353(4): Extension of the Kambe Equations with Viscous Forces

Kambe uses the Navier Stokes equation:

$$\frac{D\underline{v}}{Dt} = \frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \underline{\nabla})\underline{v} = -\frac{1}{\rho}\underline{\nabla} p = -\underline{\nabla} h \quad -(1)$$

and the continuity equation:

$$\frac{\partial \rho}{\partial t} + \underline{\nabla} \cdot (\rho \underline{v}) = 0 \quad -(2)$$

together with the entropy equation:

$$\frac{DS}{Dt} = \frac{\partial S}{\partial t} + (\underline{v} \cdot \underline{\nabla}) S = 0. \quad -(3)$$

He uses the vorticity equation:

$$\frac{\partial \underline{w}}{\partial t} + \underline{\nabla} \times (\underline{w} \times \underline{v}) = \underline{0} \quad -(4)$$

The Kambe field equations are essentially rearrangements of the above equation of fluid dynamics. The Kambe electric field is a rearrangement of eq. (1) using the definition:

$$\underline{E}_F = -\underline{\nabla} h - \frac{\partial \underline{v}}{\partial t} = (\underline{v} \cdot \underline{\nabla}) \underline{v}. \quad -(5)$$

The homogeneous field equation of Kambe is:

$$\underline{\nabla} \times \underline{E}_F + \frac{\partial \underline{w}}{\partial t} = \underline{0} \quad -(6)$$

and follows from:

$$\underline{w} = \underline{\nabla} \times \underline{v} \quad -(7)$$

and

$$\begin{aligned} \underline{\nabla} \times \underline{E}_F &= -\underline{\nabla} \times \underline{\nabla} h - \underline{\nabla} \times \left(\frac{\partial \underline{v}}{\partial t} \right) \\ &= -\frac{\partial}{\partial t} (\underline{\nabla} \times \underline{v}) \\ &= -\frac{\partial \underline{w}}{\partial t} \quad -(8) \end{aligned}$$

Q.E.D.

2) The Kambe charge q_F is defined as:

$$q_F = \underline{\nabla} \cdot \underline{E}_F = \underline{\nabla} \cdot \left((\underline{v} \cdot \underline{\nabla}) \underline{v} \right) \quad - (9)$$

The homogeneous law:

$$\underline{B}_F = \underline{\nabla} \times \underline{v} = \underline{w} \quad - (10)$$

is also a matter of definition. Finally the inhomogeneous field equation of Kambe is also a matter of definition, it is

$$a_0^2 \, \underline{\nabla} \times \underline{B}_F - \frac{\partial \underline{E}_F}{\partial t} = \underline{J}_F \quad - (11)$$

so the Kambe current $\underline{J}_F$ is <u>defined</u> as:

$$\underline{J}_F = a_0^2 \, \underline{\nabla} \times (\underline{\nabla} \times \underline{v}) - \frac{\partial}{\partial t} \left((\underline{v} \cdot \underline{\nabla}) \underline{v} \right) \quad - (12)$$

It is assumed in Eq. (11) that a_0 is the speed of sound. More generally, the Navier Stokes equation is:

$$\frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \underline{\nabla}) \underline{v} = - \underline{\nabla} h - \underline{\nabla} \phi + \underline{f}_{visc} \quad - (13)$$

where ϕ is the gravitational potential and where $\underline{f}_{visc}$ is the viscous force. So the definition of the electric field is, more generally,

$$\underline{E}_F = - \underline{\nabla} h - \underline{\nabla} \phi + \underline{f}_{visc} - \frac{\partial \underline{v}}{\partial t} \quad - (14)$$
$$= (\underline{v} \cdot \underline{\nabla}) \underline{v}$$

In order to put eq. (14) in the format of electrodynamics it is expressed as:

$$\underline{E}_F = -\underline{\nabla}\,\Phi - \frac{\partial \underline{v}}{\partial t} \quad -(15)$$

where Φ is the Kambe scalar potential and $\underline{v}$ is the Kambe vector potential.

So by definition:

$$\boxed{-\underline{\nabla}\,\Phi = -\underline{\nabla} h - \underline{\nabla}\phi + \underline{f}_{visc}} \quad -(16)$$

The viscous force may be expressed in general by:

$$\underline{f}_{visc} = \mu\nabla^2\underline{v} + \left(\underline{\nabla}\left(\underline{\nabla}\cdot\underline{v}\right)\right)\left(\mu+\mu'\right) \quad -(17)$$

Therefore:

$$\boxed{\Phi = h + \phi + \left(\mu+\mu'\right)\underline{\nabla}\cdot\underline{v} - \phi_1} \quad -(17)$$

where

$$\underline{\nabla}\phi_1 := \mu\nabla^2\underline{v} \quad -(18)$$

With the definitions (10) and (15) the Kambe equations can be developed into wave equations.

Finally, the most general vorticity equation is:

$$\frac{d\underline{w}}{dt} = \underline{\nabla}\times\left(\underline{v}\times\underline{w}\right) + \frac{1}{\rho^2}\underline{\nabla}\rho\times\underline{\nabla}p + \frac{\mu}{\rho}\nabla^2\underline{w} \quad -(18)$$

but this is not used by Kambe to derive his field equations.