395(6): Corrections to the Dipole Magnetic Potential

The dipole magnetic potential is

$$\underline{A} = \frac{\mu_0}{4\pi} \frac{\underline{m} \times \underline{r}}{|\underline{r}|^3} \quad -(1)$$

and its Cartesian components are:

$$A_X = \frac{\mu_0}{4\pi} \frac{(m_Y Z - m_Z Y)}{(X^2+Y^2+Z^2)^{3/2}} \quad -(2)$$

$$A_Y = \frac{\mu_0}{4\pi} \frac{(m_Z X - m_X Z)}{(X^2+Y^2+Z^2)^{3/2}} \quad -(3)$$

$$A_Z = \frac{\mu_0}{4\pi} \frac{(m_X Y - m_Y X)}{(X^2+Y^2+Z^2)^{3/2}} \quad -(4)$$

Therefore:

$$\langle \Delta A_X \rangle = \frac{1}{3} \Big(\frac{1}{2!} \langle \delta\underline{r} \cdot \delta\underline{r} \rangle \nabla^2 A_X + \frac{1}{4!} \langle \delta\underline{r} \cdot \delta\underline{r}\, \delta\underline{r} \cdot \delta\underline{r} \rangle \nabla^4 A_X + \frac{1}{6!} \langle \delta\underline{r} \cdot \delta\underline{r}\, \delta\underline{r} \cdot \delta\underline{r}\, \delta\underline{r} \cdot \delta\underline{r} \rangle \nabla^6 A_X + \cdots \Big) \quad -(5)$$

Note that there is a typographical error in Eq. (5) of Note 395(5), there is a factor 1/3 missing on the RHS, and this has been reinstated in Eq. (5) of this note.

Computer algebra can be used to evaluate Eq. (5), using:

$$\nabla^2 A_x = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) A_x \quad -(6)$$

where A_x is given by Eq. (2). Similarly:

$$\nabla^4 A_x = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\left(\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) A_x\right) \quad -(7)$$

and so on.

The correction to the complete vector potential is:

$$\langle \Delta \underline{A} \rangle = \langle \Delta A_x \rangle \underline{i} + \langle \Delta A_y \rangle \underline{j} + \langle \Delta A_z \rangle \underline{k} \quad -(8)$$

By definition:

$$\langle \Delta A \rangle = \underline{A} - \underline{A}_0 \quad -(9)$$

where $\underline{A}_0$ is the potential in the hypothetical absence of the vacuum.