257(3): Preparatory Calculation for the Lamb Shift.

In this calculation we begin with the Dirac type hamiltonian of UFT 253, eq. (41):

$$H\psi = \left(e\phi + mc^2 + \frac{1}{2m}\underline{\sigma}\cdot\underline{p}\,\underline{\sigma}\cdot\underline{p} + \frac{e}{4m^2c^2}\underline{\sigma}\cdot\underline{p}\,\phi\,\underline{\sigma}\cdot\underline{p}\right)\psi \quad -(1)$$

The relevant spin orbit term is:

$$H_1\psi = \frac{e}{4m^2c^2}\left(\underline{\sigma}\cdot\underline{p}\,\phi\,\underline{\sigma}\cdot\underline{p}\right)\psi \quad -(2)$$

where

$$\underline{p}\psi = -i\hbar\underline{\nabla}\psi \quad -(3)$$

so:

$$H_1\psi = -\frac{ie\hbar}{4m^2c^2}\underline{\sigma}\cdot\underline{\nabla}\phi\,\underline{\sigma}\cdot\underline{p}\,\psi \quad -(4)$$

$$= -\frac{ie\hbar}{4m^2c^2}\underline{\sigma}\cdot\underline{\nabla}\left(\phi\,\underline{\sigma}\cdot\underline{p}\,\psi\right)$$

$$= -\frac{ie\hbar}{4m^2c^2}\left(\underline{\nabla}\left(\underline{\sigma}\cdot\underline{p}\right)\phi\psi + \underline{\sigma}\cdot\underline{p}\,\underline{\nabla}\cdot(\phi\psi)\right)$$

Now use:

$$\underline{\nabla}(\phi\psi) = \psi\underline{\nabla}\phi + \phi\underline{\nabla}\psi \quad -(5)$$

so:

$$H_1\psi = -\frac{ie\hbar}{4m^2c^2}\left(\underline{\sigma}\cdot\underline{\nabla}\phi\,\underline{\sigma}\cdot\underline{p}\,\psi + \phi\,\underline{\sigma}\cdot\underline{\nabla}\psi\,\underline{\sigma}\cdot\underline{p} + \cdots\right) \quad -(6)$$

2) The Coulomb potential is used as follows:

$$\phi = -\frac{e}{4\pi\epsilon_0 r} \quad -(7)$$

So

$$\underline{\nabla}\phi = \frac{e}{4\pi\epsilon_0 r^3}\underline{r} \quad -(8)$$

and

$$H_1\phi = -\frac{ie^2\hbar}{16\pi\epsilon_0 m^2c^2}\left(\frac{1}{r^3}\underline{\sigma}\cdot\underline{r}\,\underline{\sigma}\cdot\underline{p}\,\phi - \frac{1}{r}\underline{\sigma}\cdot\underline{\nabla}\phi\,\underline{\sigma}\cdot\underline{p}\right) \quad -(9)$$

Now use the Pauli algebra:

$$\underline{\sigma}\cdot\underline{r}\,\underline{\sigma}\cdot\underline{p} = \underline{r}\cdot\underline{p} + i\underline{\sigma}\cdot\underline{r}\times\underline{p} \quad -(10)$$

$$= \underline{r}\cdot\underline{p} + i\underline{\sigma}\cdot\underline{L}$$

where the orbital angular momentum is:

$$\underline{L} = \underline{r}\times\underline{p} \quad -(11)$$

So:

$$\mathrm{Re}(H_1\phi) = \frac{e^2\hbar}{16\pi\epsilon_0 m^2c^2r^3}\underline{\sigma}\cdot\underline{L}\,\phi + \dots \quad -(12)$$

where

$$\underline{\sigma}\cdot\underline{L} = \int \phi^*\,\hat{\underline{\sigma}}\cdot\hat{\underline{L}}\,\phi\,d\tau \quad -(13)$$

The spin angular momentum operator is:

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$$\underline{\hat{S}} = \frac{\hbar}{2}\,\underline{\hat{\sigma}} \quad \text{— (14)}$$

So

$$\underline{\hat{\sigma}} \cdot \underline{\hat{L}} = \frac{2}{\hbar}\,\underline{\hat{S}} \cdot \underline{\hat{L}} \,. \quad \text{— (15)}$$

In our representation: — (16)

$$\underline{\hat{S}} \cdot \underline{\hat{L}}\,\phi = \frac{\hbar^2}{2}\Big(j(j+1) - \ell(\ell+1) - s(s+1)\Big)\phi$$

where j, ℓ and s are quantum numbers.

Therefore:

$$\underline{\sigma} \cdot \underline{L} = \langle \underline{\hat{\sigma}} \cdot \underline{\hat{L}} \rangle$$

$$= \hbar\Big(\big(j(j+1) - \ell(\ell+1) - s(s+1)\big)\int \phi^* \phi \, d\tau$$

$$= \hbar\Big(j(j+1) - \ell(\ell+1) - s(s+1)\Big) \quad \text{— (17)}$$

Therefore:

$$\mathrm{Re}\left(\hat{H}_1 \phi\right) = \frac{e^2 \hbar^2}{16\pi\epsilon_0 m^2 c^2}\Big(j(j+1) - \ell(\ell+1) - s(s+1)\Big)\int \frac{\phi\phi^*}{r^3}\, d\tau$$

— (18)

The Lamb shift is the effect of the vacuum or aether or spacetime on this calculation. In the H atom:

$$\int \frac{\phi \phi^* d\tau}{r^3} = \frac{1}{a_0^3 n^3 \ell\left(\ell+\frac{1}{2}\right)(\ell+1)} \quad -(19)$$

where

$$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{me^4} \quad -(20)$$

is the Bohr radius.

The Lamb shift is a difference of about a gigahertz between the $^2S_{1/2}$ and $^2P_{1/2}$ levels of the H atom. This is not predicted by the above Dirac theory. The $^2S_{1/2}$ level is about a gigahertz higher than the $^2P_{1/2}$ level.

The next note will begin to develop an explanation for this radiative correction using fluid electrodynamics and the potential of fluid spacetime.
