) Calculation of the Lense Thirring Precession of the Earth

From ECE 2 theory the earth's gravitomagnetic field is:

$$\underline{\Omega} = \frac{2G}{c^2 r^3}\left(\underline{L} - 3\left(\underline{L} \cdot \frac{\underline{r}}{r}\right)\frac{\underline{r}}{r}\right) \quad -(1)$$

Here $\underline{L}$ is the angular momentum of the Earth, and $\underline{r}$ the vector joining the earth's centre to Gravity Probe B. Here G is Newton's constant and c is the vacuum speed of light. If

$$\underline{L} \perp \underline{r} \quad -(2)$$

then:

$$\underline{\Omega} = \frac{2G}{c^2 r^3}\underline{L} \quad -(3)$$

The Lense Thirring precession is the Larmor precession due to the torque between $\underline{\Omega}$ and a gyroscope on board the Gravity Probe B spacecraft. This gyroscope is a gravitomagnetic dipole moment, $\underline{m}$. The torque is:

$$\underline{T_V} = \underline{m} \times \underline{\Omega} \quad -(4)$$

giving the Larmor precession frequency

$$\omega_L = g_L \frac{\Omega}{2} \quad -(5)$$

in radians per second. Here g_L is the gravitomagnetic Landé factor.

Now assume that:

$$\omega_L = \frac{\Omega}{2} \quad -(6)$$

i.e

$$g_L = 1 \quad -(7)$$

2)

The earth is regarded as a sphere of moment of inertia:

$$I = \frac{2}{5} M r^2 \quad - (8)$$

The magnitude of its angular momentum is:

$$L = \omega I \quad - (9)$$

where ω is its angular velocity:

$$\omega = \frac{2\pi}{T} \quad - (10)$$

where T is the time taken T for one rotation. This is 24 hours, or 8.640×10^4 seconds

The Lense Thirring precession is the gravito-magnetic Larmor precession, and is given by:

$$\Omega = \frac{\pi}{5} \frac{MG}{c^2} \frac{1}{r} \cdot \frac{1}{T} \quad - (11)$$

$$= \frac{\pi r g}{5 c^2 T}$$

where

$$g = \frac{MG}{r^2} \quad - (12)$$

is the acceleration due to gravity. Gravity Probe B is $\cancel{6.9571 \times}$ 7.02×10^6 metres above the centre of the Earth, so using:

$$|g| = 9.80665 \, ms^{-2} \quad - (13)$$
$$c = 2.997925 \times 10^8 \, ms^{-1} \quad - (14)$$
$$T = 8.640 \times 10^4 \, \text{seconds}$$

gives

$$\Omega = 5.570 \times 10^{-15} \, rads^{-1} - (15)$$

This must be converted to radians per year as follows:

3)

$$\Omega = 365.25 \times 3600 \times 24 \times 5.570 \times 10^{-15} \text{ rad per year}$$
$$= 1.609 \times 10^{-7} \text{ radians per year} - (16)$$

Finally: $1 \text{ radian} = 2.06271 \times 10^{5} \text{ arc seconds} - (17)$

So $\boxed{\Omega = 3.32 \times 10^{-2} \text{ arc seconds per year}} - (18)$

assuming that Gravity Probe B orbits is such a way that

$$\underline{r} \cdot \underline{k} = 0. - (19)$$

This means that the orbit is perpendicular to the axis of rotation of the earth if eqn. (19) is true. The experimental result from Gravity Probe B is:

$$\Omega (\text{exptl.}) = 4.09 \times 10^{-2} \text{ arc seconds a year}$$
$$- (19)$$

This assumes that the experimental result is seen correctly extracted from the noise. The geodedic precession from Gravity Probe B is

$$\Omega (\text{geodedic}) = 6.6144 \text{ arc seconds a year}$$
$$- (20)$$

and it is advantageous to attempt to calculate this geodedic precession in a closely analogous manner to Lense Thiring precession. This will be the subject of the next note.

—————