406(6): Calculation of the Spin Connection for Light Deflection due to Gravitation

The basic equation is:

$$\omega_r \phi_0 = m \frac{d}{dt}\left(\gamma v_N\right) + \frac{\partial \phi_0}{\partial r} \quad -(1)$$

where

$$\phi_0 = -\frac{mMG}{r} \quad -(2)$$

so

$$\omega_r \frac{MG}{r} = \frac{d}{dt}\left(\gamma v_N\right) + \frac{MG}{r^2} \quad -(3)$$

From previous work:

$$\frac{d}{dt}\left(\gamma v_N\right) = \gamma^3 \frac{dv_N}{dt} + \frac{MG}{r^2} \quad -(4)$$

where

$$\gamma = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} \quad -(5)$$

In Cartesian coordinates:

$$\gamma^3 \ddot{r} = \frac{MG}{r}\left(\omega_r - \frac{1}{r}\right) \quad -(6)$$

In previous work we have considered the equation

$$\gamma^3 \ddot{r} = -\frac{MG}{r^2} \quad -(7)$$

and have shown that this gives a precessing ellipse, or in general a precessing conical section. When considering light deflection due to gravitation:

$$\ddot{r} = 0 \quad -(8)$$

because the speed of light c cannot be exceeded, so c cannot change. Therefore:

$$\omega_r = \frac{1}{r} \quad -(9)$$

for light deflection due to gravitation.

From previous work:

$$\frac{1}{r} = \frac{2}{3}\,\frac{\langle \delta\underline{r} \cdot \delta\underline{r} \rangle}{r^3} \quad -(10)$$

and

$$\frac{\langle \delta\underline{r} \cdot \delta\underline{r} \rangle}{r^2} \doteq \frac{3}{2} \quad -(11)$$

In this case $\langle^{r} \delta\underline{r} \cdot \delta\underline{r} \rangle$ is at a maximum:

$$\langle \delta\underline{r} \cdot \delta\underline{r} \rangle_{max} = \frac{3}{2} r^2 \quad -(12)$$