378(2): Development of the Field Equations in Cartesian Components.

Consider the field equations in the format of eq. (9) of Note 378(1):

$$\underline{g} = \frac{\underline{\kappa}}{\kappa^2}\, \underline{\nabla} \cdot \underline{g} \quad -(1)$$

where

$$\kappa^2 = \kappa_x^2 + \kappa_y^2, \quad -(2)$$

$$\underline{g} = \ddot{X}\underline{i} + \ddot{Y}\underline{j} \quad -(3)$$

$$\underline{\kappa} = \kappa_x \underline{i} + \kappa_y \underline{j} \quad -(4)$$

and

$$\underline{\nabla} \cdot \underline{g} = \underline{\kappa} \cdot \underline{g} = \kappa_x \ddot{X} + \kappa_y \ddot{Y} \quad -(5)$$

It follows that:

$$\ddot{X} = \left(\kappa_x \ddot{X} + \kappa_y \ddot{Y}\right) \frac{\kappa_x}{\kappa^2} \quad -(6)$$

$$\ddot{Y} = \left(\kappa_x \ddot{X} + \kappa_y \ddot{Y}\right) \frac{\kappa_y}{\kappa^2}. \quad -(7)$$

Therefore:

$$\ddot{X}\left(1 - \frac{\kappa_x^2}{\kappa^2}\right) = \frac{\kappa_x \kappa_y}{\kappa^2} \ddot{Y} \quad -(8)$$

$$\ddot{Y}\left(1 - \frac{\kappa_y^2}{\kappa^2}\right) = \frac{\kappa_x \kappa_y}{\kappa^2} \ddot{X} \quad -(9)$$

i.e.

$$\ddot{X} \kappa_y^2 = \kappa_x \kappa_y \ddot{Y} \quad -(10)$$

$$\ddot{Y} \kappa_x^2 = \kappa_x \kappa_y \ddot{X} \quad -(11)$$

Therefore from both eqs. (10) and (11):

$$\boxed{\kappa_Y \ddot{X} = \kappa_X \ddot{Y}} \quad -(12)$$

This is the result of the field equations:

$$\underline{\nabla} \cdot \underline{g} = \underline{\kappa} \cdot \underline{g} = 4\pi G \rho_m \quad -(13)$$

and

$$\underline{\nabla} \times \underline{\kappa} = \underline{\kappa} \times \underline{g} = \underline{0} \quad -(14)$$

Eq. (12) is a pure field equation. It is true for all force equations, and shows that the components κ_X and κ_Y define the relation between $\ddot{X}$ and $\ddot{Y}$.

Force Equation Examples.

1) Forward Precession from ECE2 Relativity

$$\ddot{X} = \frac{mG}{\gamma (X^2+Y^2)^{3/2}} \left(\frac{\dot{X}\dot{Y}Y + X\dot{X}^2}{c^2} - X \right) \quad -(15)$$

$$\ddot{Y} = \frac{mG}{\gamma (X^2+Y^2)^{3/2}} \left(\frac{\dot{Y}\dot{X}X + Y\dot{Y}^2}{c^2} - Y \right) \quad -(16)$$

2) Retrograde Precession from ECE2 Relativity

$$\ddot{X} = - \frac{mGX}{\gamma^3 (X^2+Y^2)^{3/2}} \quad -(17)$$

$$\ddot{Y} = - \frac{mGY}{\gamma^3 (X^2+Y^2)^{3/2}} \quad -(18)$$

3) where

$$\gamma = \left(1 - \frac{\dot{x}^2 + \dot{y}^2}{c^2}\right)^{-1/2} \quad -(19)$$

3) <u>Newtonian Orbit</u>

$$\ddot{x} = -\frac{mGX}{(x^2+y^2)^{3/2}} \quad -(20)$$

$$\ddot{y} = -\frac{mGY}{(x^2+y^2)^{3/2}} \quad -(21)$$

It follows from eqs. (20), (21) and (12) that

$$\kappa_y X = \kappa_x Y \quad -(22)$$

It follows from eqs. (17), (18) and (12) that:

$$\kappa_y X = \kappa_x Y \quad -(23)$$

It follows from eqs. (15), (16) and (12) that:

$$\left(\frac{\dot{x}\dot{y}Y + X\dot{x}^2}{c^2} - X\right)\kappa_y = \left(\frac{\dot{y}\dot{x}X + Y\dot{y}^2}{c^2} - Y\right)\kappa_x \quad -(24)$$

Eqs. (22) and (23) are consistent with the results of previous notes:

$$X = -\frac{\kappa_x}{\kappa^2} \; ; \; Y = -\frac{\kappa_y}{\kappa^2} \quad -(25)$$

$$\kappa_x = -\frac{X}{X^2+Y^2} \; ; \; \kappa_y = -\frac{Y}{X^2+Y^2} \quad -(26)$$

4) for retrograde precession and the Newtonian orbit.

Role of the Spin Connection of ECE2 Relativity

Newtonian

The force equations are:

$$\ddot{y} = -mG \frac{\kappa_y}{\kappa_x} \frac{x}{(x^2+y^2)^{3/2}} \quad -(27)$$

$$\ddot{x} = -mG \frac{\kappa_x}{\kappa_y} \frac{y}{(x^2+y^2)^{3/2}} \quad -(28)$$

The orbit is found by solving these equations simultaneously with κ_x and κ_y as input parameters.

Retrograde Precession

The force equations are:

$$\ddot{y} = -\frac{mG}{\gamma^3} \frac{\kappa_y}{\kappa_x} \frac{x}{(x^2+y^2)^{3/2}} \quad -(29)$$

$$\ddot{x} = -\frac{mG}{\gamma^3} \frac{\kappa_x}{\kappa_y} \frac{y}{(x^2+y^2)^{3/2}} \quad -(30)$$

The orbit is given by solving these simultaneously with κ_x and κ_y as input parameters. $\quad -(31)$

Forward Precession

The force equations are:

$$\ddot{x} = \frac{mG}{\gamma(x^2+y^2)^{3/2}} \frac{\kappa_x}{\kappa_y} \left(\frac{\dot{y}\dot{x}x + y\dot{y}^2}{c^2} - y \right)$$

$$\ddot{y} = \frac{mG}{\gamma(x^2+y^2)^{3/2}} \frac{\kappa_y}{\kappa_x} \left(\frac{\dot{x}\dot{y}y + x\dot{x}^2}{c^2} - x \right) \quad -(32)$$

). The structure of the $\underline{\kappa}$ vector is:

$$\underline{\kappa} = 2\left(\frac{\underline{q}}{r^{(0)}} - \underline{\omega}\right) \quad -(33)$$

from UFT318. So:

$$\kappa_x = 2\left(\frac{q_x}{r^{(0)}} - \omega_x\right) \quad -(34)$$

$$\kappa_y = 2\left(\frac{q_y}{r^{(0)}} - \omega_y\right) \quad -(35)$$

Here $\underline{q}$ is the tetrad vector, $r^{(0)}$ has the units of metres, and $\underline{\omega}$ is the spin connection vector.

Conclusion

The observed orbit can be described by "orbital engineering" with parameter $\underline{\kappa}$. There are three parameters: $r^{(0)}$, $\underline{q}$ and $\underline{\omega}$.