Note 345(8): General Theory of Geodetic Precession

As in previous notes the general expression for the precession is:

$$\Omega = \frac{MG}{2c^2 r} \left| \underline{\omega} - 3\underline{n}(\underline{\omega} \cdot \underline{n}) \right| \quad -(1)$$

For the geodetic precession measured by Gravity Probe B the polar orbit is defined by:

$$\underline{r} = Y\underline{j} + Z\underline{k}. \quad -(2)$$

The orbit is completed once every 90 minutes, giving an angular velocity of:

$$\omega = \frac{2\pi}{90 \times 60} = 1.164 \times 10^{-3} \text{ rad s}^{-1} \quad -(3)$$

As seen from a frame of reference on Gravity Probe B, the earth rotates at a given angular velocity, generating the angular momentum:

$$\underline{L} = M r \underline{\omega}. \quad -(4)$$

If it is assumed that:

$$\underline{\omega} = \omega_X \underline{i} \quad -(5)$$

then:

$$\Omega = \frac{MG\omega}{2c^2 r} \quad -(6)$$

For the earth:

$$\frac{MG}{2c^2} = 2.2175 \times 10^{-3} \text{ m.} \quad -(7)$$

If it is assumed that r is the distance from the centre of the earth to Gravity Probe B, then:

$$r = 7.02 \times 10^{6} \text{ m} \quad -(8)$$

2) This gives:

$$\Omega = \frac{2.2175 \times 10^{-3} \times 1.164 \times 10^{-3}}{7.02 \times 10^{6}} \quad -(9)$$

$$= 3.677 \times 10^{-13} \text{ radians per second}$$

The experimental claim by Stanford / NASA is:

$$\Omega(\text{geodetic}) = 6.6144 \text{ arc seconds per year}$$

$$= 1.0161 \times 10^{-12} \text{ radian per second.} \quad -(10)$$

The theory is quite close to the experimental claim so the theory is on the right track. It has been assumed that the angular momentum needed for eq. (1) is generated by a static earth in a rotating frame. The latter is the passive rotation equivalent to the active rotation of Gravity Probe B around the centre of the earth in polar orbit once every 90 minutes.

If it is assumed that the frame rotation is described more generally by:

$$\underline{\omega} = \omega_X \underline{i} + \omega_Y \underline{j} + \omega_Z \underline{k} \quad -(11)$$

and

$$\underline{r} = Y \underline{j} + Z \underline{k} \quad -(12)$$

then computer algebra can be used to evaluate the magnitude:

$$x = |\underline{\omega} - 3\underline{n}(\underline{\omega} \cdot \underline{n})| \quad -(13)$$

using eqs. (11) and (12).

3) Eq. (11) assumes that the rotating frame is not in the plane of the polar orbit, i.e. it has j and k components as well as an ω'_x component. The magnitude of $\underline{\omega}$ is:

$$\omega = \left(\omega_x^2 + \omega_y^2 + \omega_z^2\right)^{1/2} \quad -(14)$$

In the standard model the geodetic precession is described by rotating the "Schwarzschild" metric. However this metric is fundamentally incorrect because it is a solution of a geometry without torsion.

Using eqs. (1) and (14) exact agreement between theory and experiment can be obtained.