342(3): Light Deflection by a Gravitational Field

In the first approximation consider a Newtonian theory in which the orbit is the conic section:

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(1)$$

where α is the half right latitude and ϵ the eccentricity in plane polar coordinates (r, θ). The Newtonian velocity of the orbit is:

$$v_N^2 = MG\left(\frac{2}{r} - \frac{1}{a}\right) \quad -(2)$$

where M is the mass of the attracting object, such as the sun:

$$M = 1.989 \times 10^{30} \text{ kg} \quad -(3)$$

The semi major axis for a hyperbola is:

$$a = \frac{\alpha}{1-\epsilon^2} \quad -(4)$$

At closest approach:

$$r = R_0, \quad \cos\theta = 1 \quad -(5)$$

so

$$\alpha = R_0(1+\epsilon) \quad -(6)$$

It follows that:

$$v_N^2 = \frac{MG}{R_0}\left(2-(1-\epsilon)\right) \quad -(7)$$

$$= \frac{MG}{R_0}(1+\epsilon)$$

In light deflection by the sun:

$$\epsilon \gg 1 \quad -(8)$$

so:

$$v_N^2 \sim \frac{MG}{R_0}\,\epsilon \quad -(9)$$

The angle of deflection is:

$$\Delta\xi = \frac{2}{\epsilon} \quad -(10)$$

so

$$\Delta\xi = \frac{2MG}{R_0 v_N^2} \quad -(11)$$

and depends only on the attracting mass M.

The experimental value is:

$$\Delta\xi = \frac{4MG}{R_0 c^2} \quad -(12)$$

which is known as "twice Newton". A photon grazing the sun is usually assumed to travel at c for all practical purposes. The relativistic velocity of a particle grazing the sun is:

$$\underline{v} = \gamma \underline{v}_N \quad -(13)$$

where

$$\gamma = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} \quad -(14)$$

the Lorentz factor. It follows that

$$v^2 = \left(1 - \frac{v_N^2}{c^2}\right)^{-1} v_N^2 \quad -(15)$$

so

$$v_N^2 = \frac{v^2}{1 + \frac{v^2}{c^2}} \quad -(16)$$

3) It follows from eq. (16) that

$$v^2 \xrightarrow[v \to c]{} 2 v_N^2 \quad -(17)$$

i.e.

$$v_N^2 \longrightarrow \frac{c^2}{2} \quad -(18)$$

so eq. (11) becomes:

$$\Delta \phi = \frac{4MG}{R_0 c^2} \quad -(19)$$

Q.E.D. This is exactly the experimental result.

The mass of the sun is assumed to be made up of a graviton gas obeying the Planck distribution. In the uncovered Planck distribution the number N of gravitons in a volume $\bar{V}$ is:

$$\frac{N}{\bar{V}} = 2.029 \times 10^{7} T^{3} \quad -(20)$$

The volume of the sun is:

$$\bar{V} = 1.414 \times 10^{27} m^3 \quad -(21)$$

and the surface temperature of the sun is:

$$T = 5.778 \times 10^{3} K \quad -(22)$$

So the number of gravitons in the sun is:

$$N = 5.536 \times 10^{45} \quad -(23)$$

The mass of the sun is:

$$M = N m \quad -(24)$$

+) where m is the graviton mass, so:

$$\boxed{m = 3.59 \times 10^{-16} \text{ kgm}} \quad -(25)$$

The graviton mass is:

$$m = \frac{\langle \hbar\omega \rangle}{\gamma c^2} \quad -(26)$$

where the uncorrected Planck distribution gives:

$$\langle \hbar\omega \rangle = \hbar\omega \frac{x}{1-x} \quad -(27)$$

where

$$x = \frac{\hbar\omega}{kT} \quad -(28)$$

In arriving at eq. (20), it has been assumed that the mass of the sun is ~~a~~ made up of gravitons of all frequencies, so:

$$N = \int_0^\infty \frac{8\pi}{c^3}\left(\frac{f^2 df}{x-1}\right) \quad -(29)$$

where

$$\omega = 2\pi f \quad -(30)$$

so

$$N = \left(\frac{2\zeta(3)}{\pi^2}\right)\left(\frac{k}{c\hbar}\right)^3 T^3 \quad -(31)$$

where k is the Boltzmann constant and T the temperature, $\zeta(3)$ the third zeta function.

5) More accurately eq. (26) is:

$$\langle \hbar\omega \rangle = c^2 \langle \gamma m \rangle \quad -(32)$$

which is the mean energy of one quanta regarded as a Planck oscillator. The mean relativistic energy of the quanta is:

$$\langle E \rangle = \langle \gamma m c^2 \rangle \quad -(33)$$
$$= \langle \hbar\omega \rangle$$

Exactly the same type of theory applies to a gas of photons of mass m.

If this calculation is repeated for a mass of 1 kgm occupying a volume of 1 m^3 at 293 K, the number of quanta is:

$$N = 2.029 \times 10^7 \, T^3 V$$
$$= 5.104 \times 10^{14} \quad -(34)$$

giving a quanta mass of

$$m = 1.96 \times 10^{-14} \text{ kgm} \quad -(35)$$

The number of quanta per unit volume depends on the cube of temperature.
