05(1): Precessions of a Satellite such as Gravity Probe B

) Thomas Precession

Consider the ECE2 covariant metric in plane polar coordinates (r, ϕ):

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 \quad -(1)$$

Thomas precession is due to the frame rotation:

$$\phi' = \phi + \omega t \quad -(2)$$

as in UFT110, a heavily studied paper that appears on the first page of Google with keywords " Thomas precession derivation ". It follows from eq. (2) that:

$$d\phi' = d\phi + \omega\, dt \quad -(3)$$

and

$$d\phi'^2 = d\phi^2 + 2\omega\, d\phi\, dt + \omega^2 dt^2 \quad -(4)$$

so

$$ds'^2 = (c^2 - r^2\omega^2)\, dt^2 - 2\omega r^2 d\phi\, dt - dr^2 \quad -(5)$$

The orbit of Gravity Probe B is nearly circular, so:

$$v = \omega r \quad -(6)$$

where v is the orbital velocity, so:

$$ds'^2 = \left(1 - \frac{v^2}{c^2}\right)\left(c^2 dt^2 - 2r^2 \Omega\, d\phi\, dt\right) - dr^2 - r^2 d\phi^2 \quad -(7)$$

where

$$\Omega = \omega \left(1 - \frac{v^2}{c^2}\right)^{-1} \quad -(8)$$

is the relativistic angular velocity.

The infinitesimal of time changes to:

$$dt' = \left(1 - \frac{v^2}{c^2}\right)^{1/2} dt \quad -(9)$$

Therefore in a rotation of 2π, the Thomas precession is:

$$\Delta\phi = \Omega dt' - \omega t$$

$$= 2\pi \left(\left(1 - \frac{v^2}{c^2}\right)^{-1/2} - 1 \right) \quad -(10)$$

For the approximately circular orbit of GPB:

$$v^2 = \frac{MG}{r} \quad -(11)$$

where M is the mass of the Earth, and r is the distance of Gravity Probe B to the centre of the earth:

$$M = 5.98 \times 10^{24} \text{ kg} \quad -(12)$$

$$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \quad -(13)$$

$$r = 7.0274 \times 10^{6} \text{ m} \quad -(14)$$

So the Thomas precession of GPB is:

$$\Delta\phi = 2\pi \left(\left(1 - \frac{MG}{rc^2}\right)^{-1/2} - 1 \right) \quad -(15)$$

$$\sim \pi \frac{MG}{rc^2}$$

$$= 1.984 \times 10^{-9} \text{ radians per orbit}$$

The orbit of Gravity Probe B takes 90 minutes to complete.

Lense-Thirring Precession at the Equator.

At the equator, the magnitude of the gravitomagnetic field is:

$$\Omega = \frac{2}{5} \frac{MGR^2\omega_E}{c^2 r^3} \quad -(16)$$

where R is the radius of the Earth, r is given by eq. (14), and ω_E is the angular velocity of the Earth. As in UFT110:

$$\Omega = 1.52 \times 10^{-14} \text{ rad s}^{-1} \quad -(17)$$

The Lense Thirring precession per orbit is

$$\Delta \phi_{LT} = \frac{1}{2} \Omega t \quad -(18)$$

where

$$t = 90 \text{ minutes} = 5{,}400 \text{ seconds} \quad -(19)$$

so

$$\Delta \phi_{LT} = 4.10 \times 10^{-11} \text{ radians per orbit} \quad -(20)$$

The Lense Thirring precession is roughly two orders of magnitude smaller than the Thomas precession.

3) Einsteinian Orbital Precession

This is
$$\Delta \phi_E = \frac{3MG}{ac^2(1-\epsilon^2)} \quad -(21)$$

from the method of Marion and Thornton, and

$$\Delta \phi_E = \frac{3MG}{ac^2(1-\epsilon)} \quad -(22)$$

from the apsidal method for an orbit of 2π radians. The eccentricity of Gravity Probe B is:

$$\epsilon = 0.0014 \quad -(23)$$

and

$$a \sim r \quad -(24)$$

So

$$\Delta \phi_E = 1.894 \times 10^{-9} \text{ radians per orbit.}$$

4) Note that for a nearly circular orbit the Thomas precession is

$$\Delta\phi_T = \frac{\pi MG}{rc^2} \quad -(17)$$

and the Einsteinian precession is

$$\Delta\phi_E = \frac{3MG}{rc^2} \quad -(18)$$

and they are almost the same, because:

$$\pi = 3.1415927 \quad -(19)$$

It is well known that the Einstein theory is totally incorrect, as is the classic UFT88, so Thomas precession, a planetary precession is Thomas precession, a correct theory. Thomas precession is correctly ECE2 covariant.

The next note will evaluate the spin connections for these precessions, and consider geodetic precession using the method of UFT345.

Gravity Probe B can be severely criticised on the grounds that it claims to see a small precession, the Lense Thirring precession, but this is two orders of magnitude smaller than the Thomas precession. The standard model never considers the Thomas precession of planets.