405(2): Orbital Precession as a Thomas Precession

The Thomas precession for a rotation of $2\pi$ is:

$$\Delta\phi = 2\pi\left(\left(1-\frac{v^2}{c^2}\right)^{-1/2} - 1\right) \quad -(1)$$

where v is the orbital velocity of the rotation of the metric.
The experimentally observed change is:

$$\Delta\phi = 2\pi\left(\frac{3MG}{ac^2(1-\epsilon)} - 1\right) \quad -(2)$$

where M is the mass of the attracting object, G the Newton constant, a is the semi major axis of the orbit and $\epsilon$ is its eccentricity. The result (2) is obtained from the original method of UFT 403. Therefore the orbital velocity of the rotation of the ECE2 metric is given by:

$$\left(1-\frac{v^2}{c^2}\right)^{-1/2} = \frac{3MG}{ac^2(1-\epsilon)} \quad -(3)$$

If

$$v \lll c \quad -(4)$$

$$\frac{1}{2}\frac{v^2}{c^2} \sim \frac{3MG}{ac^2(1-\epsilon)} \quad -(5)$$

and

$$\boxed{v^2 \sim \frac{6MG}{a(1-\epsilon)}} \quad -(6)$$

The ECE2 covariant metric is defined by the infinitesimal line element:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 \quad -(7)$$

in plane polar coordinates. Thomas precession is the frame rotation:

$$\phi' = \phi + \omega t \quad -(8)$$

where

$$v = \omega r. \quad -(9)$$

This frame rotation results in the relativistic angular
velocity:

$$\Omega = \omega \left(1 - \frac{v^2}{c^2}\right)^{-1} \quad -(10)$$

and the time dilation:

$$dt' = \left(1 - \frac{v^2}{c^2}\right)^{1/2} dt \quad -(11)$$

so

$$\Delta\phi = \Omega dt' - \omega dt$$
$$= 2\pi \left( \left(1 - \frac{v^2}{c^2}\right)^{-1/2} - 1 \right) \quad -(12)$$

in one orbit of $2\pi$ radians, Q.E.D.

This is any planar orbit of a mass m around a mass M, and any observable precession can be thought of as an ECE2 covariant Thomas precession. Each observable precession has its own v.

The Thomas precession is attributed to the ECE2 covariant force law:

$$\underline{F} = -\underline{\nabla}\phi + \underline{\omega}\phi_0 \quad -(13)$$

in a self consistent way, because the force law (13) and the line element (7) are both ECE2 covariant. They are defined in a space of finite torsion and curvature.

The Thomas precession is due to the vacuum force:

$$\underline{F}(vac) = \underline{\omega}\phi_0 \quad -(14)$$

where $\underline{\omega}$ is the spin connection vector and where $\phi$ is the gravitational potential:

$$\phi_0 = -\frac{mG}{r} \quad -(15)$$

As shown in UFT 403 the original method produces:

$$\Delta\phi = \frac{r^2}{2}\left(\frac{\omega}{r} - \frac{\partial\omega}{\partial r}\right) \quad -(16)$$

This is the rotation defined by:

$$\phi \sim \pi\left(1 + \frac{6mG}{c^2 a(1-\epsilon)}\right) \quad -(17)$$

$$= \pi + \Delta\phi$$

If there were no precession the angle $\phi$ between the two apsides of an ellipse is $\pi$. From eq. (17):

$$\Delta\phi := \Delta\phi = \frac{6mG\pi}{c^2 a(1-\epsilon)} = 2\pi\left(\frac{3mG}{ac^2(1-\epsilon)}\right) \quad -(18)$$

as in eq. (2), Q.E.D.

From Eqs. (16) and (18):

$$\Delta\phi = \frac{r^2}{2}\left(\frac{\omega}{r} - \frac{\partial\omega}{\partial r}\right) = \frac{6\pi mG}{c^2 a(1-\epsilon)} \quad -(19)$$

$$= 2\pi\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

In general:

$$\boxed{\frac{r^2}{2}\left(\frac{\omega}{r} - \frac{\partial\omega}{\partial r}\right) = 2\pi\left(1 - \frac{v^2}{c^2}\right)^{-1/2}} \quad -(20)$$

So the magnitude of the spin connection is defined by the velocity $v$ of the rotation of the ECE2 covariant metric.

Finally from UFT403:

$$\Delta\phi = \frac{4}{3}\frac{\langle \delta\underline{r} \cdot \delta\underline{r} \rangle}{r^2} - \frac{1}{3r}\frac{\partial}{\partial r}\langle \delta\underline{r} \cdot \delta\underline{r} \rangle \quad -(21)$$

where:

$$\frac{\partial}{\partial r}\langle \delta\underline{r} \cdot \delta\underline{r} \rangle \sim \frac{4}{a}\langle \delta\underline{r} \cdot \delta\underline{r} \rangle \quad -(22)$$

So

$$\boxed{\begin{aligned}\Delta\phi &= \frac{4}{3}\frac{\langle \delta\underline{r} \cdot \delta\underline{r} \rangle}{r}\left(\frac{1}{r} - \frac{1}{a}\right) \\ &= 2\pi\left(1 - \frac{v^2}{c^2}\right)^{-1/2}\end{aligned}} \quad -(23)$$

This is true in general for any precession.

For gravitational precession:

$$\Delta\phi = \frac{4}{3}\frac{\langle \delta\underline{r} \cdot \delta\underline{r} \rangle}{r}\left(\frac{1}{r} - \frac{1}{a}\right) = \frac{6\pi MG}{ac^2(1-\epsilon)} \quad -(24)$$

At the perihelion:

$$r = \frac{\alpha}{1+\epsilon} \quad -(25)$$

$$\sim \frac{a}{1+\epsilon}$$

x a near circular orbit. So:

$$\frac{4}{3}\frac{\langle \delta\underline{r} \cdot \delta\underline{r} \rangle}{a^2}\epsilon = \frac{6\pi MG}{ac^2(1-\epsilon)} \quad -(25)$$