538(5): "Artificial" Recovery of Fine Structure

This proceeds as in immediately preceding papers of the UFT series by considering the accurate equation:

$$(H - e\phi_W - mc^2)\phi = \frac{ie\hbar}{m} \underline{\sigma} \cdot \underline{\nabla} \left(\frac{1}{1+\gamma} \right) \underline{\sigma} \cdot \underline{W} \phi + \dots \quad -(1)$$

using:

$$\gamma = \left(1 - \frac{p_0^2}{m^2 c^2} \right)^{-1/2} \quad -(2)$$

with

$$H_0 = \frac{p_0^2}{2m} + U, \quad -(3)$$

$$U = e\phi_W. \quad -(4)$$

Here H_0 is the non-relativistic hamiltonian and $\underline{p_0}$ the non-relativistic momentum.

So as in recent UFT papers:

$$\frac{1}{1+\gamma} = \frac{1}{1 + \left(1 - \frac{p_0^2}{m^2c^2} \right)^{-1/2}} \quad -(5)$$

$$= \frac{1}{1 + \left(1 - \frac{2(H_0 - U)}{mc^2} \right)^{-1/2}}$$

$$\xrightarrow[v \ll c]{} \frac{1}{2}$$

2) This is an "artificial" reinstatement of U in order to obtain spin-orbit fine structure. Obviously, U does not appear in Eq. (2). However, it is always true in non-relativistic classical dynamics that:

$$p_0^2 = 2(H_0 - U) \quad -(6)$$

if there is an interaction potential U present. When an electron interacts with the vacuum:

$$U = e\phi_W = \hbar\Omega^0 c \quad -(7)$$

For:

$$2(H_0 - U) << mc^2 \quad -(8)$$

$$\frac{1}{1+\gamma} \approx \frac{1}{1+1+\frac{H_0-U}{mc^2}} = \frac{1}{2+\frac{H_0-U}{mc^2}}$$

$$= \frac{1}{2\left(1+\frac{H_0-U}{2mc^2}\right)} \sim \frac{1}{2}\left(1-\left(\frac{H_0-U}{2mc^2}\right)\right)$$

$$= \frac{1}{2}\left(1+\frac{U}{2mc^2}\right) - \frac{1}{2}\frac{H_0}{2mc^2} \quad -(9)$$

Therefore:

$$(H - e\phi_W - mc^2)\psi \sim \frac{ie\hbar}{2m}\left(\underline{\sigma}\cdot\underline{\nabla}\left(1+\frac{U}{2mc^2}\right)\underline{\sigma}\cdot\underline{W}\psi - \frac{H_0}{2mc^2}\underline{\sigma}\cdot\underline{\nabla}\,\underline{\sigma}\cdot\underline{W}\psi\right) \quad -(10)$$

3) Eq. (10) restores the spin orbit term:

$$\left(H - e\phi_W - mc^2\right)\phi_{SO} = \frac{ie\hbar}{4m^2c^2}\, \underline{\sigma} \cdot \underline{\nabla} U\, \underline{\sigma} \cdot \underline{W}\, \phi \quad \text{—(11)}$$

and adds another term:

$$\left(H - e\phi_W - mc^2\right)\phi = -\frac{ie\hbar}{2m}\frac{H_0}{2mc^2}\, \underline{\sigma} \cdot \underline{\nabla}\, \underline{\sigma} \cdot \underline{W}\, \phi \quad \text{—(12)}$$

whose real part is:

$$\left(H - e\phi_W - mc^2\right)\phi = \frac{e\hbar H_0}{4m^2c^2}\, \underline{\sigma} \cdot \underline{\nabla} \times \underline{W}\, \phi \quad \text{—(13)}$$

which should appear in ESR and NMR.