342(4): Complete Details of the Newtonian Theory of Light Deflection due to Gravitation

These complete details are needed for an understanding that light deflection depends only on M, the mass of the attracting object. On the classical level these details are the same for any M, including the gravitational mass.

The starting point is the expression for velocity in plane polar coordinates, (r, θ):

$$v_N^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 \quad -(1)$$

On the Newtonian level the orbit is a conic section:

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(2)$$

where α is the half right latitude and ϵ the eccentricity.

Therefore:

$$\frac{dr}{d\theta} = \frac{\epsilon r^2 \sin\theta}{\alpha} \quad -(3)$$

and

$$\cos\theta = \frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right) \quad -(4)$$

From a lagrangian analysis:

$$\frac{d\theta}{dt} = \frac{L}{mr^2} \quad -(5)$$

where L is the angular momentum, a constant of motion. Now use:

$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} \quad -(6)$$

2) From these equations:

$$v_N^2 = \frac{L^2}{m}\left(\frac{1}{r^2} + \left(\frac{\epsilon}{d}\right)^2 \sin^2\theta\right) - (7)$$

Now use:

$$\sin^2\theta = 1 - \cos^2\theta - (8)$$

to find that:

$$v_N^2 = \frac{L^2}{m}\left(\frac{1}{r^2} + \left(\frac{\epsilon}{d}\right)^2\left(1 - \frac{1}{\epsilon^2}\left(\frac{d}{r} - 1\right)^2\right)\right) - (9)$$

$$= \frac{L^2}{m}\left(\frac{\epsilon^2 - 1}{d^2} + \frac{2}{dr}\right)$$

Now use:

$$L^2 = m^2 MG, - (10)$$

a result of Newtonian dynamics, to find that:

$$\boxed{v_N^2 = MG\left(\frac{\epsilon^2 - 1}{d} + \frac{2}{r}\right)} - (11)$$

It is seen that m has cancelled out of the calculation, leaving only M.

At closest approach:

$$r = R_0 = \frac{d}{1+\epsilon} - (12)$$

because

$$\cos\theta = 1, \quad \theta = 0. - (13)$$

3).

So at closest approach:

$$d = R_0 (1+\epsilon) \quad -(14)$$

and

$$v_N^2 = \frac{MG}{R_0}\left(\frac{\epsilon^2-1}{(\epsilon+1)} + 2\right)$$

$$= \frac{MG}{R_0}(\epsilon+1) \quad -(15)$$

The angle of deflection is:

$$\Delta\zeta = \frac{2}{\epsilon} \quad -(16)$$

For an orbit of very high eccentricity, such as that of a photon grazing the sun:

$$\epsilon \gg 1 \quad -(17)$$

so:

$$\boxed{\Delta\zeta = \frac{2MG}{R_0 v_N^2}} \quad -(18)$$

The experimental result is:

$$\Delta\zeta = \frac{4MG}{R_0 c^2} \quad -(19)$$

Therefore the <u>Newtonian</u> theory <u>fails</u> completely. The Newtonian theory is based on a classical hamiltonian and lagrangian:

$$H = T + U \quad -(20)$$

$$\mathcal{L} = T - U \quad -(21)$$

where T is the kinetic energy and U the potential energy. In ECE2 special relativity:

$$H = E + U \quad -(22)$$

and

$$\mathcal{L} = -\frac{mc^2}{\gamma} - U \quad -(23)$$

where

$$E = \gamma mc^2 \quad -(24)$$

is the total relativistic energy and where:

$$\gamma = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} \quad -(25)$$

is the Lorentz factor.

It was shown in UFT328 that simultaneous solution of eqs. (22) and (23) lead to precession of the perihelion of the orbit, as observed experimentally. The Newtonian theory does not lead to perihelion precession.

The metric corresponding to eqs. (22) and (23) is deduced from the infinitesimal line element:

$$c^2 d\tau^2 = (c^2 - v_N^2) dt^2 \quad -(26)$$

where τ is the proper time. The Lorentz factor is defined by:

5)

$$\gamma = \frac{dt}{d\tau} = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} \quad -(27)$$

directly from the line element.

Note carefully that the Lorentz factor is defined by the Newtonian velocity (1) for a planar orbit.

In Newtonian dynamics:

$$\underline{F} = \frac{d\underline{p}_N}{dt} = m\frac{d\underline{v}_N}{dt} \quad -(28)$$

where $\underline{p}_N$ is the classical momentum:

$$\underline{p}_N = m\underline{v}_N \quad -(29)$$

However the force $\underline{F}$ of eq. (28) is covariant only under a Galilean transform. In order to ensure that momentum is covariant under the Lorentz transformation, the momentum must be defined as:

$$\underline{p} = \gamma m \underline{v}_N \quad -(30)$$

$$= m\underline{v}$$

Here $\underline{p}$ is the relativistic momentum and $\underline{v}$ is the relativistic velocity:

$$\boxed{\underline{v} = \gamma \underline{v}_N} \quad -(31)$$

These concepts are difficult to grasp, and special relativity is highly counter intuitive. The equations must be followed rather than intuition.

From eqs. (30) and (27) it follows that:

$$p^2 c^2 = \gamma^2 m^2 c^4 \left(1 - \frac{1}{\gamma^2}\right)$$

$$= E^2 - E_0^2 \quad -(32)$$

i.e.

$$\boxed{E^2 = p^2 c^2 + m^2 c^4} \quad -(33)$$

This is the Einstein energy equation. Note carefully that in eq. (33):

$$E = \gamma m c^2, \quad \underline{p} = \gamma m \underline{v}_N, \quad -(34)$$

and

$$E_0 = m c^2 \quad -(35)$$

, the rest energy. This does not exist in classical physics.

Note carefully also that the Einstein energy equation is based on:

$$\underline{v} = \gamma \underline{v}_N \quad -(36)$$

So eq. (36) is the Einstein energy equation. It can be written as:

$$\boxed{v_N^2 = \frac{v^2}{1 + \frac{v^2}{c^2}}} \quad -(37)$$

7) and this is the tried result of Lorentz covariance.

Recall that ECE2 is a theory developed in a space with finite T and R, but which is Lorentz covariant. Therefore ECE2 unifies special and general relativity.

The observed particle velocity in a relativistic ECE2 orbit is $\underline{v}$. A particle of mass m orbits M at $\underline{v}$. If it is assumed that:

$$v^2 \rightarrow c^2 \quad -(38)$$

then for eq. (37), the Einstein energy equation,

$$v_N^2 \rightarrow \frac{c^2}{2} \quad -(39)$$

This means that the classical angle of deflection, eq. (18), becomes:

$$\boxed{\Delta\phi \xrightarrow[v_N^2 \rightarrow c^2/2]{} \frac{4MG}{c^2 R_0}} \quad -(40)$$

which is exactly the experimental result, QED.

The Einstein energy equation gives the exactly correct deflection due to gravitation. This is a new and important result in physics.

The result (39) imposes an upper bound on the Lorentz factor:

$$\gamma = \left(1 - \frac{1}{2}\right)^{-1/2} = \sqrt{2} \quad -(41)$$

The usual interpretation of special relativity

) is that:

$$v_N \to ? \; c \quad -(42)$$

and that

$$\gamma \to ? \; \infty \quad -(43)$$

This leads to the false claim that a particle travelling at the speed of light must be massless. The old interpretation leads to the further difficulty that for a particle travelling at the speed of light:

$$E = \hbar\omega = \gamma m c^2 = ? \; 0 . \infty c^2 \quad -(44)$$

which is mathematically indeterminate.

Another obvious difficulty with the old theory is that from eq. (18):

$$\Delta \xi \xrightarrow[v_N \to \infty]{} ? \; 0 \quad -(44)$$

so that the old interpretation collapses.

As shown in UFT150 and UFT155, and in many other papers, the old Einstein theory of general relativity collapses. This leads to the only possible interpretation:

$$v_N^2 \to \frac{c^2}{2}, \quad \gamma \to \sqrt{2} \quad -(45)$$

This means that a particle with finite mass m can propagate at c. This is actually well known experimentally in accelerating electrons to c.

The next note will deal with the calculation of averaged mass in a gravitational gas.
