

356(1) : Velocity Fields Generated by Plane Waves and $\underline{B}^{(3)}$ Field

Consider plane waves of electric field strength $\underline{E}^{(1)}$ in volts per metre and magnetic flux density $\underline{B}^{(1)}$ in Tesla. Here:

$$\underline{E}^{(1)} = \underline{E}^{(2)*} \quad - (1)$$

$$\underline{B}^{(1)} = \underline{B}^{(2)*} \quad - (2)$$

and where * denotes complex conjugate. Therefore:

$$\underline{E}^{(1)} = \frac{\underline{E}^{(0)}}{\sqrt{2}} (\underline{i} - \underline{j}) e^{i\phi} \quad - (3)$$

$$\underline{E}^{(2)} = \frac{\underline{E}^{(0)}}{\sqrt{2}} (\underline{i} + \underline{j}) e^{-i\phi} \quad - (4)$$

$$\underline{B}^{(1)} = \frac{\underline{B}^{(0)}}{\sqrt{2}} (\underline{ii} + \underline{j}) e^{i\phi} \quad - (5)$$

$$\underline{B}^{(2)} = \frac{\underline{B}^{(0)}}{\sqrt{2}} (-\underline{ii} + \underline{j}) e^{-i\phi} \quad - (6)$$

$$\underline{B}^{(3)} = \underline{B}^{(0)} \underline{k} \quad - (7)$$

$$\text{where: } \phi = \omega t - kZ \quad - (8)$$

Here ω is the angular frequency of the wave at an instant t and k is the magnitude of its wave number at a position Z along the \underline{k} axis.
Each of these electric and magnetic waves

2) and $\underline{B}^{(3)}$ field generates a velocity field \underline{v} in the latter. Denote:

$$\alpha = \left(\frac{\rho_m}{\rho} \right) \quad - (9)$$

This is the ratio of effective mass to charge density in a material through which the plane wave is propagating. This is a simple first approximation, more accurately a plane wave propagating in a medium of refractive index n must be considered and that can be done at a later stage.

The simplest calculation is:

$$\underline{B}^{(3)} = B^{(0)} \underline{k} = \alpha \underline{\nabla} \times \underline{v}^{(3)} \quad - (10)$$

so $\underline{v}^{(3)}$ has components in the \underline{i} and \underline{j} directions:

$$\underline{v}^{(3)} = \frac{B^{(0)}}{2\alpha} (-Y \underline{i} + X \underline{j}) \quad - (11)$$

and is a divergenceless function that can be graphed straightforwardly. We also have:

$$\underline{B}^{(1)} = \frac{B^{(0)}}{\sqrt{2}} (\underline{i}\underline{i} + \underline{j}\underline{j}) e^{i\phi} = \alpha \underline{\nabla} \times \underline{v}^{(1)} \quad - (12)$$

$$\underline{B}^{(2)} = \frac{B^{(0)}}{\sqrt{2}} (-\underline{i}\underline{i} + \underline{j}\underline{j}) e^{-i\phi} = \alpha \underline{\nabla} \times \underline{v}^{(2)} \quad - (13)$$

3) and it is also straightforward to use computer algebra to work out $\underline{v}^{(1)}$ and $\underline{v}^{(2)}$.

Finally:

$$\underline{E}^{(1)} = \frac{E^{(0)}}{\sqrt{2}} (\underline{i} - i\underline{j}) e^{i\phi} = x(\underline{v}^{(1)} \cdot \underline{\nabla}) \underline{v}^{(1)} \quad (14)$$

$$\text{and } \underline{E}^{(2)} = \frac{E^{(0)}}{\sqrt{2}} (\underline{i} + i\underline{j}) e^{-i\phi} x(\underline{v}^{(2)} \cdot \underline{\nabla}) \underline{v}^{(2)} \quad (15)$$

and in this case also it is not difficult to work out the velocity fields set up in the latter by the electric component of plane waves.

A static electric field can be defined by:

$$\underline{E} = -e \frac{\underline{e}_r}{4\pi\epsilon_0 r^2} \quad (16)$$

and the velocity flow set up by this field is defined by:

$$\underline{E} = -e \frac{\underline{e}_r}{4\pi\epsilon_0 r^2} = \left(\frac{\rho_n}{\rho}\right) (\text{material}) (\underline{v} \cdot \underline{\nabla}) \underline{v} \quad (17)$$

where \underline{e}_r is the radial unit vector.

Finally, a static magnetic field is defined by:

$$\nabla \times \underline{B} = \mu_0 \underline{J}(\text{material}) \quad (18)$$

where:

$$\underline{B}(\text{material}) = \left(\frac{\mu_r}{\mu_0} \right) (\text{material}) \nabla \times \underline{A}(\text{vector})$$

so

$$\nabla \times \left(\left(\frac{\mu_r}{\mu_0} \right) (\text{material}) \nabla \times \underline{A}(\text{vector}) \right) = \mu_0 \underline{J}(\text{material}) \quad (19)$$

If it is assumed that $(\mu_r / \mu_0)(\text{material})$ is independent of \underline{r} , then:

$$\left(\frac{\mu_r}{\mu_0} \right) (\text{material}) \nabla \times (\nabla \times \underline{A}(\text{vector})) = \mu_0 \underline{J}(\text{material}) \quad (20)$$

which can be solved for a given $\underline{J}(\text{material})$.

Any type of electric or magnetic field can be considered in general, including pulsed fields.