

Q4): Effect of Vacuum on the Dipole Magnetic flux Density

Consider the magnetic vector potential:

$$\underline{A}_0 = \frac{\mu_0}{4\pi} \frac{\underline{m} \times \underline{r}}{r^3} \quad (1)$$

In the usual magnetic flux density  $\underline{B}_0$  of the standard model

$$\underline{B}_0 = \nabla \times \underline{A} \quad (2)$$

In ECE2 theory:

$$\underline{B} = \nabla \times \underline{A} - \underline{\omega} \times \underline{A} \quad (3)$$

here  $\underline{\omega}$  is the vector spin connection. Here:

$$\underline{B}(\text{vac}) = -\underline{\omega} \times \underline{A} \quad (4)$$

i.e. the vacuum correction of  $\underline{B}$ .

So ECE2 theory automatically considers the vacuum correction:

$$\underline{B} = \underline{B}_0 + \underline{B}(\text{vac}) \quad (5)$$

From vector analysis:

$$\underline{B}_0 = -\frac{\mu_0}{4\pi} \underline{m} \nabla^3 \left( \frac{1}{r} \right) + \frac{\mu_0}{4\pi r^3} \left( 3\underline{m} \cdot \frac{\underline{r} \underline{r}}{r^3} - \underline{m} \right) \quad (6)$$

which

$$\underline{m} = m_x \underline{i} + m_y \underline{j} + m_z \underline{k} \quad (7)$$

The magnetic dipole moment and

$$\underline{r} = x \underline{i} + y \underline{j} + z \underline{k} \quad (8)$$

Therefore it follows that:

$$\text{If we use: } \nabla^2 \left( \frac{1}{r} \right) = -\delta - (9)$$

then:

$$\therefore \underline{\underline{\mu}}_0 = \frac{\mu_0}{4\pi} \left[ \frac{3(m_x \underline{x} + m_y \underline{y} + m_z \underline{z})}{(\underline{x}^2 + \underline{y}^2 + \underline{z}^2)^{5/2}} (\underline{x_i} + \underline{y_j} + \underline{z_k}) - \frac{(m_x \underline{i} + m_y \underline{j} + m_z \underline{k})}{(\underline{x}^2 + \underline{y}^2 + \underline{z}^2)^{3/2}} \right] - (10)$$

so the scalar components of  $\underline{\underline{B}}_0$  are: -(11)

$$B_{0x} = \left[ \frac{3(m_x \underline{x} + m_y \underline{y} + m_z \underline{z}) \underline{x}}{(\underline{x}^2 + \underline{y}^2 + \underline{z}^2)^{5/2}} - \frac{m_x}{(\underline{x}^2 + \underline{y}^2 + \underline{z}^2)^{3/2}} \right].$$

$$B_{0y} = \left[ \frac{3(m_x \underline{x} + m_y \underline{y} + m_z \underline{z}) \underline{y}}{(\underline{x}^2 + \underline{y}^2 + \underline{z}^2)^{5/2}} - \frac{m_y}{(\underline{x}^2 + \underline{y}^2 + \underline{z}^2)^{3/2}} \right] - (12)$$

$$B_{0z} = \left[ \frac{3(m_x \underline{x} + m_y \underline{y} + m_z \underline{z}) \underline{z}}{(\underline{x}^2 + \underline{y}^2 + \underline{z}^2)^{5/2}} - \frac{m_z}{(\underline{x}^2 + \underline{y}^2 + \underline{z}^2)^{3/2}} \right] - (13)$$

The isotropically averaged effect of the vacuum  
for example the  $x$  component is given by the  
perturbational Taylor series:

$$\langle \Delta B_{\text{Box}} \rangle = \left\langle \frac{1}{2!} \left( \delta_x \frac{\partial}{\partial x} + \delta_y \frac{\partial}{\partial y} + \delta_z \frac{\partial}{\partial z} \right) \left( \delta_x \frac{\partial B_{\text{Box}}}{\partial x} + \delta_y \frac{\partial B_{\text{Box}}}{\partial y} + \delta_z \frac{\partial B_{\text{Box}}}{\partial z} \right) \right. \\ + \left. \delta_z \frac{\partial B_{\text{Box}}}{\partial z} \right) + \frac{1}{4!} \left( \delta_x \frac{\partial}{\partial x} + \delta_y \frac{\partial}{\partial y} + \delta_z \frac{\partial}{\partial z} \right)^2 \left[ \left( \delta_x \frac{\partial}{\partial x} + \delta_y \frac{\partial}{\partial y} + \delta_z \frac{\partial}{\partial z} \right) \right. \\ \left. \left. \left[ \left( \delta_x \frac{\partial}{\partial x} + \delta_y \frac{\partial}{\partial y} + \delta_z \frac{\partial}{\partial z} \right) \left( \delta_x \frac{\partial B_{\text{Box}}}{\partial x} + \delta_y \frac{\partial B_{\text{Box}}}{\partial y} + \delta_z \frac{\partial B_{\text{Box}}}{\partial z} \right) \right] \right] \right\rangle \\ - (14)$$

In three dimensions:

$$\langle \underline{\Delta B}_0 \rangle = \langle \Delta B_{\text{Box}} \rangle_i + \langle \Delta B_{\text{Ort}} \rangle_j + \langle \Delta B_{\text{Orz}} \rangle_k - (15)$$

However, from eq. (5):

$$\langle \underline{\Delta B}_0 \rangle = \underline{B} - \underline{B}_0 = \langle \underline{B}(\text{vac}) \rangle - (16)$$

$$= - \underline{\omega} \times \underline{A}_0$$

So the isotropically averaged vacuum magnetic flux density is:

$$\langle \underline{B}(\text{vac}) \rangle = \langle \Delta B_{\text{Box}} \rangle_i + \langle \Delta B_{\text{Ort}} \rangle_j + \langle \Delta B_{\text{Orz}} \rangle_k$$

$$= - \underline{\omega} \times \underline{A}_0 - (17)$$

So the spin comedia vector  $\underline{\omega}$  can be found.