344(3): Larmor Precession Theory and Gravitomagnetism.

In electromagnetism, the magnetic dipole moment is defined by:

$$\underline{m} = -\frac{e}{2m}\,\underline{L} \quad -(1)$$

where $\underline{m}$ is the magnetic dipole moment, e the charge and m the mass, for example of an electron, which carries the charge $-e$, and $\underline{L}$ is the orbital angular momentum. A magnetic dipole moment $\underline{m}$ in the magnetic flux density $\underline{B}$ produces the torque:

$$Tq = \underline{m} \times \underline{B} \quad -(2)$$

The effect of such a torque is animated in the animation by Chris Pelkie and myself from Cornell Theory Center, based on my molecular dynamics code on www.aias.us. I was the pioneer of molecular dynamics in the presence of torques induced by certain types of external field.

The animation shows the effect of eq. (2) directly, and shows the precessional motion directly due to this torque.

The Larmor precession frequency is

$$\omega_L = \frac{eg}{2m}\,B \quad -(3)$$

where g is the Landé factor. In the classical physics of for example the Zeeman effect,

$$g = 1 \quad -(4)$$

but in the anomalous Zeeman effect it becomes different from one. This well known theory can be adapted directly

) for gravitomagnetism. The magnetic dipole moment $\underline{m}$ is replaced by the gravitomagnetic dipole moment $\underline{m}_g$, and the angular momentum $\underline{L}$ is that of a system such as a planet in orbit. The charge $-e$ is replaced by a mass m, so:

$$\boxed{\underline{m}_g = \frac{1}{2}\underline{L}} \quad -(5)$$

so the gravitomagnetic dipole moment is directly proportional to the orbital angular momentum, a constant of motion. The orbital angular momentum is defined by:

$$\underline{L} = \underline{r} \times \underline{p} \quad -(6)$$

$$= m\,\underline{r} \times \underline{v}$$

where $\underline{v}$ is the orbital velocity of an object such as a planet, and m is the mass of the planet. Therefore:

$$\underline{m}_g = \frac{m}{2}\,\underline{r} \times \underline{v} \quad -(7)$$

For a planar orbit, $\underline{L}$ is directed in the Z axis perpendicular to the plane of the orbit, but note carefully that this is no longer true for an orbit in three dimensions (see UFT papers on three dimensional orbits). Therefore for a planar orbit:

$$\underline{L} = mrv\,\underline{k} \quad -(8)$$

and

3)

$$\boxed{\begin{aligned}\underline{m_g} &= \frac{1}{2} m r v \underline{k} \\ &= \text{constant of motion}\end{aligned}} \quad -(9)$$

Therefore the orbit of a planet can be thought of as generating a constant gravitomagnetic dipole moment.
This classical theory can be developed to an ECE2 type of relativistic theory. Note that the magnitude of the gravitomagnetic dipole moment is:

$$m_g = \frac{1}{2} m r v \quad -(10)$$

and is directly proportional to the orbital velocity. Here $\underline{r}$ is the vector joining a mass m orbiting a mass M, for example the Earth around the Sun.
A torque is formed between the gravitomagnetic dipole moment $\underline{m_g}$ and a gravitomagnetic field:

$$\boxed{\underline{Tq} = \underline{m_g} \times \underline{\Omega}} \quad -(11)$$

and this results in the gravitomagnetic Larmor precession frequency:

$$\boxed{\omega_g = \frac{1}{2} g_m \Omega} \quad -(12)$$

4) Here $\underline{\Omega}$ is the gravitomagnetic field in radians per second.

Therefore any orbital motion is accompanied by a Larmor precession, specifically a gravitomagnetic Larmor precession. The Lense Thirring precession is an example of this. In the Lense Thirring precession the gravitomagnetic field is due to a spinning sphere, the Earth for example. The magnetic equivalent of this is the well known magnetic flux density of a charged and spinning sphere. The gravitomagnetic dipole moment in the Lense Thirring effect is the gyroscope on board a spacecraft such as Gravity Probe B.

In the dipole approximation in electrodynamics, the magnetic flux density due to a magnetic dipole moment $\underline{m}$ is:

$$\underline{B} = \frac{\mu_0}{4\pi r^3}\left(3\left(\underline{m}\cdot\hat{\underline{r}}\right)\hat{\underline{r}} - \underline{m}\right) \quad -(13)$$

where

$$\hat{\underline{r}} = \underline{r} / r \quad -(14)$$

and μ_0 is the S.I. permeability in vacuo.

In direct analogy:

$$\underline{\Omega} = \frac{\mu_{0g}}{4\pi r^3}\left(3\left(\underline{m}_g\cdot\hat{\underline{r}}\right)\hat{\underline{r}} - \underline{m}_g\right) \quad -(15)$$

) where

$$\mu_{0g} = \frac{4\pi G}{c^2} \quad -(16)$$

, the gravitomagnetic permeability in vacuo, and

$$\underline{m_g} = \frac{1}{2} m r v \underline{k} \quad -(17)$$

Therefore the Larmor precession frequency is:

$$\omega_g = \frac{1}{2} g_r |\underline{\Omega}| \quad -(18)$$

$$= \frac{g_r \mu_{0g}}{8\pi r^3} \left| 3\left(\underline{m_g} \cdot \underline{\hat{r}}\right)\underline{\hat{r}} - \underline{m_g} \right|$$

i.e.

$$\omega_g = \frac{g_r G}{2c^2 r^3} \left| 3\left(\underline{m_g} \cdot \underline{\hat{r}}\right)\underline{\hat{r}} - \underline{m_g} \right| \quad -(19)$$

From Eqs. (9) and (19):

$$\omega_g = \frac{g_r v m G}{4c^2 r^2} \left| 3\left(\underline{k} \cdot \underline{\hat{r}}\right)\underline{\hat{r}} - \underline{k} \right| \quad -(20)$$

Units Check

ω_g = rad s^{-1}; $mG/c^2 = m$, $v = m s^{-1}$, $c^2 r^2 = m^3 s^{-2}$

so $\omega_g = \frac{r v}{r^2} = s^{-1}$ ✓

For planar orbits:

$$\underline{k} \cdot \underline{\hat{r}} = 0 \quad -(21)$$

b) so $$\left| 3(\underline{k}\cdot\underline{\hat{r}})\underline{\hat{r}} - \underline{k} \right| = 1 \quad -(22)$$

It follows that:

$$\boxed{\omega_g = \frac{g_r v m G}{4c^2 r^2}} \quad -(23)$$

For orbits around the sun of mass $\underline{M}$:

$$\omega_g = \frac{g_r v \underline{M} G}{4c^2 r^2} \quad -(24)$$

where r is the distance of the planet to the sun.

Therefore an initially elliptical orbit will precess, and this is observed as the precession of the perihelion.