363(3): Modifications to the Hooke / Newton Inverse Square Law due to Fluid Spacetime

In classical dynamics the acceleration in plane polar coordinates is given by the Cartan derivative:

$$\frac{D}{Dt}\begin{bmatrix}\dot{r}\\ r\dot{\theta}\end{bmatrix} = \frac{\partial}{\partial t}\begin{bmatrix}\dot{r}\\ r\dot{\theta}\end{bmatrix} + \begin{bmatrix}0 & -\dot{\theta}\\ \dot{\theta} & 0\end{bmatrix}\begin{bmatrix}\dot{r}\\ r\dot{\theta}\end{bmatrix} \quad -(1)$$

where:

$$\underline{a} = \frac{d\underline{v}}{dt} = \frac{d}{dt}\left(\dot{r}\,\underline{e}_r + r\dot{\theta}\,\underline{e}_\theta\right) \quad -(2)$$

so

$$\frac{D\dot{r}}{Dt} = \frac{\partial\dot{r}}{\partial t} - r\dot{\theta}^2 \quad -(3)$$

and

$$\frac{D(r\dot{\theta})}{Dt} = \frac{\partial(r\dot{\theta})}{\partial t} + \dot{\theta}\dot{r} = r\ddot{\theta} + 2\dot{r}\dot{\theta} \quad -(4)$$

so:

$$\underline{a} = \left(\ddot{r} - \omega^2 r\right)\underline{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\underline{e}_\theta \quad -(5)$$

For the Newtonian orbit:

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(6)$$

it has been shown in previous UFT papers that:

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \quad -(7)$$

so

$$\underline{a} = \left(\ddot{r} - \omega^2 r\right)\underline{e}_r \quad -(8)$$

2) Therefore from the equivalence principle:

$$\underline{F} = m\underline{a} = m\left(\ddot{r} - \omega^2 r\right)\underline{e}_r = -\frac{mMG}{r^2}\underline{e}_r \quad -(9)$$

In the presence of fluid spacetime:

$$\frac{D}{Dt}\begin{bmatrix}\dot{r} \\ r\dot{\theta}\end{bmatrix} = \frac{\partial}{\partial t}\begin{bmatrix}\dot{r} \\ r\dot{\theta}\end{bmatrix} + \begin{bmatrix}0 & -\dot{\theta} \\ \dot{\theta} & 0\end{bmatrix}\begin{bmatrix}\dot{r} \\ r\dot{\theta}\end{bmatrix} + \begin{bmatrix}\Omega^1{}_{01} & \Omega^1{}_{02} \\ \Omega^2{}_{01} & \Omega^2{}_{02}\end{bmatrix}\begin{bmatrix}\dot{r} \\ r\dot{\theta}\end{bmatrix} \quad -(10)$$

where the spin connection induced by a fluid spacetime, vacuum or aether is:

$$\begin{bmatrix}\Omega^1{}_{01} & \Omega^1{}_{02} \\ \Omega^2{}_{01} & \Omega^2{}_{02}\end{bmatrix} = \begin{bmatrix}\frac{\partial v_r}{\partial r} & \frac{1}{r}\frac{\partial v_r}{\partial \theta} \\ \frac{\partial v_\theta}{\partial r} & \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}\end{bmatrix}. \quad -(11)$$

Therefore:

$$\frac{D\dot{r}}{Dt} = \frac{d\dot{r}}{dt} - r\dot{\theta}^2 + \Omega^1{}_{01}\dot{r} + \Omega^1{}_{02}r\dot{\theta} \quad -(12)$$

$$\frac{D(r\dot{\theta})}{Dt} = \frac{d(r\dot{\theta})}{dt} + \dot{\theta}\dot{r} + \Omega^2{}_{01}\dot{r} + \Omega^2{}_{02}r\dot{\theta} \quad -(13)$$

It follows that the force between m orbiting M is:

3)

$$\underline{F} = m\underline{a} \quad -(14)$$

where:

$$\underline{a} = \left(\ddot{r} - \omega^2 r + \Omega^1{}_{01}\dot{r} + \Omega^1{}_{02} r\dot{\theta}\right)\underline{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta} + \Omega^2{}_{01}\dot{r} + \Omega^2{}_{02} r\dot{\theta}\right)\underline{e}_\theta \quad -(15)$$

and this force is the general result for classical dynamics in the presence of a fluid vacuum.

The inverse square law is no longer true and the force between m and M is no longer central, there is also a component in $\underline{e}_\theta$.

If it is assumed that the vacuum is a small perturbation of the Newtonian orbit, then to an excellent approximation:

$$\ddot{r} - \omega^2 r \doteq -\frac{MG}{r^2} \quad -(16)$$

and

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} \doteq 0 \quad -(17)$$

so in this approximation:

$$\underline{a} = \left(-\frac{MG}{r^2} + \Omega^1{}_{01}\dot{r} + \Omega^1{}_{02} r\dot{\theta}\right)\underline{e}_r + \left(\Omega^2{}_{01}\dot{r} + \Omega^2{}_{02} r\dot{\theta}\right)\underline{e}_\theta \quad -(18)$$

4) From a lagrangian analysis in the Newtonian approximation (16):

$$\omega = \dot{\theta} = \frac{L}{mr^2} \quad - (19)$$

where L is the constant angular momentum. Eq. (19) is the result of a Lagrangian analysis. In the same approximation (16):

$$\dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} \quad - (20)$$

$$= \frac{L}{mr^2}\frac{dr}{d\theta}$$

Therefore:

$$\underline{a} = \left(-\frac{mG}{r^2} + \Omega^1{}_{01}\frac{L}{mr^2}\frac{dr}{d\theta} + \Omega^1{}_{02}\frac{L}{mr} \right)\underline{e}_r$$

$$+ \left(\Omega^2{}_{01}\frac{L}{mr^2}\frac{dr}{d\theta} + \Omega^2{}_{02}\frac{L}{mr} \right)\underline{e}_\theta \quad - (21)$$

In the same approximation (16):

$$\frac{dr}{d\theta} = \frac{\epsilon r^2}{\alpha}\sin\theta \quad - (20)$$

Therefore the inverse square law is changed to

$$\underline{F} = m\underline{a} \quad -(21)$$

where:

$$\underline{a} = \left(-\frac{mG}{r^2} + \Omega'_{01} L \frac{\epsilon}{\alpha} \sin\theta + \Omega'_{02} \frac{L}{mr} \right) \underline{e}_r + \left(\Omega^2_{01} L \frac{\epsilon}{\alpha} \sin\theta + \Omega^2_{02} \frac{L}{mr} \right) \underline{e}_\theta \quad -(22)$$

From eq. (6):

$$\cos\theta = \frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right) \quad -(23)$$

so

$$\sin\theta = \left(1 - \cos^2\theta\right)^{1/2} = \left(1 - \frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right)\right)^{1/2} \quad -(24)$$

Therefore the $\underline{e}_r$ and $\underline{e}_\theta$ components can be graphed in terms of r. In this approximation this is the force law responsible for a precessing orbit.