388(3): Development of the Lindstrom Constraint

The Lindstrom constraint is:

$$\phi^2 = c^2 A^2 \quad -(1)$$

and has been deduced by Lindstrom from the totally antisymmetric torsion tensor of UFT 354. The four potential is:

$$A^\mu = \left(\frac{\phi}{c}, \underline{A} \right) \quad -(2)$$

Eq. (1) is a generalization of the Lorenz condition:

$$\underline{\nabla} \cdot \underline{A} + \frac{1}{c^2} \frac{\partial \phi}{\partial t}, \quad -(3)$$

a generalization needed for the law of conservation of antisymmetry. The latter demands that the solution of eq. (3) be eq. (1).

The continuity equation is:

$$\frac{\partial \rho}{\partial t} + \underline{\nabla} \cdot \underline{J} = 0 \quad -(4)$$

i.e.

$$\partial_\mu J^\mu = 0 \quad -(5)$$

where

$$J^\mu = (c\rho, \underline{J}) \quad -(6)$$

and

$$\partial_\mu = \left(\frac{1}{c} \frac{\partial}{\partial t}, \underline{\nabla} \right) \quad -(7)$$

Here ρ is the charge density and $\underline{J}$ is the current density. The use of the Lorenz gauge (3) in the field equations leads to:

$$\Box A^\mu = \mu_0 J^\mu \quad -(8)$$

d

$$\Box \phi = \rho / \epsilon_0 \quad -(9)$$

$$\Box \underline{A} = \mu_0 \underline{J} \quad -(10)$$

The Lorenz gauge follows from the continuity equation in eq. (4): From eqs. (9) and (10)

$$\frac{1}{\mu_0} \underline{\nabla} \cdot \Box \underline{A} + \epsilon_0 \frac{\partial}{\partial t} \Box \rho = 0 \quad -(11)$$

which:

$$\underline{\nabla} \cdot \Box \underline{A} = \Box (\underline{\nabla} \cdot \underline{A}) \quad -(12)$$

$$\frac{\partial}{\partial t} \Box \rho = \Box \frac{\partial \rho}{\partial t} \quad -(13)$$

Eq. (11) is therefore:

$$\Box \left(\frac{1}{\mu_0} \underline{\nabla} \cdot \underline{A} + \epsilon_0 \frac{\partial \phi}{\partial t} \right) = 0 \quad -(14)$$

Using

$$\epsilon_0 \mu_0 = \frac{1}{c^2} \quad -(15)$$

it follows that

$$\Box \left(\underline{\nabla} \cdot \underline{A} + \frac{1}{c^2} \frac{\partial \phi}{\partial t} \right) = 0 \quad -(16)$$

so the Lorenz gauge is a possible solution of eq. (16):

$$\underline{\nabla} \cdot \underline{A} + \frac{1}{c^2} \frac{\partial \phi}{\partial t} = 0 \quad -(17)$$

Q.E.D.

The complete set of wrisymmetry laws is therefore:

3)

$$\phi^2 = c^2 A^2 \quad -(18)$$

$$\underline{E} = -\underline{\nabla}\phi + \underline{\omega}\phi = -\frac{\partial \underline{A}}{\partial t} - \omega_0 \underline{A} \quad -(19)$$

$$\frac{\partial A_z}{\partial y} + \frac{\partial A_y}{\partial z} = \omega_y A_z + \omega_z A_y \quad -(20)$$

$$\frac{\partial A_x}{\partial z} + \frac{\partial A_z}{\partial x} = \omega_z A_x + \omega_x A_z \quad -(21)$$

$$\frac{\partial A_y}{\partial x} + \frac{\partial A_x}{\partial y} = \omega_x A_y + \omega_y A_x \quad -(22)$$

Procedure for Computation

1) Assume:

$$\underline{J}(\text{observed}) = \underline{J} + \underline{J}_1 \sim \underline{J} \quad -(23)$$

as in the previous note.

2) Compute $\underline{A}$ from:

$$\Box \underline{A} = \mu_0 \underline{J} \quad -(24)$$

3) Compute $\underline{\omega}$ from eqs. (20) to (22).

4) Compute ω_0 from:

$$\underline{E} = c\left(\mp \underline{\nabla} A \pm \underline{\omega} A\right) \quad -(25)$$

$$= -\frac{\partial \underline{A}}{\partial t} - \omega_0 \underline{A}$$

where

$$A = \left(A_x^2 + A_y^2 + A_z^2\right)^{1/2} \quad -(26)$$

5) Compute ϕ from:

$$\phi^2 = c^2 A^2 \quad -(27)$$

) Compute the charge density from

$$\rho = \epsilon_0 \Box \phi \quad -(28)$$

) Compute the interaction electric field strength from:

$$\underline{E_1} = \underline{\omega} \phi = \pm \underline{\omega} c A \quad -(29)$$

) Compute the interaction flux density from:

$$\underline{B_1} = -\underline{\omega} \times \underline{A} \quad -(30)$$

Interpretations

1) If eq. (1) is interpreted as a solution of eq. (3) then

$$\underline{\nabla} \cdot \underline{A} \pm \frac{1}{c} \frac{\partial A}{\partial t} = 0 \quad -(31)$$

and

$$A^\mu = \left(\pm A, \underline{A} \right) \quad -(32)$$

Eq. (32) shows that the radiation gauge:

$$\phi = 0, \quad \underline{\nabla} \cdot \underline{A} = 0 \quad -(33)$$

violates conservation of antisymmetry. If a current density $\underline{J}$ is measured in a circuit, then

$$\Box \underline{A} = \mu_0 \underline{J}(\text{exp}) \quad -(34)$$

$$\Box A = \pm \frac{\rho(\text{exp})}{c \epsilon_0} \quad -(35)$$

So there is a relation between $\rho(\text{exp})$ and $\underline{J}(\text{exp})$.

2) If eq. (1) is interpreted as entirely replacing the Lorenz gauge then the field equations develop in a different way:

$$\underline{E} = -\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t} \quad -(36)$$

$$= \mp c\,\underline{\nabla}A - \frac{\partial \underline{A}}{\partial t}$$

and

$$\underline{B} = \underline{\nabla} \times \underline{A} \quad -(37)$$

with

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(38)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(39)$$

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(40)$$

$$\underline{\nabla} \times \underline{B} + \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} \quad -(41)$$

This set of equations leads back to

$$\Box A^\mu = \mu_0 J^\mu \quad -(42)$$

$$\Box \phi = \rho / \epsilon_0 \quad -(43)$$

$$\Box \underline{A} = \mu_0 \underline{J} \quad -(44)$$

if it is assumed that:

$$\underline{\nabla} \cdot \underline{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0 \quad -(45)$$

with

$$\phi^2 = c^2 A^2 \quad -(46)$$

If assumption (45) is not made, a different set of equations emerges for $\underline{A}$ and ϕ. This will be shown in the next note.