344(4): Perihelion Precession and Gravitomagnetic Torque

Consider the torque between the gravitomagnetic dipole moment of a planet in orbit about the sun, and the gravitomagnetic field of the sun:

$$\underline{Tq} = \underline{m}_g \times \underline{\Omega}_{sun} \quad -(1)$$

$$= \frac{1}{2}\underline{L} \times \underline{\Omega}_{sun}$$

where $\underline{L}$ is the angular momentum of the planet in orbit. Consider the sun to be a rotating sphere with the gravitomagnetic field:

$$\underline{\Omega}_{sun} = \frac{2G}{c^2 r^3}\left(\underline{L}_s - 3\left(\underline{L}_s \cdot \frac{\underline{r}}{r}\right)\frac{\underline{r}}{r}\right) \quad -(2)$$

where r is the radius of the sun, and $\underline{L}_s$ is its angular momentum about the axis of rotation. Eq. (2) is the same as that used in Note 344(1) for the Lense–Thirring effect.

Assume that:

$$\underline{r} \perp \underline{L}_s \quad -(3)$$

then

$$\underline{\Omega}_{sun} = \frac{2G}{c^2 r^3}\underline{L}_s \quad -(4)$$

and

$$\boxed{\underline{Tq} = \frac{G}{c^2 r^3}\,\underline{L} \times \underline{L}_s} \quad -(5)$$

2) In order for this torque to exist, $\underline{L}$ and $\underline{L}_s$ must not be parallel.

From the site:

solarscience.msfc.nasa.gov/sunturn.shtml

the sun rotates on its axis once in 27 days. The rotation axis is tilted by 7.25° from the axis of the Earth's orbit, so the torque (5) is non-zero.

As in Note 344(1):

$$\Omega_{sun} = \frac{MG\omega}{5c^2 r} \quad - (6)$$

where the angular velocity is:

$$\omega = \frac{2\pi}{T} \quad - (7)$$

where T is the time taken for one rotation, 27 days.

So:

$$\boxed{\Omega_{sun} = \frac{\pi}{5} \frac{r_0}{r} \frac{1}{T}} \quad - (8)$$

in radians per second. Here:

$$r_0 = \frac{2MG}{c^2} = 2.95 \times 10^3 \text{ m} \quad - (9)$$

and

$$r = 6.957 \times 10^9 \text{ m} \quad - (10)$$

In one Earth year (365.25 days):

$$\Omega_{sun} = 365.25 \times 24 \times 3600 \frac{\pi}{5} \frac{r_0}{r} \frac{1}{T} \quad - (11)$$

3)

$$= \frac{365.25 \times 24 \times 3600}{27 \times 24 \times 3600} \frac{\pi}{5} \left(\frac{2.95}{6.957} \right) \times 10^{-6}$$

$$= \frac{365.25}{27} \frac{\pi}{5} \left(\frac{2.95}{6.957} \right) \times 10^{-6} \text{ radians per year}$$

As in previous notes for UFT344, the Larmor precession frequency is:

$$\omega_L = \frac{g_{eff}}{2} \Omega_{sun} \quad -(12)$$

where g_{eff} is the effective gravitomagnetic Landé factor.

Therefore:

$$\omega_L = \frac{365.25}{27} g_{eff} \frac{\pi}{10} \left(\frac{2.95}{6.957} \right) \times 10^{-6} \text{ radians per year}$$

$$= 1.802\, g_{eff} \times 10^{-6} \text{ radians per year} \quad -(13)$$

The observed perihelion precession of the Earth is:

$$\omega(\text{perihelion}) = 11.45 \text{ arc seconds per year}$$

$$= \frac{11.45}{206,271} \text{ radians per year}$$

$$= 5.551 \times 10^{-5} \text{ radians per year} \quad -(14)$$

4) So $\boxed{g_{eff} = 3.08}$ — (15)

Therefore the observed perihelion precession is explained through the fact that the sun and Earth generate the torque (1), provided that the effective Landé factor is $g_{eff} = 3.08$. This is the gravitomagnetic Landé factor of the planet. In general, every object m orbiting an object M is characterized by its gravitomagnetic Landé factor.

Therefore perihelion precession is a Larmor precession at a frequency:

$$\omega_L = g_{eff} \frac{\pi}{10} \left(\frac{r_0}{r} \right) \frac{1}{T} \quad -(16)$$

In one year or 2π revolution:

$$\omega_L = 365.25 \times 3600 \times 24 \, g_{eff} \frac{\pi}{10} \left(\frac{r_0}{r} \right) \frac{1}{T}$$

radians per Earth year —(17)

This result can be expressed as:

$$\omega_L = \frac{6\pi GM}{ac^2(1-\epsilon^2)} \quad -(18)$$

Here a is the semi major axis and ϵ the eccentricity of the orbit.