# 344(1): ECE2 Theory of the Lense Thirring Precession

Reference: [1] A. I. Arbab, Astrophys. Space Sci. 330, 61–88, (2010); Google "Gravitomagnetic Field and Precession".

Consider the ECE2 gravitational equations of UFT318 assuming the absence of a gravitomagnetic monopole:

$$\underline{\nabla} \cdot \underline{g} = 4\pi G \rho_m \quad -(1)$$

$$\underline{\nabla} \times \underline{g} + \frac{\partial \underline{\Omega}}{\partial t} = \underline{0} \quad -(2)$$

$$\underline{\nabla} \cdot \underline{\Omega} = 0 \quad -(3)$$

$$\underline{\nabla} \times \underline{\Omega} - \frac{1}{c^2} \frac{\partial \underline{g}}{\partial t} = \frac{4\pi G}{c^2} \underline{J}_m \quad -(4)$$

These are Lorentz covariant structures in a space with finite torsion and curvature. They are structurally identical with the Maxwell Heaviside equations, but the latter are nineteenth century constructs. The correct equations of electromagnetism are the ECE2 equations:

$$\underline{\nabla} \cdot \underline{E} = \rho_e / \epsilon_0 \quad -(5)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(6)$$

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(7)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J}_e \quad -(8)$$

Eqs. (5) – (8) look like the MH equations, but are correctly written in a space with finite torsion and

2) curvature. Here $\underline{E}$ is the electric field strength in volts per metre, $\rho_e$ is the electric charge density, $\epsilon_0$ the (vacuum) free space permittivity, $\underline{B}$ the magnetic flux density in tesla, $\mu_0$ the vacuum permeability and $c$ the speed of light in vacuo. All the equations are written in S.I. units. Eq. (5) is the Coulomb law, eq. (6) is the Gauss law of magnetism, eq. (7) is the Faraday law of induction, and eq. (8) is the Ampère Maxwell law, all written in a space with finite torsion and curvature. Eqs (5) to (8) are rigorously Lorentz covariant.

Similarly, eqs. (1) to (4) are rigorously Lorentz covariant in a space with finite torsion and curvature. Here $\underline{g}$ is the acceleration due to gravity, $G$ is Newton's constant, $\rho_m$ is the mass density, $\underline{\Omega}$ is the gravitomagnetic field, and $J_m$ the density of mass current. It follows that the gravitational vacuum permittivity is:

$$\epsilon_{0g} = \frac{1}{4\pi G} \quad -(9)$$

and that the gravitational vacuum permeability is:

$$\mu_{0g} = \frac{4\pi G}{c^2} \quad -(10)$$

so

$$\epsilon_{0g}\,\mu_{0g} = \frac{1}{c^2} \quad -(11)$$

Similarly:

$$\epsilon_0\,\mu_0 = \frac{1}{c^2} \quad -(12)$$

The Lense Thirring precession is exactly analogous to the effect of the magnetic flux density $\underline{B}$ of a spherical, spinning object on a compass, with

) magnetic dipole moment $\underline{m}$. The spinning object does not have to be spherical, it can be any shape. The gravitomagnetic field of such an object is (1):

$$\underline{\Omega} = \frac{2G}{c^2 r^3}\left(\underline{L} - 3\left(\underline{L}\cdot\frac{\underline{r}}{r}\right)\frac{\underline{r}}{r}\right) \quad -(13)$$

where $\underline{r}$ is the radial vector joining an object to the central object. Here $\underline{L}$ is the angular momentum of the spinning object.

The gravitomagnetic field $\underline{\Omega}$ interacts with a gyroscope in an orbiting spacecraft. The gyroscope is a ring of mass, and is formally equivalent to a magnetic dipole moment. The solution (13) is formally equivalent to the solution for the magnetic flux density $\underline{B}$ of a spinning object such as the earth, acting on a compass needle. The Lense Thirring precession was originally produced by linearizing the Einstein field equation to give a structure formally identical with Eqs. (1) to (4) called the Maxwell-Einstein equations. However, the original Lense Thirring theory is incorrect because it relies on the incorrect Einstein field equation.

Arbab derived the structure (1) to (4) using quaternions, used in analogy with hydrodynamics.

Consider eq. (13) at the equatorial plane, where

$$\underline{r} \perp \underline{L}, \quad -(14)$$

then:

$$\underline{\Omega} = \frac{G}{2c^2}\frac{L}{r^3} \quad -(15)$$

7) Consider the rotating object, such as the Earth, to be a sphere, of angular momentum magnitude:

$$L = \omega I \quad -(16)$$

where the moment of inertia of the sphere is:

$$I = \frac{2}{5} M r^2 \quad -(17)$$

Here $M$ is its mass and $r$ is its radius. Therefore:

$$\Omega = \frac{M G \omega}{5 c^2 r} \quad -(18)$$

where

$$\omega T = 2\pi \quad -(19)$$

where $T$ is the rotational period.

Now use the Newtonian solution of Eq. (1):

$$\underline{F} = m \underline{g} = -\frac{m M G}{r^2} \underline{e}_r, \quad -(20)$$

so

$$g = -\frac{M G}{r^2} \quad -(21)$$

More consistently, eqs. (1) to (4) must be solved simultaneously.

Therefore the magnitude of the gravitomagnetic field is:

$$|\Omega| = \frac{2\pi r |g|}{5 c^2 T} \quad -(22)$$

in units of radians per second, i.e. angular frequency.

) The units of $\Omega$ are clear from the fact that:

$$\Omega = \left(\frac{mG}{5c^2 r}\right)\omega. \quad -(23)$$

Finally the Lense-Thirring precession is defined in the literature as:

$$\Omega_{LT} = \left|\frac{\Omega}{2}\right| \quad -(24)$$

in radians per second.

Eq. (24) appears to be a definition, perhaps introduced in the early years of the subject.

<u>Experimental Data</u>

From eqs. (22) and (24), the Lense-Thirring precession for the earth is:

$$\Omega_{LT} = \frac{\pi r |g|}{5c^2 T} \quad -(25)$$

For a Foucault pendulum on the surface of the Earth, the radius of the earth can be used, and the earth's diurnal interval $T$ of 24 hours. Therefore:

$$r = R = 6371 \text{ km} = 6.371 \times 10^6 \text{ m}$$

$$|g| = 9.80665 \text{ m s}^{-2}$$

$$c = 2.997925 \times 10^8 \text{ m s}^{-1}$$

$$T = 24 \times 3600 = 8.640 \times 10^4 \text{ s}$$

Therefore

$$\boxed{\Omega_{LT} = 5.055 \times 10^{-15} \text{ rad s}^{-1}} \quad -(26)$$

for one <u>day at</u> the equator.

6) This precession occurs in addition to the geodetic precession. The Earth's equatorial gravitomagnetic field is:

$$\boxed{\Omega = 2\Omega_{LT} = 1.011 \times 10^{-14} \text{ rad s}^{-1}} \quad -(27)$$

In the fast spinning pulsar PSR J1748 – 2446ad the gravitomagnetic field is:

$$\boxed{\Omega_{pulsar} = 1043.0 \text{ radians per second}} \quad -(28)$$

and is easily observable, thus verifying ECE2 theory because for ECE2, Eq. (28) is an exact result. In the Newtonian theory, the only equation available is:

$$\underline{\nabla} \cdot \underline{g} = 4\pi G \rho_m \quad -(29)$$

and there is no gravitomagnetic field in Newton's theory.