60(2): The New Law of Planar Orbits

This is a generalization of the Newtonian law to:

$$\underline{F} = m\underline{g} = -\frac{mM}{r^2}\,\underline{e}_r \quad -(1)$$

where:

$$r^2 = X^2 + Y^2 \quad -(2)$$

In plane polar coordinates, (r, θ):

$$X = MG\left(\frac{\dot{r}\sin\theta + r\dot{\theta}\cos\theta}{(\dot{r}^2 + r^2\dot{\theta}^2)^{3/2}}\right) \quad -(3)$$

$$Y = MG\left(\frac{r\dot{\theta}\sin\theta - \dot{r}\cos\theta}{(\dot{r}^2 + r^2\dot{\theta}^2)^{3/2}}\right) \quad -(4)$$

In general:

$$\boxed{\underline{F} = m(\underline{v}\cdot\underline{\nabla})\underline{v}} \quad -(5)$$

where $\underline{v}$ is the orbital velocity.

For example, for Newtonian dynamics:

$$r = \frac{\alpha}{1 + \epsilon\cos\theta} \quad -(6)$$

$$v^2 = MG\left(\frac{2}{r} - \frac{1}{a}\right) \quad -(7)$$