342(6). The Particular Case $\theta = 2\pi$

The basic equation is:

$$v^2 = \frac{mG}{\alpha}\left(1+\epsilon^2+2\epsilon\cos\theta_1\right)$$

$$= \frac{\frac{mG}{\alpha}\left(1+\epsilon^2+2\epsilon\cos\theta\right)}{1-\frac{mG}{c^2\alpha}\left(1+\epsilon^2+2\epsilon\cos\theta\right)} \quad -(1)$$

If

$$\theta = 2\pi \quad -(2)$$

the original ellipse:

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(3)$$

has made one complete revolution, or orbit, and r has returned to the same point. In this case:

$$1+\epsilon^2+2\epsilon\cos\theta_1 = \frac{\alpha}{mG}A \quad -(4)$$

here:

$$A = \frac{\frac{mG}{\alpha}\left(1+\epsilon\right)^2}{1-\frac{mG}{c^2\alpha}\left(1+\epsilon\right)^2} \quad -(5)$$

So:

$$\cos\theta_1 = \frac{1}{2\epsilon}\left(\frac{A\alpha}{mG}-\left(1+\epsilon^2\right)\right) \quad -(6)$$

2) i.e.

$$\cos\theta_1 = \frac{1}{2\epsilon}\left[\frac{(1+\epsilon)^2}{1-\frac{mG}{c^2 d}(1+\epsilon)^2} - (1+\epsilon^2)\right] \quad -(7)$$

$:= x$

and θ_1 is not 2π.

In the limit:

$$\frac{mG}{c^2 d} \to 0 \quad -(8)$$

it follows that:

$$\cos\theta_1 = 1 \quad -(9)$$

and in this limit

$$\theta_1 = \theta = 2\pi \quad -(10)$$

Therefore it is seen that there is movement of the orbital point in general.

From eq. (7):

$$\theta_1 = \cos^{-1} x \quad -(11)$$

$$= \theta + \Delta\theta = 2\pi + \Delta\theta$$

so

$$\boxed{\Delta\theta = \cos^{-1} x - 2\pi} \quad -(12)$$

3) This is the angle of precession for one complete orbit of 2π.

In general:

$$x = \frac{1}{2\epsilon}\left(\frac{1+\epsilon^2+2\epsilon\cos\theta}{1-\frac{mG}{c^2\alpha}\left(1+\epsilon^2+2\epsilon\cos\theta\right)} - \left(1+\epsilon^2\right)\right) \quad -(13)$$

$$= \cos\theta_1(\theta)$$

Therefore the precession at any point in the orbit is

$$\boxed{\begin{aligned}\Delta\theta &= \cos^{-1}x - \theta \\ &= \theta_1 - \theta\end{aligned}} \quad -(14)$$

The orbit in general is:

$$r = \frac{\alpha}{1+\frac{1}{2}\left(\frac{1+\epsilon^2+2\epsilon\cos\theta}{1-\frac{mG}{c^2\alpha}\left(1+\epsilon^2+2\epsilon\cos\theta\right)} - \left(1+\epsilon^2\right)\right)} \quad -(15)$$

$$\xrightarrow[\frac{mG}{c^2\alpha}\to 0]{} \frac{\alpha}{1+\epsilon\cos\theta}$$