341(2): Gravitational Rayleigh Jeans Law and Planck Distribution

The Rayleigh Jeans density of states of gravitational radiation considers gravitational waves in a given radiation volume V. There are modes of radiation. The theory is based on a d'Alembert wave equation:

$$\square f = 0 \quad -(1)$$

so assumes from the outset that the graviton mass is zero. Similarly, the original electromagnetic theory assumes that the photon mass is zero. In a rigorous theory the ECE wave equation must replace eq. (1):

$$(\square + R) q^a_\mu = 0 \quad -(2)$$

where q^a_μ is the Cartan tetrad. In ECE, the gravitational four potential is defined as:

$$Q^a_\mu = Q^{(0)} q^a_\mu \quad -(3)$$

$$= \left(\underline{\Phi}^a, -c\underline{Q}^a \right)$$

In the limit of ECE 2 special relativity:

$$\left(\square + \left(\frac{mc}{\hbar} \right)^2 \right) Q^\mu = 0 \quad -(4)$$

where m is the mass of the graviton. Here:

$$Q^\mu = \left(\underline{\Phi}, c\underline{Q} \right) - (5)$$

2) Therefore eq. (1) is developed into:

$$\left(\Box + \left(\frac{mc}{\hbar} \right)^2 \right) \underline{\Phi} = 0 \quad - (6)$$

and

$$\left(\Box + \left(\frac{mc}{\hbar} \right)^2 \right) \underline{Q} = \underline{0} \quad - (7)$$

Eq. (4) is the Proca equation of the graviton, which in ECE2 is a boson.

For a massless graviton or photon:

$$\omega = c\kappa \quad - (8)$$

where ω is the angular frequency of the radiation, c is the vacuum speed of light and where κ is the magnitude of the wavenumber of the radiation. However, for a graviton or photon with mass, the Einstein energy equation is quantized as follows:

$$E^2 = c^2 p^2 + m^2 c^4 \quad - (9)$$

where

$$E = \hbar\omega = \gamma m c^2 \quad - (10)$$

and

$$p = \hbar \underline{\kappa} = \gamma m \underline{v_0} \quad - (11)$$

Here γ is the Lorentz factor:

$$\gamma = \left(1 - \frac{v_0^2}{c^2} \right)^{-1/2} \quad - (12)$$

and $\underline{v}_0$ the velocity of the graviton or photon, defined by the Lorentz transform:

3) $$c^2 d\tau^2 = (c^2 - v_0^2)\,dt^2 \quad -(13)$$

So $$\gamma = \frac{dt}{d\tau} = \left(1 - \frac{v_0^2}{c^2}\right)^{-1/2} \quad -(14)$$

Q.E.D.

Therefore the Einstein energy equation of the graviton quantizes to:

$$\hbar^2\omega^2 = c^2\hbar^2\kappa^2 + m^2c^4 \quad -(15)$$

so $$\omega^2 = c^2\kappa^2 + \left(\frac{mc^2}{\hbar}\right)^2 \quad -(16)$$

A wave equation such as (16) has solution:

$$\Phi = \Phi_0 \exp\left(-i(\omega t - \underline{\kappa}\cdot\underline{r})\right) \quad -(17)$$

In developing the Rayleigh Jeans law these waves are contained in the volume of radiation V. This is gravitational radiation. The Rayleigh Jeans development is modified by eq. (16) to:

$$\omega^2 = c^2(\kappa^2 + \kappa_0^2) \quad -(18)$$

where $$\kappa_0 = \frac{mc}{\hbar} \quad -(19)$$

The total number of gravitational modes with frequencies less than or equal to ω is the number of ways in which the integers n_1, n_2 and n_3

) can be chosen to obey:

$$n_1^2 + n_2^2 + n_3^2 \leq \frac{\omega^2 L^2}{c^2 \pi^2} \quad -(20)$$

Here

$$\omega^2 = \frac{c^2 \pi^2}{L^2} \left(n_1^2 + n_2^2 + n_3^2 \right) = c^2 \left(\kappa^2 + \kappa_0^2 \right) \quad -(21)$$

So it is assumed that:

$$\kappa^2 + \kappa_0^2 = \frac{\pi^2}{L^2} \left(n_1^2 + n_2^2 + n_3^2 \right) \quad -(22)$$

It follows that the number of gravitational oscillators N in a volume of radiation V is:

$$\frac{N}{V} = \frac{\omega^3}{6 c^3 \pi^2}. \quad -(23)$$

The infinitesimal change from ω to $\omega + d\omega$ is:

$$dN = \frac{V}{6 c^3 \pi^2} \left((\omega + d\omega)^3 - \omega^3 \right) \quad -(24)$$

$$= \frac{V}{\pi^2 c^3} \left(\omega^2 d\omega + \omega (d\omega)^2 + \frac{1}{3} (d\omega)^3 \right)$$

Rayleigh assumed without proof that:

$$dN = \frac{V \omega^2}{\pi^2 c^3} d\omega \quad -(25)$$

In UFT291 this arbitrary assertion was

) corrected.

In this note, eq. (25) is used for the sake of argument only, and corrected in later notes

Therefore:

$$\boxed{\frac{N}{V} = \frac{1}{V}\int dN = \int \frac{\omega^2}{\pi^2 c^3}\, d\omega = \frac{\omega^3}{3\pi^2 c^3}} \quad -(26)$$

This is the density of states of gravitational radiation, the number of gravitational oscillators, N, in a volume V of gravitational radiation.

The graviton is the quantum of gravitational energy:

$$E = \hbar\omega = \gamma m c^2 \quad -(27)$$

where m is the graviton mass. Using the Maxwell-Boltzmann distribution the mean energy is:

$$\langle E \rangle = \left(\frac{x}{1-x}\right)\hbar\omega \quad -(28)$$

where

$$x = \exp\left(-\frac{\hbar\omega}{kT}\right) \quad -(29)$$

where k is Boltzmann's constant and T the temperature. The infinitesimal of energy is:

$$dU = \langle E \rangle\, dN \quad -(30)$$

So:

$$\frac{\bar{U}}{V} = \frac{1}{V}\int dU = \int \langle E \rangle \frac{dN}{V}\, d\omega \quad -(31)$$

From eqs (25) and (31):

$$\frac{\bar{U}}{V} = \int \frac{\hbar\omega^3}{\pi^2 c^3}\left(\frac{x}{1-x}\right) d\omega \quad -(32)$$

This is the energy density of gravitational radiation

If the gravitational radiation consists of all frequencies:

$$\frac{\bar{U}}{V} = \int_0^\infty \frac{\hbar\omega^3}{\pi^2 c^3}\left(\frac{x}{1-x}\right) d\omega$$
$$= \left(\frac{\pi^2}{15}\frac{k^4}{c^3\hbar^3}\right) T^4 \quad -(33)$$

This is the Stefan Boltzmann law for gravitational radiation

The number of gravitons of energy $\hbar\omega$ in a volume V of gravitational radiation is:

$$\frac{N_g}{V} = \int \frac{\langle E \rangle}{\hbar\omega}\frac{dN}{V}\, d\omega \quad -(34)$$

For radiation of all frequencies:

7)
$$N_g = \bar{V} \int_0^\infty \frac{\omega^2}{\pi^2 c^3} \left(\frac{x}{1-x} \right) d\omega$$
$$= \bar{V} \left(\frac{2\zeta(3)}{\pi^2} \left(\frac{k}{c\hbar} \right)^3 \right) T^3 \quad - (35)$$

where $\zeta(3)$ is the third zeta function.

The number of gravitons in a volume $\bar{V}$ of gravitational radiation is proportional to the cube of temperature if the radiation is of all frequencies: black body gravitational radiation.

The energy U of black body gravitational radiation in a radiation volume $\bar{V}$ is proportional to the fourth power of temperature.

Experimental Testing

The gravitational Stefan Boltzmann law means that a very hot star radiates much more energy density $U/\bar{V}$ than a cold star. Therefore there should be much more gravitational radiation from a hot star.