347(1): A Simple Derivation of the Orbital Velocity due to Precession

In UFT 346 it was shown that the gravitomagnetic field is defined by:

$$\underline{\Omega}_g = \underline{\nabla} \times \underline{v}_g \quad -(1)$$

so that any precession is defined by:

$$\Omega = \frac{1}{2} \left| \underline{\nabla} \times \underline{v}_g \right| \quad -(2)$$

$$= \frac{1}{2} \left| \underline{\Omega}_g \right|$$

If it is assumed that:

$$\underline{\Omega}_g = \Omega_g \underline{k} \quad -(3)$$

then:

$$\underline{v}_g = \frac{1}{2} \Omega_g \left(-Y \underline{i} + X \underline{j} \right) \quad -(4)$$

and

$$\underline{\nabla} \cdot \underline{v}_g = 0 \quad -(5)$$

This defines an ECE2 spacetime that corresponds to an inviscid fluid in hydrodynamics.

More generally,

$$\underline{v}_g = \frac{1}{2} \underline{\Omega}_g \times \underline{r} \quad -(6)$$

corresponds to a uniform $\underline{\Omega}_g$.

Proof

$$\underline{\nabla} \times \underline{v}_g = \frac{1}{2} \underline{\nabla} \times \left(\underline{\Omega}_g \times \underline{r} \right) \quad -(7)$$

$$= \frac{1}{2} \left(\underline{\Omega}_r \left(\underline{\nabla} \cdot \underline{r} \right) - \left(\underline{\nabla} \cdot \underline{\Omega}_g \right) \underline{r} + \left(\underline{r} \cdot \underline{\nabla} \right) \underline{\Omega}_g - \left(\underline{\Omega}_g \cdot \underline{\nabla} \right) \underline{r} \right)$$

2) Assume that: $\underline{\Omega}_g = \Omega_{gz}\,\underline{k}$ — (8)

where Ω_{gz} is independent of $\underline{r}$. Then

$$\underline{\nabla}\cdot\underline{\Omega}_g = 0 \quad -(9)$$

and

$$(\underline{r}\cdot\underline{\nabla})\,\underline{\Omega}_g = \underline{0} \quad -(10)$$

Also:

$$\underline{\Omega}_r(\underline{\nabla}\cdot\underline{r}) = 3\underline{\Omega}_r \quad -(11)$$

and

$$(\underline{\Omega}_g\cdot\underline{\nabla})\,\underline{r} = \Omega_{gz}\,\underline{k} \quad -(12)$$

So:

$$\underline{\nabla}\times\underline{v}_g = \frac{1}{2}\left(3\Omega_{gz} - \Omega_{gz}\right)\underline{k} \quad -(13)$$

$$= \Omega_{gz}\,\underline{k}$$

Q.E.D.

From eqs. (6) and (8):

$$\underline{v}_g = \frac{1}{2}\Omega_{gz}\,\underline{k}\times\underline{r} \quad -(14)$$

where

$$\underline{r} = r\,\underline{e}_r \quad -(15)$$

Here:

$$\underline{e}_r = \underline{i}\cos\theta + \underline{j}\sin\theta \quad -(16)$$

$$\underline{e}_\theta = -\underline{i}\sin\theta + \underline{j}\cos\theta \quad -(17)$$

So

$$\underline{k}\times\underline{e}_r = \underline{e}_\theta \quad -(18)$$

It follows that:

$$\underline{v}_g = \frac{1}{2}\Omega_{gz}\, r\, \underline{e}_\theta \quad - (19)$$

if

$$\underline{\Omega}_g = \Omega_{gz}\,\underline{k} \quad - (20)$$

So

$$|\underline{v}_g| = \frac{1}{2}\Omega_{gz}\, r \quad - (21)$$

The observed precession in radians per second is:

$$\Omega = \frac{1}{2}\Omega_{gz} \quad - (22)$$

So

$$\boxed{v_g = |\underline{v}_g| = \Omega r} \quad - (23)$$

For the precession of the earth's perihelion:

$$\Omega = 7.681 \times 10^{-15} \text{ rad s}^{-1} \quad - (24)$$

In a rough first approximation, assuming a circular orbit:

$$r = 1.49598 \times 10^{11} \text{ m} \quad - (25)$$

So

$$\boxed{v_g = 1.149 \times 10^{-3} \text{ ms}^{-1}} \quad - (26)$$

The orbital linear velocity of the earth about the sun is:

$$v = 2.9786 \times 10^{4} \text{ ms}^{-1} \quad - (27)$$

the precession introduces a small increment in the orbital velocity every earth year, or 2π radians