342(9): Basic Definitions

Relativistic Orbit

This is known experimentally to be:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(1)$$

where

$$x = 1 - \frac{3MG}{c^2\alpha} \quad -(2)$$

for

$$1 - x \rightarrow 0. \quad -(3)$$

It is assumed that orbit (1) is due to the ECE2 lagrangian of UFT328:

$$\mathcal{L} = -\frac{mc^2}{\gamma} - U \quad -(4)$$

and hamiltonian

$$H = \gamma mc^2 + U \quad -(5)$$

The lagrangian (4) gives:

$$L = \gamma m r^2 \dot{\theta} \quad -(6)$$
$$= \text{constant}$$

where L is the relativistic angular momentum.

Non Relativistic Orbit

This is the conic section:

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(7)$$

and is due to the lagrangian:

$$\mathcal{L}_0 = \frac{1}{2}mv_0^2 - U \quad -(8)$$

2) The Lagrangian (8) gives:

$$L_0 = mr^2 \dot{\theta}_0 = \text{constant} \quad -(9)$$

Note carefully that both L and L_0 are constants of motion. They are related by:

$$\frac{L}{L_0} = \gamma \frac{\dot{\theta}}{\dot{\theta}_0} \quad -(10)$$

$$= \text{constant}$$

The assumption: $L = \gamma L_0 \quad -(11)$

also assumes: $\dot{\theta} \overset{?}{=} \dot{\theta}_0 \quad -(12)$

but in general: $\dot{\theta} = \left(\frac{1}{\gamma} \frac{L}{L_0} \right) \dot{\theta}_0 \quad -(13)$

Therefore in the equation:

$$L^2 \left(\frac{1}{r^2} + \frac{x^2 \epsilon^2 \sin^2(x\theta)}{d^2} \right) = \frac{L_0^2 \left(\frac{1}{r^2} + \frac{\epsilon^2}{d^2} \sin^2\theta \right)}{1 - \left(\frac{L_0}{mrc} \right)^2 \left(\frac{1}{r^2} + \frac{\epsilon^2}{d^2} \sin^2\theta \right)} \quad -(14)$$

both L and L_0 are constants.