96(3): Effect of the Vacuum on the Inverse Square Laws of Physics

Consider the Hooke / Newton inverse square law:

$$\underline{F} = -mMG\frac{\underline{r}}{r^3} \quad -(1)$$

where $\underline{F}$ is the vector force between m and M, G is Newton's constant and $\underline{r}$ is the vector joining m and M. The Cartesian scalar components of $\underline{F}$ are:

$$F_X = -mMG\frac{X}{(X^2+Y^2+Z^2)^{3/2}} \quad -(2)$$

$$F_Y = -mMG\frac{Y}{(X^2+Y^2+Z^2)^{3/2}} \quad -(3)$$

$$F_Z = -mMG\frac{Z}{(X^2+Y^2+Z^2)^{3/2}} \quad -(4)$$

The isotropically averaged change in these components due to vacuum fluctuations $\delta\underline{r}$ are:

$$\langle \Delta F_X \rangle = \Big\langle \frac{1}{2!}\left(\delta X\frac{\partial}{\partial X} + \delta Y\frac{\partial}{\partial Y} + \delta Z\frac{\partial}{\partial Z}\right)\left(\delta X\frac{\partial F_X}{\partial X} + \delta Y\frac{\partial F_X}{\partial Y} + \delta Z\frac{\partial F_X}{\partial Z}\right) + \frac{1}{4!}\left(\delta X\frac{\partial}{\partial X} + \delta Y\frac{\partial}{\partial Y} + \delta Z\frac{\partial}{\partial Z}\right)\left[\left(\delta X\frac{\partial}{\partial X} + \delta Y\frac{\partial}{\partial Y} + \delta Z\frac{\partial}{\partial Z}\right)\left[\left(\delta X\frac{\partial}{\partial X} + \delta Y\frac{\partial}{\partial Y} + \delta Z\frac{\partial}{\partial Z}\right)\left(\delta X\frac{\partial F_X}{\partial X} + \delta Y\frac{\partial F_X}{\partial Y} + \delta Z\frac{\partial F_X}{\partial Z}\right)\right]\right]\Big\rangle \quad -(5)$$

$$+ \cdots$$

and similar expressions for $\langle \Delta F_Y \rangle$ and $\langle \Delta F_Z \rangle$

2) Here $\langle \ \rangle$ denotes isotropic averaging of the vacuum fluctuations.

The change in $\underline{F}$ due to the vacuum is therefore:

$$\Delta \underline{F} = \underline{F}(\underline{r} + \delta\underline{r}) - \underline{F}(\underline{r})$$
$$= \Delta F_x \underline{i} + \Delta F_y \underline{j} + \Delta F_z \underline{k} \quad -(6)$$

So:

$$\boxed{\underline{F}(\underline{r} + \delta\underline{r}) = -mMG\frac{\underline{r}}{r^3} + \Delta\underline{F}} \quad -(7)$$

is the force between m and M in the presence of the vacuum. The vacuum corrects the inverse square law. So the elliptical orbit will be changed by the vacuum. Experimentally, it is known that the orbit of m around M is a precessing ellipse, so it is concluded that the precession is caused by $\Delta\underline{F}$.

To second order:

$$\boxed{\Delta\underline{F} = \frac{1}{2!}\langle \delta\underline{r} \cdot \delta\underline{r} \rangle \left(\nabla^2 F_x \underline{i} + \nabla^2 F_y \underline{j} + \nabla^2 F_z \underline{k} \right)} \quad -(8)$$

So to second order:

$$\underline{F}(\underline{r} + \delta\underline{r}) = -mMG\frac{\underline{r}}{r^3} + \frac{1}{2!}\langle \delta\underline{r} \cdot \delta\underline{r} \rangle \left(\nabla^2 F_x \underline{i} + \nabla^2 F_y \underline{j} + \nabla^2 F_z \underline{k} \right) \quad -(9)$$

) The force law (9) must be compared with the experimental data on precession. To second order the precession is determined by $\langle \delta \underline{r} \cdot \delta \underline{r} \rangle$ of the vacuum. If the complete Taylor series (5) is used, the precession is determined by higher order isotropic averages.

From Note 377(4) the lagrangian:

$$\mathcal{L} = -mc^2\left(1 - \frac{\dot{\underline{r}} \cdot \dot{\underline{r}}}{c^2}\right)^{1/2} + \frac{mMG}{|\underline{r}|} \quad - (10)$$

when used with the Euler Lagrange equation:

$$\frac{\partial \mathcal{L}}{\partial \underline{r}} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\underline{r}}} \quad - (11)$$

gives the orbital equation:

$$\underline{F} = m\ddot{\underline{r}} = -mMG\frac{\underline{r}}{r^3}\left(1 - \frac{v_0^2}{c^2}\right)^{3/2} \quad - (12)$$

Here:

$$\underline{p} = \frac{\partial \mathcal{L}}{\partial \dot{\underline{r}}} = \gamma m \underline{v}_0 \quad - (13)$$

is the relativistic momentum, in which $\underline{v}_0$ is the Newtonian velocity. In UFT 377 it was shown that eq. (12) gives a precessing orbit.

If

$$v_0 << c \quad - (14)$$

as in planetary orbits, then:

$$\underline{F} \sim -mMG\frac{\underline{r}}{r^3}\left(1 - \frac{3}{2}\frac{v_0^2}{c^2} + \ldots\right) \quad - (15)$$

:) Comparing eqs. (7) and (12)

$$\Delta \underline{F} = \frac{3}{2}\frac{v_0^2}{c^2} mMG \frac{\underline{r}}{r^3} \quad -(16)$$

Therefore the vacuum correction in eq. (8) gives orbital precession if:

$$\frac{1}{2!}\langle \delta\underline{r}\cdot\delta\underline{r}\rangle \left(\nabla^2 F_x \underline{i} + \nabla^2 F_y \underline{j} + \nabla^2 F_z \underline{k}\right) = \frac{3}{2}\frac{v_0^2}{c^2} mMG \frac{\underline{r}}{r^3} \quad -(17)$$

Comparing term by term:

$$\langle \delta\underline{r}\cdot\delta\underline{r}\rangle \nabla^2 F_x = 3\frac{v_0^2}{c^2} mMG \frac{X}{r^3} \quad -(18)$$

$$\langle \delta\underline{r}\cdot\delta\underline{r}\rangle \nabla^2 F_y = 3\frac{v_0^2}{c^2} mMG \frac{Y}{r^3} \quad -(19)$$

$$\langle \delta\underline{r}\cdot\delta\underline{r}\rangle \nabla^2 F_z = 3\frac{v_0^2}{c^2} mMG \frac{Z}{r^3} \quad -(20)$$

where

$$r^3 = (X^2+Y^2+Z^2)^{3/2} \quad -(21)$$

Therefore the vacuum average $\langle \delta\underline{r}\cdot\delta\underline{r}\rangle$ needed for orbital precession may be found.
