342(1): Definition of Mass and Acceleration due to Gravity with an Ensemble of Gravitons

Consider the equivalent of the Coulomb law in the gravitational field equations of ECE2:

$$\underline{\nabla} \cdot \underline{g} = \underline{\kappa} \cdot \underline{g} = 4\pi G \rho_m \quad -(1)$$

as in UFT318. Here $\rho_m(\underline{r})$ is the mass density, which is defined by:

$$\rho_m(\underline{r}) = \sum_{i=1}^{N} m_i \, \delta(\underline{r} - \underline{r}_i) \quad -(2)$$

where m_i is the mass of the graviton. The total mass $\underline{M}$ of any object is:

$$\underline{M} = \int \rho_m \, dV \quad -(3)$$

where V is its volume. So according to ECE2, a mass $\underline{M}$ is an ensemble of graviton mass.

Using the divergence theorem for any vector $\underline{V}$:

$$\oint_S \underline{V} \cdot \underline{n} \, dA = \int_V \underline{\nabla} \cdot \underline{V} \, d^3 r \quad -(4)$$

it follows that:

$$\oint_S \underline{g} \cdot \underline{n} \, dA = 4\pi G \int_V \rho_m \, dV \quad -(5)$$

2) The right hand side of eq. (5) is the limit of:

$$\int \rho_m \, dV = \sum_{i=1}^{N} m_i \quad -(6)$$

So

$$\oint_S \underline{g} \cdot \underline{n} \, dA = 4\pi G \sum_{i=1}^{N} m_i \quad -(7)$$

In direct analogy, the ECE2 Coulomb law is:

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(8)$$

where ρ is the charge density. The integral form of eq. (8) is:

$$\oint_S \underline{E} \cdot \underline{n} \, dA = \frac{1}{\epsilon_0} \int_V \rho \, d^3 r \quad -(9)$$

where

$$dV = d^3 r \quad -(10)$$

The charge density is defined as:

$$\rho = \sum_{i=1}^{N} e_i \, \delta(\underline{r} - \underline{r}_i) \quad -(11)$$

where $\delta(\underline{r} - \underline{r}_i)$ is the Dirac delta function.

The electric field strength of electrostatics is:

$$\underline{E}(\underline{r}) = \frac{1}{4\pi\epsilon_0} \int \rho(\underline{r}') \frac{\underline{r} - \underline{r}'}{|\underline{r} - \underline{r}'|^3} \, d^3 r' \quad -(12)$$

Substituting (11) into (12) gives:

3)

$$\underline{E}(\underline{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} e_i \frac{\underline{r} - \underline{r}_i}{|\underline{r} - \underline{r}_i|^3} \quad -(13)$$

In direct analogy:

$$\underline{g}(r) = -G \sum_{i=1}^{n} m_i \frac{\underline{r} - \underline{r}_i}{|\underline{r} - \underline{r}_i|^3} \quad -(14)$$

where the minus sign in gravitational theory is a matter of convention, denoting attraction.

Using:

$$\rho_m(\underline{r}) = \sum_{i=1}^{n} m_i \, \delta(\underline{r} - \underline{r}_i) \quad -(15)$$

it is concluded that:

$$\underline{g}(r) = -G \int \rho_m(\underline{r}') \frac{\underline{r} - \underline{r}'}{|\underline{r} - \underline{r}'|^3} d^3 r' \quad -(16)$$

and

$$\underline{M} = \int \rho_m \, d^3 r \quad -(17)$$

For one graviton interacting with a mass m_1:

$$\underline{F} = m_1 \underline{g} = -\frac{m m_1 G}{r^2} \underline{e}_r \quad -(18)$$

where m is the mass of the graviton. Therefore for a mass density (15) of gravitons interacting with

t) with a mass m_1:

$$\underline{F} = m_1 \underline{g} = -m_1 G \int \rho_m(\underline{r}') \frac{\underline{r}-\underline{r}'}{|\underline{r}-\underline{r}'|^3} d^3 r' \quad -(19)$$

Light deflection due to gravitation can be thought of in terms of eq. (19), where m_1 is photon mass, and where the mass of the attracting object, such as the sun, is a linear superposition of gravitons of mass m_i and mass density:

$$\rho_m(\underline{r}) = \sum_{i=1}^{n} m_i \delta(\underline{r} - \underline{r}_i) \quad -(20).$$

and mass:

$$\underline{M} = \int \rho_m d^3 r. \quad -(21)$$

Any mass $\underline{M}$ is the mass density of gravitons integrated over the volume occupied by the mass $\underline{M}$.

For a spherical mass M it can be shown that the potential a distance R from the centre of the sphere is

$$\phi = -\frac{MG}{R} \quad -(22)$$

This will be shown in the next note. So the essence of light deflection by gravitation is the interaction of a graviton and photon.