

1) Plane Wave Solution for the S_{pi} Connection

In the case of electromagnetic:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad - (1)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad - (2)$$

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \quad - (3)$$

$$\underline{E} = - \frac{\partial \underline{A}}{\partial t} - \omega_0 \underline{A} \quad - (4)$$

and by antisymmetry:

$$\left(\frac{\partial}{\partial t} - \omega_y \right) A_z = - \left(\frac{\partial}{\partial z} - \omega_z \right) A_y \quad - (5)$$

$$\left(\frac{\partial}{\partial z} - \omega_z \right) A_x = - \left(\frac{\partial}{\partial x} - \omega_x \right) A_z \quad - (6)$$

$$\left(\frac{\partial}{\partial x} - \omega_x \right) A_y = - \left(\frac{\partial}{\partial y} - \omega_y \right) A_x \quad - (7)$$

From eqs. (2) to (4):

$$\frac{\partial}{\partial t} (\underline{\omega} \times \underline{A}) + \underline{\nabla} \times (\omega_0 \underline{A}) = \underline{0} \quad - (8)$$

Consider the plane wave:

$$\underline{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} - i \underline{j}) \exp(-i(\omega t - k z)) \quad - (9)$$

The antisymmetry equations (5) to (7) reduce to a simpler set of equations:

$$\frac{\partial A_x^{(1)}}{\partial z} = \omega_z A_x^{(1)} \quad - (10)$$

$$\frac{\partial A_y^{(1)}}{\partial z} = \omega_z A_y^{(1)} \quad - (11)$$

$$-\omega_x A_y^{(1)} = \omega_y A_x^{(1)} \quad - (12)$$

where

$$A_x^{(1)} = \frac{A^{(0)}}{\sqrt{2}} \exp(-i(\omega t - \kappa z)) \quad - (13)$$

$$A_y^{(1)} = -i \frac{A^{(0)}}{\sqrt{2}} \exp(-i(\omega t - \kappa z)) \quad - (14)$$

$$A_z^{(1)} = 0 \quad - (15)$$

From eqs. (10) and (13):

$$\omega_z = i\kappa \quad - (16)$$

From eqs. (11) and (14):

$$\omega_z = \kappa \quad - (17)$$

So

$$\omega_z = 0 \quad - (18)$$

is the only possibility.

From eqs. (12) to (14):

$$i\omega_x = \omega_y \quad - (19)$$

Eqs. (18) and (19) are satisfied by:

$$\underline{\omega}^{(2)} = \frac{\omega^{(0)}}{\sqrt{2}} \begin{pmatrix} i \\ 1 \\ 0 \end{pmatrix} \exp(i(\omega t - \kappa z)) \quad - (20)$$

1) Derive the gauge part of the spin correction for vector a plane wave.

In order to find ω_0 , it is necessary to solve:

$$\frac{d}{dt} (\underline{\omega}^{(2)} \times \underline{A}^{(1)}) + \underline{\nabla} \times (\omega_0 \underline{A}^{(1)}) = \underline{0} \quad - (21)$$

Here:

$$\underline{\omega}^{(2)} \times \underline{A}^{(1)} = \frac{\omega^{(0)} A^{(0)}}{2} \begin{vmatrix} i & j & k \\ 1 & i & 0 \\ 1 & -i & 0 \end{vmatrix} \quad - (22)$$

$$= -i \omega^{(0)} A^{(0)} \underline{k}$$

So:

$$\frac{d}{dt} (\underline{\omega}^{(2)} \times \underline{A}^{(1)}) = \underline{0} \quad - (23)$$

and

$$\underline{\nabla} \times (\omega_0 \underline{A}^{(1)}) = \underline{0} \quad - (24)$$

This means that:

$$\omega_0 \underline{\nabla} \times \underline{A}^{(1)} + (\underline{\nabla} \omega_0) \times \underline{A}^{(1)} = \underline{0} \quad - (25)$$

in which

$$\underline{\nabla} \times \underline{A}^{(1)} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x^{(1)} & A_y^{(1)} & 0 \end{vmatrix} \quad - (26)$$

So.

$$\omega_0 \nabla \times \underline{A}^{(1)} = -\omega_0 \frac{\partial A_y^{(1)}}{\partial z} \underline{i} + \omega_0 \frac{\partial A_x^{(1)}}{\partial z} \underline{j} \quad (27)$$

Assume that $\omega_0 = \omega^{(1)} \exp(-i(\omega t - kz)) \quad (28)$

then $\nabla \omega_0 = \frac{\partial \omega_0}{\partial z} \underline{k} \quad (29)$

and $\nabla \omega_0 \times \underline{A}^{(1)} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0 & \frac{\partial \omega_0}{\partial z} \\ A_x^{(1)} & A_y^{(1)} & 0 \end{vmatrix} \quad (29)$

$$= -\frac{\partial \omega_0}{\partial z} A_y^{(1)} \underline{i} - \frac{\partial \omega_0}{\partial z} A_x^{(1)} \underline{j} \quad (30)$$

From eqs (27) and (30):

$$\omega_0 \frac{\partial A_y^{(1)}}{\partial z} = \frac{\partial \omega_0}{\partial z} A_y^{(1)} \quad (31)$$

$$\omega_0 \frac{\partial A_x^{(1)}}{\partial z} = -\frac{\partial \omega_0}{\partial z} A_x^{(1)} \quad (32)$$

W. L. R. $\frac{\partial A_x^{(1)}}{\partial z} = i\kappa A_x^{(1)}; \frac{\partial A_y^{(1)}}{\partial z} = i\kappa A_y^{(1)} \quad (33)$

so $\frac{\partial \omega_0}{\partial z} = i\kappa \quad (34)$

$$\frac{\partial \omega_0}{\partial z} = -i\kappa \quad (35)$$

and

i.e $\omega_0 = \omega^{(1)} = 0 \quad (36)$

5) Summary

If:
$$\underline{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} - i\underline{j}) \exp(-i(\omega t - kz)) - (37)$$

then a possible solution of eqs. (1) to (4) is:

$$\underline{\omega}^{(2)} = \frac{\omega^{(0)}}{\sqrt{2}} (\underline{i} + i\underline{j}) \exp(i(\omega t - kz)) - (38)$$

so

$$i\omega_x = \omega_y - (39)$$

$$\omega_z = 0 - (40)$$

so the space part of the spin curvatures is a plane wave.
The time part of the spin curvatures is:

$$\omega_0 = 0 - (41)$$

The electric field strength $\underline{E}^{(1)}$ and the magnetic flux density $\underline{B}^{(1)}$ are also plane waves.

The spin curvatures plane wave is a wave of matter as spacetime. Eqs. (1) to (4) are the homogeneous field equations of the spacetime as matter. The plane wave (38) is, the same for gravitation and electromagnetic.