357(4): Lamb Shift from the Potential of Fluid Electrodynamics.

In the Dirac theory the following energy levels are degenerate:

$$2S_{1/2} \; (n = 2, \ell = 0, j = 1/2) \quad -(1)$$

$$2P_{1/2} \; (n = 2, \ell = 1, j = 1/2) \quad -(2)$$

for the H atom. Experimentally however, $2S_{1/2}$ is shifted to higher frequency by 1057.864 MHz. (see for example UFT 340).

This shift was first explained by Bethe by assuming that the Coulomb potential between the electron and proton fluctuates due to the influence of the surrounding spacetime or aether or vacuum, so

$$\bar{U} = U(\underline{r} - \delta\underline{r}) \quad -(3)$$

The additional potential energy induced in the H atom by the surrounding spacetime is:

$$\Delta U = \bar{U}(\underline{r} + \delta\underline{r}) - U(\underline{r}) \quad -(4)$$

where $\underline{r}$ is the vector joining the electron and proton (see UFT 340).

From a Maclaurin expansion

$$\langle \Delta U \rangle = \frac{1}{6} \langle (\delta r)^2 \rangle \nabla^2 U \quad -(5)$$

For the $2S_{1/2}$ orbital of the H atom:

$$\langle \nabla^2 \bar{U} \rangle = \left\langle \nabla^2 \left(\frac{-e^2}{4\pi\epsilon_0 r} \right) \right\rangle \quad -(6)$$

2) is an expectation value that is worked out with the $^2S_{1/2}$ orbital of

$$\langle \nabla^2 U \rangle = \int \psi^* \nabla^2 \left(\frac{-e^2}{4\pi\epsilon_0 r} \right) \psi \, d\tau \quad -(7)$$

The rigorous calculation involves the Dirac wave function, and gives:

$$\langle \nabla^2 U \rangle = \frac{e^2}{8\pi\epsilon_0 a_0^3} \quad -(8)$$

where ϵ_0 is the vacuum permittivity and a_0 the Bohr radius. Here $-e$ is the charge on the electron. Therefore the additional potential energy induced in the $^2S_{1/2}$ orbital by the surrounding spacetime is

$$\langle \Delta U \rangle = \frac{1}{6} \langle (\delta r)^2 \rangle \frac{e^2}{8\pi\epsilon_0 a_0^3}$$

$$= \frac{e^2}{48\pi\epsilon_0 a_0^3} \langle (\delta r)^2 \rangle \quad -(9)$$

in units of joules.

In fluid electrodynamics this additional energy is explained by the scalar potential Φ

3) of the fluid dynamical vacuum. For one electron this induces in the $2S_{1/2}$ orbital the potential:

$$\underline{\Phi}_W = \frac{m}{e}\underline{\Phi} \quad -(10)$$

and the potential energy:

$$\boxed{U_W = e\underline{\Phi}_W = m\underline{\Phi}} \quad -(11)$$

where m is the mass of the electron.

$$\boxed{\text{So: } U_W = m\underline{\Phi} = \frac{e^2}{48\pi\epsilon_0 a_0^3}\langle(\delta r)^2\rangle} \quad -(12)$$

and this is the explanation of the Lamb shift in fluid electrodynamics.

The spacetime or vacuum potential is part of the four vector:

$$V^\mu = \left(\frac{\underline{\Phi}}{a_0}, \underline{V}\right) \quad -(13)$$

where a_0 is the assumed constant speed of sound and $\underline{V}$ is the velocity field of the spacetime or vacuum. The Lorenz condition gives:

$$\frac{1}{a_0^2}\frac{\partial\underline{\Phi}}{\partial t} + \underline{\nabla}\cdot\underline{V} = 0 \quad -(14)$$

4) From Eq. (12) the spacetime or vacuum potential needed to explain the Lamb shift in fluid electrodynamics is:

$$\boxed{\bar{\Phi} = \frac{e^2}{48\pi\epsilon_0 m a_0^3} \langle(\delta r)^2\rangle} \quad -(15)$$

where

$$e = 1.60219 \times 10^{-19}\ C$$
$$4\pi\epsilon_0 = 1.112650 \times 10^{-10}\ J^{-1} C^2 m^{-1}$$
$$m = 9.10953 \times 10^{-31}\ kg$$
$$a_0 = 5.29177 \times 10^{-11}\ m$$

The units of $\bar{\Phi}$ are $m^2 s^{-2}$, so the units of $m\bar{\Phi}$ are joules.

The measured Lamb shift is

$$f = 1.057864 \times 10^9\ Hz \quad -(16)$$

so

$$\omega = 2\pi f = 6.64675 \times 10^9\ rad\ s^{-1} \quad -(17)$$

and

$$U_m = m\bar{\Phi} = \hbar\omega \quad -(18)$$

It follows that

$$\bar{\Phi} = \frac{\hbar\omega}{m} = \frac{6.62618 \times 10^{-34} \times 6.64675 \times 10^9}{9.10953 \times 10^{-31}}$$

$$= 4.834779 \times 10^6\ m^2 s^{-2}$$

$$-(18)$$