac{1}{r_0^2}\right)}, \quad -(36)$$

$$v^2 \xrightarrow[r\to\infty]{} \frac{L^2 m^2(r)}{m^2 r_0^2}\left(1+\frac{L^2}{m^2c^2r_0^2}\right)^{-1} \quad -(37)$$

If the velocity v becomes a constant as $r \to \infty$ then:

$$\boxed{m^2(r) \xrightarrow[r\to\infty]{} \text{constant}} \quad -(38)$$

From eq. (32), using:

$$\frac{1}{r^2} \xrightarrow[r\to\infty]{} 0 \quad -(39)$$

then:

$$v^2 \xrightarrow[r\to\infty]{} \frac{L^2 m(r)}{\gamma^2 m^2 r^4}\left(\frac{dr}{d\phi}\right)^2 \quad -(40)$$

Now assume that:

$$v^2 \xrightarrow[r\to\infty]{} \text{constant} := A \quad -(41)$$

so the orbit must be:

$$\frac{dr}{d\phi} = \frac{\gamma^2 m^2 r^4 A}{m(r) L^2} \quad -(42)$$

where

$$\gamma^2 = \left(m(r) - \frac{v^2}{m(r)c^2}\right)^{-1} \quad -(43)$$

In the approximation:

6)

$$v \lll c \quad - (44)$$

then:

$$\gamma^2 \sim \frac{1}{m(r)} \quad - (45)$$

and

$$\frac{d\phi}{dr} = \frac{L\, m(r)}{m r^2 A^{1/2}} \quad - (46)$$

so

$$\phi = \frac{L}{m A^{1/2}} \int \frac{m(r)}{r^2}\, dr \quad - (47)$$

In the limit:

$$m(r) \rightarrow 1 \quad - (48)$$

Eq. (47) gives the spiral:

$$\phi = -\frac{L}{m A^{1/2}} \frac{1}{r}. \quad - (49)$$

Otherwise the most general orbit that gives a constant v as $r \rightarrow \infty$ is given by eq. (47). By varying $m(r)$, different shapes of galaxies can be produced.

More generally, from eqs. (42) and (43):

$$\left(\frac{dr}{d\phi}\right)^2 = \frac{m^2 r^4 A}{m(r)\left(m(r) - \frac{v^2}{m(r) c^2}\right)} \quad - (50)$$

Eq. (50) has been derived in the limit:

$$v^2 \longrightarrow A = \text{constant} \quad - (51)$$

so

$$\left(\frac{dr}{d\phi}\right)^2 = \frac{m^2 r^4 A}{m(r)\left(m(r) - \frac{A}{m(r) c^2}\right)} \quad - (52)$$

7) It follows that:

$$\frac{d\phi}{dr} = \left(\frac{m(r)}{m^2 r^4 A} \left(m(r) - \frac{A}{m(r) c^2} \right) \right)^{1/2} \quad -(53)$$

and

$$\phi = \frac{1}{m} \int \left(\frac{m(r)}{A r^4} \left(m(r) - \frac{A}{m(r) c^2} \right) \right)^{1/2} dr \quad -(54)$$

Eq. (54) gives a generalized spiral structure. For different $m(r)$, different whirlpool galaxies emerge.
