41(5): Force Based Evaluation of the Gyroscope

In classical dynamics the force based calculation is:

$$\underline{F} = m\left(\frac{d\underline{v}}{dt} + \underline{\omega} \times \underline{v}\right)_m = -mg\,\underline{k} \quad -(1)$$

where the subscript m denotes moving frame, the frame defined by the principal moments of inertia of the gyroscope. Here

$$\underline{v} = \left(\frac{d\underline{r}}{dt} + \underline{\omega} \times \underline{r}\right)_m \quad -(2)$$

The force in the lab frame is:

$$\underline{F} = -mg\,\underline{k} \quad -(3)$$

where m is the mass of the gyroscope and g the acceleration due to gravity.

The velocity in the moving frame is:

$$\underline{v} = v_1\underline{e}_1 + v_2\underline{e}_2 + v_3\underline{e}_3 \quad -(4)$$

and

$$\underline{\omega} \times \underline{v} = \begin{vmatrix} \underline{e}_1 & \underline{e}_2 & \underline{e}_3 \\ \omega_1 & \omega_2 & \omega_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \quad -(5)$$

So the force components in the moving frame are:

$$F_1 = \frac{dv_1}{dt} + (\omega_2 v_3 - \omega_3 v_2) \quad -(6)$$

$$F_2 = \frac{dv_2}{dt} + (\omega_3 v_1 - \omega_1 v_3) \quad -(7)$$

$$F_3 = \frac{dv_3}{dt} + (\omega_1 v_2 - \omega_2 v_1) \quad -(8)$$

and $$F^2 = F_1^2 + F_2^2 + F_3^2 = F_X^2 + F_Y^2 + F_Z^2 \quad -(9)$$

This is the result of transformation of frames. The force magnitude squared in the lab and moving frames are always equal.

In the special case of eq. (3),

$$m^2 g_Z^2 = \left(F_1^2 + F_2^2 + F_3^2\right) \quad -(10)$$

In terms of Euler angles:

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} \cos\psi\cos\phi - \cos\theta\sin\phi\sin\psi & \cos\psi\sin\phi + \cos\theta\cos\phi\sin\psi & \sin\psi\sin\theta \\ -\sin\psi\cos\phi - \cos\theta\sin\phi\cos\psi & -\sin\psi\sin\phi + \cos\theta\cos\phi\cos\psi & \cos\psi\sin\theta \\ \sin\theta\sin\phi & -\sin\theta\cos\phi & \cos\theta \end{bmatrix} \begin{bmatrix} F_X \\ F_Y \\ F_Z \end{bmatrix} . \quad -(11)$$

So the motion is exceedingly intricate. However, it is clear from eqs. (9) and (10) that classical dynamics never gives a counter-gravitational force.

In fluid dynamics in the lab frame:

$$\underline{F} = m\left(\frac{d\underline{v}}{dt} + (\underline{v}\cdot\underline{\nabla})\underline{v}\right) \quad -(12)$$

where $$\underline{v} = \underline{v}(t, \underline{r}(t)) \quad -(13)$$

is the velocity field. We have:

$$\left(\frac{d\underline{v}}{dt}\right)_{lab} = \left(\frac{d\underline{v}}{dt} + \underline{\omega} \times \underline{v}\right)_{moving} \quad -(14)$$

There is an excess force:

$$\underline{F}_{lab} = m(\underline{v} \cdot \underline{\nabla})\underline{v} \quad -(15)$$

In order to observe counter gravitation a force of the following type must exist:

$$\underline{F}_{lab} = F_Z \underline{k} \quad -(16)$$

In general:

$$\underline{F} = m(\underline{v} \cdot \underline{\nabla})\underline{v}$$

$$= m\left(v_X \frac{\partial}{\partial X} + v_Y \frac{\partial}{\partial Y} + v_Z \frac{\partial}{\partial Z}\right)\left(v_X \underline{i} + v_Y \underline{j} + v_Z \underline{k}\right) \quad -(17)$$

so:

$$F_Z = \left(v_X \frac{\partial}{\partial X} + v_Y \frac{\partial}{\partial Y} + v_Z \frac{\partial}{\partial Z}\right) v_Z \quad -(18)$$

It is possible that Laithwaite and Shipov may be observing this force. It can be plotted using gnuplot.