352(7): General Wave Equation

In general:

$$\nabla^2 \phi_W + \frac{\partial}{\partial t}\left(\underline{\nabla}\cdot\underline{W}\right) = -\left(\frac{\rho}{\epsilon_0}\right)_{vac} \quad -(1)$$

and

$$\Box \underline{W} + \underline{\nabla}\left(\underline{\nabla}\cdot\underline{W}\right) + \frac{1}{c^2}\frac{\partial \phi_W}{\partial t} = \mu_0 \underline{J}\,(vac) \quad -(2)$$

Eq. (1) may be rewritten as:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\phi_W - \frac{\partial}{\partial t}\left(\underline{\nabla}\cdot\underline{W}\right) = \frac{1}{c^2}\frac{\partial^2 \phi_W}{\partial t^2} + \left(\frac{\rho}{\epsilon_0}\right)_{vac} \quad -(3)$$

$$-(4)$$

so

$$\Box \phi_W + \frac{1}{c^2}\frac{\partial^2 \phi_W}{\partial t^2} - \frac{\partial}{\partial t}\left(\underline{\nabla}\cdot\underline{W}\right) = \left(\frac{\rho}{\epsilon_0}\right)_{vac} \quad -(5)$$

Now define:

$$\underline{J}\,\text{circuit} := \frac{1}{\mu_0}\left(\underline{\nabla}\left(\underline{\nabla}\cdot\underline{W}\right) + \frac{1}{c^2}\frac{\partial \phi_W}{\partial t}\right)$$

and

$$\rho\,\text{circuit} := -\epsilon_0\left(\frac{1}{c^2}\frac{\partial^2 \phi_W}{\partial t^2} + \frac{\partial}{\partial t}\,\underline{\nabla}\cdot\underline{W}\right) \quad -(6)$$

then if:

$$J^\mu(\text{circuit}) = \left(c\rho(\text{circuit}), \underline{J}(\text{circuit})\right) \quad -(7)$$

$$\Box W^\mu + \mu_0 J^\mu(\text{circuit}) = \mu_0 J^\mu(\text{vacuum}) \quad -(8)$$