339(1): Development of the γ Factor in Relativistic Quantum Mechanics.

Consider the hamiltonian of ECE general relativity:

$$H = \gamma mc^2 + U. \quad -(1)$$

It follows that:

$$\gamma = \frac{H - U}{mc^2}. \quad -(2)$$

Consider now the de Broglie / Einstein equation:

$$\hbar\omega = \gamma mc^2 \quad -(3)$$

then

$$\gamma = \frac{\hbar\omega}{mc^2}. \quad -(4)$$

It follows that: $\boxed{\hbar\omega = H - U}. \quad -(5)$

In other words:

$$E = \gamma mc^2 = H - U = \hbar\omega \quad -(6)$$

Therefore the quantum of energy $\hbar\omega$ is the quantum of the total relativistic energy.

For a particle at rest:

$$E = H - U \rightarrow mc^2 = \hbar\omega_0 \quad -(7)$$

As in UFT338, the correct equation of relativistic quantum mechanics is:

$$2)\quad (H - e\phi_W - mc^2)\phi = \frac{ie\hbar}{m}\,\underline{\sigma}\cdot\underline{\nabla}\left(\frac{\underline{\sigma}\cdot\underline{p}\,\phi}{1+\gamma}\right) + \dots \qquad -(8)$$

If γ has no dependence on $\underline{r}$, eq. (8) can be written as:

$$(H - e\phi_W - mc^2)\phi = \frac{ie\hbar}{m(1+\gamma)}\,\underline{\sigma}\cdot\underline{\nabla}\left(\underline{\sigma}\cdot\underline{p}\,\phi\right) + \dots \qquad -(9)$$

However if γ depends on $\underline{r}$, there is an additional term from the Leibnitz Theorem, giving:

$$(H - e\phi_W - mc^2)\phi = \frac{ie\hbar}{m(1+\gamma)}\,\underline{\sigma}\cdot\underline{\nabla}\left(\underline{\sigma}\cdot\underline{p}\,\phi\right) \qquad -(10)$$

$$+ \frac{ie\hbar}{m}\,\underline{\sigma}\cdot\underline{\nabla}\left(\frac{1}{1+\gamma}\right)\underline{\sigma}\cdot\underline{p}\,\phi + \dots$$

The first term on the right hand side of eq. (10) gives

$$g = 1+\gamma = 1 + \frac{H-U}{mc^2} = 1 + \frac{\hbar\omega}{mc^2} \qquad -(11)$$

The second term can be developed as:

$$(H - e\phi_W - mc^2)\phi = \frac{ie\hbar}{m}\,\underline{\sigma}\cdot\underline{\nabla}\left(1 + \frac{H-U}{mc^2}\right)^{-1}\underline{\sigma}\cdot\underline{p}\,\phi \qquad -(12)$$

$$) = \frac{ie\hbar^2 c}{m}\underline{\sigma}\cdot\underline{\nabla}\left(H+mc^2-U\right)^{-1}\underline{\sigma}\cdot\underline{p}\,\phi + \dots \quad -(13)$$

The Dirac procedure is to use:

$$H = ?\, mc^2 \quad -(14)$$

However this gives:

$$H_0 = H - mc^2 = ?\ 0 \quad -(15)$$

Eq. (14) happens to give the correct spin orbit fine structure of atomic spectra, but this is not only purely empirical but self contradictory. This is because eq. (14) means:

$$U = ?\ 0 \quad -(16)$$

but the fine structure is obtained from:

$$\left(H - e\phi_W - mc^2\right)\phi = \frac{ie\hbar^2 c}{m}\underline{\sigma}\cdot\underline{\nabla}\left(2mc^2-U\right)^{-1}\underline{\sigma}\cdot\underline{p}\,\phi + \dots$$

$$\sim \frac{ie\hbar}{2mc^2}\underline{\sigma}\cdot\underline{\nabla}\left(1-\frac{U}{2mc^2}\right)^{-1}\underline{\sigma}\cdot\underline{p}\,\phi + \dots \quad -(17)$$

The Thomas factor of $\frac{1}{2}$ is a result of the unphysical equation (14). Clearly, $U = 0$ in eq. (16) and $U \neq 0$ in eq. (17).

The correct equation is equation (12), in which

$$H = \text{constant} \quad -(18)$$

Therefore computer algebra should be used to

4) evaluate:

$$f(r) = \underline{V}\left(1 + \frac{H - U}{mc^2}\right)^{-1} \quad -(19)$$

In the hydrogen atom:

$$U = -\frac{e^2}{4\pi\epsilon_0 r} \quad -(20)$$

This should give an expression:

$$(H - e\phi_W - mc^2)\phi = \frac{ie\hbar}{2mc^2}\,\underline{\sigma}\cdot f(r)\,\underline{\sigma}\cdot\underline{p}\,\phi + \cdots \quad -(21)$$

$$= \frac{ie\hbar}{2mc^2} f_1(r)\,\underline{\sigma}\cdot\underline{r}\,\underline{\sigma}\cdot\underline{p}\,\phi$$

whose real part is the spin orbit term:

$$\mathrm{Re}(H - e\phi_W - mc^2)\phi = -\frac{e\hbar}{2mc^2} f_1(r)\,\underline{\sigma}\cdot\underline{L}\,\phi \quad -(22)$$

Finally, the function:

$$f_1(r) = f(H) \quad -(23)$$

can be chosen to give the correct spin orbit structure.
This means that:

$$f_1(r) = f_1(H) = \frac{1}{8\pi\epsilon_0 m r^3} \quad -(24)$$

and H adjusted to give this result.