341(3): The Corrected Rayleigh Jeans Density of States for Gravitational Radiation.

As in UFT 291, the complete expression for the Rayleigh Jeans density of states is:

$$\frac{dN}{V} = \frac{\omega^2}{\pi^2 c^3} d\omega + \frac{\omega}{\pi^2 c^3} (d\omega)^2 + \frac{(d\omega)^3}{3\pi^2 c^3} \quad -(1)$$

$$:= \frac{1}{V}\left(dN_1 + dN_2 + dN_3 \right)$$

Rayleigh retained only the first term on the right hand side, so

$$\frac{dN}{V}(\text{Rayleigh}) = \frac{\omega^2}{\pi^2 c^3} d\omega \quad -(2)$$

Apparently, Rayleigh assumed that:

$$(d\omega)^3 << (d\omega)^2 << d\omega \quad -(3)$$

However, the correct method is first given in UFT 291 for electromagnetic radiation, adapted here for gravitational radiation.

We have:

$$\frac{dN_2}{V} = \frac{\omega}{\pi^2 c^3} (d\omega)^2 \quad -(4)$$

and

$$\frac{dN_3}{V} = \frac{(d\omega)^3}{3\pi^2 c^3} \quad -(5)$$

Therefore:

$$\left(\frac{dN_2}{V}\right)^{1/2} = \left(\frac{\omega}{\pi^2 c^3}\right)^{1/2} d\omega \quad -(6)$$

and

$$\left(\frac{dN_3}{V}\right)^{1/3} = \frac{d\omega}{(3\pi^2 c^3)^{1/3}} \quad -(7)$$

Therefore:

$$\frac{1}{V^{1/2}} \int (dN_2)^{1/2} = \int \left(\frac{\omega}{\pi^2 c^3}\right)^{1/2} d\omega = \frac{2\omega^{3/2}}{3(\pi^2 c^3)^{1/2}} \quad -(8)$$

and

$$\frac{N_A}{V} = \frac{1}{V}\left(\int (dN_2)^{1/2}\right)^2 = \frac{4\omega^3}{9\pi^2 c^3} \quad -(9)$$

Also:

$$\frac{1}{V^{1/3}} \int (dN_3)^{1/3} = \int \frac{d\omega}{(3\pi^2 c^3)^{1/3}} = \frac{\omega}{(3\pi^2 c^3)^{1/3}}, \quad -(10)$$

So

$$\frac{N_B}{V} = \frac{1}{V}\left(\int (dN_3)^{1/3}\right)^3 = \frac{\omega^3}{3\pi^2 c^3} \quad -(11)$$

so the corrected Rayleigh Jeans density of states is

$$\boxed{\frac{N}{V} = \frac{1}{V}\left(N_1 + N_2 + N_3\right) = \frac{10}{9}\frac{\omega^3}{\pi^2 c^3}} \quad -(12)$$

The Rayleigh Jeans density of states is:

$$\frac{N}{V} = \frac{\omega^3}{3\pi^2 c^3} \quad -(13)$$

) The above method uses the results:

$$\int (dN_2)^{1/2} = N_2^{1/2} \quad -(14)$$

and

$$\int (dN_3)^{1/3} = N_3^{1/3} \quad -(15)$$

In order to prove that:

$$\int (dx)^{1/2} = x^{1/2} \quad -(16)$$

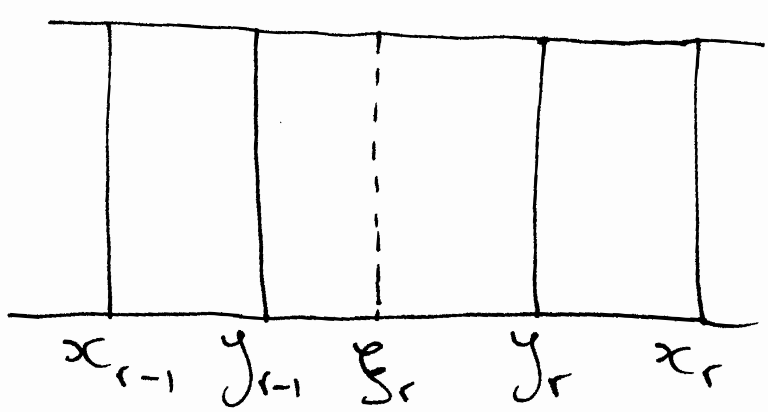
it is necessary to use the definition:

$$\int f(x)\,dx = \lim_{\substack{n \to \infty \\ \delta_r \to 0}} \sum_{r=1}^{n} f(\xi_r)\,\delta_r \quad -(17)$$

(G. Stephenson, "Mathematical Methods for Science Students" Longmans, 1968), in the special case:

$$f(\xi_r) = 1 \quad -(18)$$

Fig(1)



Here:

$$\delta_r = x_r - x_{r-1} \quad -(19)$$

Therefore:

4) $$\int dx = x = (x_1 - x_0) + (x_2 - x_1) + \dots + (x_n - x_{n-1})$$
$$n \to \infty, \quad \delta_r \to 0 \qquad -(20)$$

Similarly:
$$\int (dx)^{1/2} = (x_1 - x_0)^{1/2} + (x_2 - x_1)^{1/2} + \dots + (x_n - x_{n-1})^{1/2}$$
$$:= y_1 - y_0 + y_2 - y_0 + \dots + y_n - y_{n-1}$$
$$n \to \infty, \quad \delta_{r1} \to 0 \qquad -(21)$$

where
$$\delta_{r1} = y_r - y_{r-1} \qquad -(22)$$

Finally define:
$$x^{1/2} = (x_1 - x_0)^{1/2} + \dots + (x_n - x_{n-1})^{1/2} \qquad -(23)$$

so
$$\int (dx)^{1/2} = x^{1/2} \qquad -(23)$$

Q.E.D.

Similarly
$$\int (dx)^{1/3} = x^{1/3} \qquad -(24)$$

and
$$\boxed{\int (dx)^{1/n} = x^{1/n}} \qquad -(25)$$

Therefore the conversed density of states, eq. (12), must be used for gravitational radiation.