342(7): General Expression for Precession

The relativistic velocity is:

$$\underline{v} = \frac{d\underline{r}}{d\tau} \quad -(1)$$

In plane polar coordinates:

$$\underline{v} = \frac{d}{d\tau}(r\underline{e}_r)$$

$$= \frac{dr}{d\tau}\underline{e}_r + r\frac{d\underline{e}_r}{d\tau} \quad -(2)$$

$$= \frac{dr}{d\tau}\underline{e}_r + r\frac{d\theta}{d\tau}\underline{e}_\theta$$

Here τ is the proper time.

So: $$v^2 = \left(\frac{dr}{d\tau}\right)^2 + r^2\left(\frac{d\theta}{d\tau}\right)^2 \quad -(3)$$

Now use: $$\frac{dr}{d\tau} = \frac{dr}{d\theta}\frac{d\theta}{d\tau} \quad -(4)$$

to find: $$v^2 = \left(\frac{d\theta}{d\tau}\right)^2\left(r^2 + \left(\frac{dr}{d\theta}\right)^2\right) \quad -(5)$$

$$= \left(\frac{dt}{d\tau}\right)^2\left(\frac{d\theta}{dt}\right)^2\left(r^2 + \left(\frac{dr}{d\theta}\right)^2\right)$$

$$= \gamma^2\left(\frac{d\theta}{dt}\right)^2\left(r^2 + \left(\frac{dr}{d\theta}\right)^2\right)$$

$$= \frac{L_r^2}{m^2 r^4}\left(r^2 + \left(\frac{dr}{d\theta}\right)^2\right)$$

2) where

$$L_r = \gamma \, L \quad -(6)$$

is the relativistic angular momentum, a constant of motion.

Therefore:

$$\frac{L_r^2}{m^2 r^4}\left(r^2+\left(\frac{dr}{d\theta}\right)^2\right) = \frac{\frac{mG}{\alpha}\left(1+\epsilon^2+2\epsilon\cos\theta\right)}{1-\frac{mG}{\alpha c^2}\left(1+\epsilon^2+2\epsilon\cos\theta\right)} \quad -(7)$$

Therefore this equation can be solved for $dr/d\theta$ using computer algebra, using L_r as a constant to be determined by the orbit. For a static ellipse:

$$\frac{dr}{d\theta} = \frac{\epsilon r^2}{\alpha}\sin\theta \quad -(8)$$

From eq. (7):

$$\left(\frac{dr}{d\theta}\right)^2 = \frac{\frac{m^2 r^4 mG}{\alpha L_r^2}\left(1+\epsilon^2+2\epsilon\cos\theta\right)}{1-\frac{mG}{\alpha c^2}\left(1+\epsilon^2+2\epsilon\cos\theta\right)} - r^2 \quad -(9)$$

Eq. (9) is the general equation for $dr/d\theta$ in ECE2 relativity. It is known from UFT328

3) that ECE2 produces a precessing ellipse from a simultaneous solution of the hamiltonian and lagrangian of ECE2.

Therefore eq. (9) must produce a precessing ellipse, because it is based on the same theory.

The two functions (8) and (9) can be graphed and compared at particular points in the orbit. For example after one complete orbit:

$$\theta = 2\pi \quad -(10)$$

and at this point, eq. (8) gives:

$$\frac{dr}{d\theta} = 0 \quad -(11)$$

but eq. (9) gives:

$$\left(\frac{dr}{d\theta}\right)^2 = \frac{\frac{m^2 r^4 MG}{\alpha L_r^2}\left(1+2\epsilon+\epsilon^2\right)}{1-\frac{MG}{\alpha c^2}\left(1+\epsilon^2+2\epsilon\right)} - r^2 \quad -(12)$$

and θ is no longer independent of r. The form of eq. (12) at $\theta = 2\pi$ is:

$$\left(\frac{dr}{d\theta}\right)^2 = Ar^4 - r^2 \quad -(13)$$

so

$$\theta = \int \frac{dr}{(Ar^4 - r^2)^{1/2}} \quad -(14)$$

The next note will develop this analysis in more detail.