344(5): Thomas Precession as a Larmor Precession

As in UFT110, the Thomas angular velocity is

$$\Omega_T = \omega \left(1 - \frac{v^2}{c^2} \right)^{-1} \quad -(1)$$

where

$$v = r\omega \quad -(2)$$

and is the result of rotating a Minkowski metric at the angular frequency:

$$\omega = \frac{d\theta}{dt} = \frac{v}{r}. \quad -(3)$$

The gravitomagnetic ECE2 field equations are based on a Minkowski metric in a space with finite torsion and curvature, so they are Lorentz covariant and automatically relativistic. The relativistic Thomas precession theory therefore applies.

Consider the torque produced by the gravitomagnetic field of the sun and the earth's gravitomagnetic dipole moment. This torque produces a Larmor precession in the orbit of the earth, which can be well approximated by a circle. A Minkowski frame fixed in the Earth rotates in a circle, and this produces Thomas precession.

So the Larmor precession and the Thomas precession are the same in this approximation, that of regarding the Earth's orbit as being approximately circular.

In the dipole approximation the gravitomagnetic field

) of the sun is:

$$\underline{\Omega}_{sun} = \frac{MG}{c^2R^3}\left(3\left(\underline{m}_g \cdot \frac{\underline{R}}{R}\right)\frac{\underline{R}}{R} - \underline{m}_g\right) \quad -(4)$$

The field at the Earth is defined by (4) the earth to sun distance $\underline{R}$. The gravitomagnetic dipole moment of the sun is

$$\underline{m}_g = \frac{1}{2}\underline{L} \quad -(5)$$

where $\underline{L}$ is the sun's angular momentum:

$$\underline{L} = \underline{r}_s \times \underline{p}_s \quad -(6)$$
$$= M v_s r_s$$

where v_s is the orbital velocity of the sun, M is its mass, and r_s is its radius.

If it is assumed for the sake of simplicity that:

$$\underline{m}_g \cdot \frac{\underline{R}}{R} = 0 \quad -(7)$$

then

$$\Omega_{sun} = |\underline{\Omega}_{sun}| = \frac{GL}{2c^2R^3} = \frac{1}{2}\left(\frac{MG}{c^2}\right)\frac{v_s r_s}{R^3} \quad -(8)$$

in radians per second. The torque produced by $\underline{\Omega}_g$ and the gravitomagnetic dipole moment of the Earth is:

$$\underline{T_q} = \underline{m}_g(\text{earth}) \times \underline{\Omega}_{sun} \quad - (9)$$

where $\underline{m_g}(\text{earth})$ is the gravitomagnetic dipole moment of the earth:

$$\underline{m}_g(\text{earth}) = \frac{1}{2}\underline{L}(\text{earth}) \quad - (10)$$

here $\underline{L}(\text{earth})$ is the orbital angular momentum of the earth. As in previous notes the torque (9) produces the Larmor precession frequency:

$$\Omega_L = \frac{1}{2} g_L \Omega_{sun} \quad - (11)$$

i.e.

$$\Omega_L = \frac{g_L}{4}\left(\frac{MG}{c^2}\right)\frac{v_s r_s}{R^3} \quad - (12).$$

where g_L is the effective Landé factor, or more accurately the gravitomagnetic Landé factor.

Equating eqs (1) and (12):

$$\Omega_T = \Omega_L \quad - (13)$$

so

$$\omega\left(1 - \frac{v^2}{c^2}\right)^{-1} = g_L\left(\frac{MG}{4c^2}\right)\frac{v_s r_s}{R^3} \quad - (14)$$

and the effective Landé factor can be found.

The Thomas precession per earth year is the

+) precession of the orbit after a revolution of 2π radians has been completed. This precession is known experimentally to be:

$$\Omega(\text{exp}) = \frac{6\pi MG}{ac^2(1-\epsilon^2)} \quad -(15)$$

for an elliptical orbit. In the approximation of a circular orbit:

$$\Omega(\text{exp}) = \frac{6\pi MG}{Rc^2} \quad -(16)$$

for a revolution of 2π.

The Thomas precession for a revolution of 2π is

$$\Omega_T = \omega\left(1-\frac{v^2}{c^2}\right)^{-1} \quad -(17)$$

$$= \frac{v}{R}\left(1-\frac{v^2}{c^2}\right)^{-1}$$

Here:

$$r_0 = \frac{2MG}{c^2} = 2.95 \times 10^3 \text{ m}, \quad -(18)$$

$$R = 1.496 \times 10^9 \text{ m} \quad -(19)$$

and

$$v = 3 \times 10^4 \text{ m s}^{-1} \quad -(20)$$

if v is assumed to be the orbital velocity of the Earth in orbit around the sun. $\quad -(21)$

It follows that:

$$\Omega(\text{exp}) = 5.916 \times 10^{-6} \text{ radians in } 2\pi$$

and

$$\Omega_T = 2.005 \times 10^{-5} \text{ radians in } 2\pi$$

This is quite good agreement. The perihelion precession is approximately a Thomas precession.

If it is assumed that the velocity of the Thomas precession is the effective velocity v_{eff}, then, for

$$v_{eff} << c \quad -(22)$$

$$v_{eff} = 5.916 \times 10^{-6} R \quad -(23)$$
$$= 8.850 \times 10^{4} \, m s^{-1}$$

From eq. (21):

$$\Omega(\text{exp}) = 0.2951 \, \Omega_T \quad -(24)$$

Therefore in eq. (14):

$$\Omega(\text{exp}) = 0.2951 \left(\frac{mG}{4c^2} \right) \frac{v_s r_s}{R^3} \gamma_L \quad -(25)$$

$$= 5.916 \times 10^{-5}$$

The orbital velocity of the sun is:

$$v_s = 7.189 \times 10^{3} \, km/hr = 1.997 \times 10^{3} \, m s^{-1}$$

and the radius of the sun is:

$$r_s = 6.957 \times 10^{8} \, m \quad -(26)$$

Therefore:

$$\gamma_L = 1.638 \times 10^{8} \quad -(27)$$