363(1): Orbital Precession due to the Fluid Vacuum

As shown in Note 362(5), the orbital velocity in the presence of a fluid spacetime, aether or vacuum is given in component form by:

$$\frac{D}{Dt}\begin{bmatrix} r \\ 0 \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} r \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & -\dot{\theta} \\ \dot{\theta} & 0 \end{bmatrix}\begin{bmatrix} r \\ 0 \end{bmatrix} + \begin{bmatrix} \Omega^1{}_{01} & \Omega^1{}_{02} \\ \Omega^2{}_{01} & \Omega^2{}_{02} \end{bmatrix}\begin{bmatrix} \dot{r} \\ r\dot{\theta} \end{bmatrix} \quad -(1)$$

in the notation of note 362(5). In the absence of the fluid vacuum:

$$\frac{D}{Dt}\begin{bmatrix} r \\ 0 \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} r \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & -\dot{\theta} \\ \dot{\theta} & 0 \end{bmatrix}\begin{bmatrix} r \\ 0 \end{bmatrix} \quad -(2)$$

It is shown in this note that eq. (1) produces orbital precession due to the fluid spacetime, aether or vacuum.

In eq. (1), the Cartan spin connection matrix is defined by:

$$\Omega^a{}_b = \begin{bmatrix} \Omega^1{}_{01} & \Omega^1{}_{02} \\ \Omega^2{}_{01} & \Omega^2{}_{02} \end{bmatrix} = \begin{bmatrix} \frac{\partial v_r}{\partial r} & \frac{1}{r}\frac{\partial v_r}{\partial \theta} \\ \frac{\partial v_\theta}{\partial r} & \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} \end{bmatrix} \quad -(3)$$

Eq. (2) gives the Coriolis velocity components:

$$v_r = \dot{r} \quad -(4)$$

$$v_\theta = \dot{\theta} r = \omega r, \quad -(5)$$

this familiar result can be thought of as the result in the absence of a fluid vacuum.

Eq. (1) gives the effect on the orbit of a fluid vacuum:

2)

$$V_r = \dot{r} + \Omega^1{}_{01}\dot{r} + \Omega^1{}_{02}\, r\dot{\theta} \quad -(6)$$

$$V_\theta = \omega r + \Omega^2{}_{01}\dot{r} + \Omega^2{}_{02}\, r\dot{\theta}. \quad -(7)$$

Therefore:

$$V_r = (1+\Omega^1{}_{01})\dot{r} + \Omega^1{}_{02}\,\omega r \quad -(8)$$

$$V_\theta = (1+\Omega^2{}_{02})\,\omega r + \Omega^2{}_{01}\dot{r} \quad -(9)$$

For a Newtonian orbit:

$$V_r = \dot{r} = \frac{dr}{dt} \quad -(10)$$

and

$$V_\theta = \omega r \quad -(11)$$

Use:

$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = \frac{L}{mr^2}\frac{dr}{d\theta} \quad -(12)$$

and

$$V_\theta = r\frac{d\theta}{dt} \quad -(13)$$

It follows ~~that~~:

$$V_r = \frac{d\theta}{dt}\frac{dr}{d\theta} \quad -(14)$$

and that:

$$\frac{V_r}{V_\theta} = \frac{1}{r}\frac{dr}{d\theta} \quad -(15)$$

Now denote V_r and V_θ of a precessing orbit by $V_r(P)$ and $V_\theta(P)$ and those of a Newtonian orbit by $V_r(N)$ and $V_\theta(N)$. Then eqs. (8) and (9) become:

$$V_r(P) = (1+\Omega^1{}_{01})V_r(N) + \Omega^1{}_{02}V_\theta(N) \quad -(16)$$

$$V_\theta(P) = (1+\Omega^2{}_{02})V_\theta(N) + \Omega^2{}_{01}V_r(N) \quad -(17)$$

From eq. (15):

$$\frac{V_r(N)}{V_\theta(N)} = \frac{1}{r}\frac{dr}{d\theta} \quad -(18)$$

where

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(19)$$

So

$$\frac{dr}{d\theta} = \frac{\epsilon r^2}{\alpha}\sin\theta \quad -(20)$$

so

$$\frac{V_r(N)}{V_\theta(N)} = \frac{\epsilon r}{\alpha}\sin\theta \quad -(21)$$

in which

$$\sin\theta = \left(1-\cos^2\theta\right)^{1/2} \quad -(22)$$

$$= \left(1-\frac{1}{\epsilon^2}\left(\frac{\alpha}{r}-1\right)^2\right)^{1/2}$$

so

$$\frac{V_r(N)}{V_\theta(N)} = \frac{\epsilon r}{\alpha}\left(1-\frac{1}{\epsilon^2}\left(\frac{\alpha}{r}-1\right)^2\right)^{1/2} \quad -(23)$$

and

$$\frac{V_\theta(N)}{V_r(N)} = \frac{\alpha}{\epsilon r}\left(1-\frac{1}{\epsilon^2}\left(\frac{\alpha}{r}-1\right)^2\right)^{-1/2} \quad -(24)$$

If we denote:

$$A = \frac{\epsilon r}{\alpha}\left(1-\frac{1}{\epsilon^2}\left(\frac{\alpha}{r}-1\right)^2\right)^{1/2} \quad -(25)$$

then eqs (16) and (17) become relations between $V_r(P)$ and $V_\theta(P)$ and $V_r(N)$ and $V_\theta(N)$:

$$V_R(\underline{P}) = \left(1 + \Omega'_{01} + \frac{\Omega'_{02}}{A}\right) V_R(N) \quad -(28)$$

nd

$$V_\theta(\underline{P}) = \left(1 + \Omega^2_{02} + A\Omega^2_{01}\right) V_\theta(N) \quad -(29)$$

It follows that:

$$\frac{V_R(\underline{P})}{V_\theta(\underline{P})} = B \frac{V_R(N)}{V_\theta(N)} \quad -(28)$$

where:

$$B = \frac{1 + \Omega'_{01} + \frac{\Omega'_{02}}{A}}{1 + \Omega^2_{02} + A\Omega^2_{01}} \quad -(29)$$

so

$$\frac{V_R(\underline{P})}{V_\theta(\underline{P})} = AB \quad -(30)$$

Now use:

$$\underline{v}(\underline{P}) = v_r(\underline{P})\,\underline{e}_r + V_\theta(\underline{P})\,\underline{e}_\theta \quad -(31)$$

$$= \dot{r}(\underline{P})\,\underline{e}_r + \omega(\underline{P})\,r(\underline{P})\,\underline{e}_\theta$$

then:

$$\frac{V_R(\underline{P})}{V_\theta(\underline{P})} = \left(\frac{dr}{d\theta}\right)_{\underline{P}} \quad -(32)$$

nd

$$\boxed{\left(\frac{dr}{d\theta}\right)_{\underline{P}} = B\left(\frac{dr}{d\theta}\right)_N} \quad -(33)$$

) If the precessing orbit is modelled by

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(34)$$

with

$$x = 1 + \frac{3MG}{c^2\alpha} \quad -(35)$$

then

$$\left(\frac{dr}{d\theta}\right)_P = \frac{x\epsilon r^2}{\alpha}\sin(x\theta) \quad -(36)$$

and

$$\left(\frac{dr}{d\theta}\right)_N = \frac{\epsilon r^2}{\alpha}\sin\theta \quad -(37)$$

To an excellent approximation:

$$\sin(x\theta) \sim \sin\theta \quad -(38)$$

in the solar system, where x is very close to unity.

Therefore

$$\left(\frac{dr}{d\theta}\right)_P = x\left(\frac{dr}{d\theta}\right)_N \quad -(39)$$

Comparing eqs. (33) and (39):

$$\boxed{x = B} \quad -(40)$$

in the approximation (38).

From the experimental data it is known that x is the constant (35) for a given half right latitude α.

6) Therefore one possible solution is:

$$\frac{1+\Omega^1{}_{01}}{1+\Omega^2{}_{02}} = x = 1 + \frac{3mG}{c^2\alpha} \quad -(34)$$

and:

$$\Omega^1{}_{02} = \Omega^2{}_{01} = 0 \quad -(35)$$

It has been shown that orbital precession to any precision is produced by a fluid spacetime, vacuum or aether.
