399(1): Spin Connection Resonance due to Vacuum Fluctuations

Assume that:

$$\delta \underline{r} = \delta \underline{r}(0) \exp(-i \omega_0 t) \quad -(1)$$

where $\delta \underline{r}$ is the vacuum fluctuation, taking place with angular frequency ω and wave vector $\underline{\kappa}$. The vacuum force is then:

$$\underline{F}(r) = -m \omega_0^2 \delta \underline{r} = m \frac{d^2 \delta \underline{r}}{dt^2} \quad -(2)$$

$$= -e \underline{E}(vac)$$

where $\underline{E}(vac)$ is the fluctuating vacuum field.

Therefore:

$$\underline{E}(vac) = \frac{m}{e} \omega_0^2 \delta \underline{r}(0) e^{-i\omega_0 t} \quad -(3).$$

However, in ECE2:

$$\underline{E}(vac) = \underline{\omega} \phi_0 \quad -(4)$$

where $\underline{\omega}$ is the spin connection vector and ϕ_0 is the potential in the absence of the vacuum. So:

$$\underline{\omega} \phi_0 = \frac{m}{e} \omega_0^2 \delta \underline{r}(0) e^{-i\omega_0 t} \quad -(5)$$

and

$$\frac{\partial^2}{\partial t^2} (\underline{\omega} \phi_0) = \frac{m}{e} \omega_0^4 \delta \underline{r}(0) e^{-i\omega_0 t}$$

$$:= \underline{A} e^{-i\omega_0 t} \quad -(6)$$

where

$$\underline{A} = \frac{m}{e} \omega_0^4 \delta \underline{r}(0) = \text{constant} \quad -(7)$$

2) Therefore:

$$\underline{\omega}\frac{\partial^2\phi_0}{\partial t^2} + 2\frac{\partial\phi_0}{\partial t}\frac{\partial\underline{\omega}}{\partial t} + \left(\frac{\partial^2\underline{\omega}}{\partial t^2}\right)\phi_0 = \underline{A}e^{-i\omega_0 t} \quad -(8)$$

For ϕ_0 X component:

$$\frac{\partial^2\phi_0}{\partial t^2} + \frac{2}{\omega_x}\frac{\partial\omega_x}{\partial t}\frac{\partial\phi_0}{\partial t} + \frac{1}{\omega_x}\frac{\partial^2\omega_x}{\partial t^2}\phi_0 = \frac{A_x e^{-i\omega_0 t}}{\omega_x} \quad -(9)$$

and similarly for Y and Z.

For Euler Bernoulli resonance to occur eq. (9) has to be of the format:

$$\ddot{x} + 2\beta\dot{x} + \omega_1^2 x = A\cos\omega_0 t \quad -(10)$$

so

$$\frac{2}{\omega_x}\frac{\partial\omega_x}{\partial t} = 2\beta \quad -(11)$$

$$\omega_1^2 = \frac{1}{\omega_x}\frac{\partial^2\omega_x}{\partial t^2} \quad -(12)$$

Therefore

$$\frac{\partial^2\omega_x}{\partial t^2} = \omega_1^2\omega_x \quad -(13)$$

and

$$\frac{\partial\omega_x}{\partial t} = \beta\omega_x \quad -(14)$$

A solution of eq (13) is:

$$\omega_x = \omega(0)\exp\left(i\left(\omega_1 t - \underline{\kappa}\cdot\underline{r}\right)\right) \quad -(15)$$

Using this solution, β is imaginary, so β is zero in the physical interpretation, using the usual rule that physical part is the real part.

So eq. (9) becomes: — (16)

$$\frac{d^2\phi_0}{dt^2} + \omega_1^2 \phi_0 = A_{X}^{\omega(0)} \exp\left(-i\left(\omega_0 t + \omega_1 t - \underline{\kappa} \cdot \underline{r}\right)\right)$$

Taking the real part:

$$\frac{d^2\phi_0}{dt^2} + \omega_1^2 \phi_0 = A_{X}^{\omega(0)} \cos\left((\omega_0 + \omega_1)t - \underline{\kappa} \cdot \underline{r}\right) \quad -(17)$$

The usual Euler - Bernoulli structure is:

$$\frac{d^2\phi_0}{dt^2} + \omega_1^2 \phi_0 = A \cos \omega_0 t, \quad -(18)$$

and at

$$\omega_0 = \omega_1 \quad -(19)$$

the potential becomes infinite:

$$\phi_0 \rightarrow \infty \quad -(20)$$

It is clear that eq. (18) can be obtained from eq. (17) when:

$$\underline{\kappa} \cdot \underline{r} := \omega_2 t \quad -(21)$$

where ω_2 is an angular frequency. The resonance occurs when

$$\boxed{\omega_0 = \omega_2} \quad -(22)$$

and infinite energy is taken from the vacuum.