372(7): Non-relativistic Limit of Note 372(6) — (1)

The lagrangian is:

$$\mathcal{L} = \frac{1}{2} m \left(\dot{r}^2 + r^2 \left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta \right) \right) + \frac{e^2}{4\pi\epsilon_0 r}.$$

The three Euler Lagrange equations are the same as those in Note 372(6). The quantization equations are:

$$i\hbar \frac{\partial \psi}{\partial t} = E\psi \quad -(2)$$

$$-i\hbar \frac{\partial \psi}{\partial r} = m\dot{r}\psi \quad -(3)$$

$$-\frac{i\hbar}{r} \frac{\partial \psi}{\partial \theta} = mr\dot{\theta}\psi \quad -(4)$$

$$-\frac{i\hbar}{r\sin\theta} \frac{\partial \psi}{\partial \phi} = mr\dot{\phi}\sin\theta\,\psi \quad -(5)$$

Now assume: $\psi = \psi(r)\psi(\theta,\phi)$ — (6)

and eqs. (3) to (5) reduce to:

$$-i\hbar \frac{d\psi(r)}{dr} = m\dot{r}\psi(r) \quad -(7)$$

$$-i\hbar \frac{d\psi(\theta,\phi)}{d\theta} = mr^2\dot{\theta}\psi(\theta,\phi) \quad -(8)$$

$$-i\hbar \frac{d\psi(\theta,\phi)}{d\phi} = mr^2\dot{\phi}\sin^2\theta\,\psi(\theta,\phi) \quad -(9)$$