347(3): The Gravitomagnetic Minimal Prescription.

Consider the minimal prescription in electrodynamics:

$$\underline{p} \rightarrow \underline{p} - e\underline{A} \quad -(1)$$

In ECE2 theory this goes to:

$$\underline{p} \rightarrow \underline{p} - e\underline{W} \quad -(2)$$

where $-e$ is the charge on the electron. In the presence of $\underline{W}$ the hamiltonian of a free particle becomes:

$$H = \frac{1}{2m}\left(\underline{p} - e\underline{W}\right)\cdot\left(\underline{p} - e\underline{W}\right)$$

$$= \frac{p^2}{2m} + \frac{e^2W^2}{2m} - \frac{e}{m}\,\underline{p}\cdot\underline{W} \quad -(3)$$

The magnetic field is defined by:

$$\underline{B} = \underline{\nabla}\times\underline{W} \quad -(4)$$

and for a uniform magnetic field:

$$\underline{W} = \frac{1}{2}\underline{B}\times\underline{r}. \quad -(5)$$

Therefore:

$$H_1 = -\frac{e}{m}\,\underline{p}\cdot\underline{W} = -\frac{e}{m}\,\underline{p}\cdot\underline{B}\times\underline{r}$$

$$= -\frac{e}{2m}\,\underline{L}\cdot\underline{B} \quad -(6)$$

where

$$\underline{L} = \underline{p}\times\underline{r} \quad -(7)$$

is the orbital angular momentum. The magnetic dipole moment is defined as:

$$\underline{m} = \frac{e}{2m}\,\underline{L} \quad -(8)$$

so:

$$H_1 = -\underline{m}\cdot\underline{B} \quad -(9)$$

2) The torque generated between $\underline{m}$ and $\underline{B}$ is:

$$\underline{Tq} = \underline{m} \times \underline{B} \quad -(10)$$

and the Larmor precession frequency is:

$$\omega = \frac{eB}{2m} \quad -(11)$$

as observed in electron spin resonance (ESR).

This well known theory can be developed for any astronomical precession as follows.

The precession is due to the gravitomagnetic field:

$$\underline{\Omega}_g = \underline{\nabla} \times \underline{v}_g \quad -(12)$$

where

$$\underline{W}_g = \underline{v}_g \quad -(13)$$

Therefore the gravitomagnetic field is a vorticity in ECE2 spacetime.

The gravitomagnetic minimal prescription is:

$$\underline{p} \rightarrow \underline{p} + m\underline{v}_g \quad -(14)$$

for a free particle in the presence of the potential defined in Eq. (13). The free particle hamiltonian becomes:

$$H = \frac{1}{2m} \left(\underline{p} + m\underline{v}_g \right) \cdot \left(\underline{p} + m\underline{v}_g \right) \quad -(15)$$

where:

$$\underline{v}_g = \frac{1}{2} \underline{\Omega}_g \times \underline{r} \quad -(16)$$

for a uniform gravitomagnetic field $\underline{\Omega}_g$. Therefore the free particle hamiltonian becomes:

$$H = \frac{p^2}{2m} + \frac{1}{2} m v_g^2 + \frac{1}{2} \underline{L} \cdot \underline{\Omega}_g \quad -(17)$$

where $\underline{L}$ is defined by eq. (7). The factor 1/2 in the last term of eq. (17) takes the place of $-e/2m$ in electrodynamics, so the gravitomagnetic Larmor precession frequency is:

$$\boxed{\Omega = \frac{1}{2}\Omega_g} \quad -(18)$$

In the presence of a precession Ω in radians per second the orbital hamiltonian is changed from:

$$H = \frac{p^2}{2m} + U \quad -(19)$$

to

$$\boxed{H = \frac{p^2}{2m} + \frac{1}{2}mv_g^2 + \Omega L + U} \quad -(20)$$

and the lagrangian becomes:

$$\boxed{\mathcal{L} = \frac{p^2}{2m} + \frac{1}{2}mv_g^2 + \Omega L - U} \quad -(21)$$

The orbital characteristics can now be worked out by development of standard methods, to give the precessing orbit.