338(4): A Simple and Accurate Theory of the Anomalous g Factor of the Electron.

Consider the relevant term of the Dirac theory:

$$(H - e\phi_W - mc^2)\phi = ie\hbar\,\underline{\sigma}\cdot\underline{\nabla}\left(\left(\frac{c^2\,\underline{\sigma}\cdot\underline{W}}{H - e\phi_W + mc^2}\right)\phi\right) + \dots \quad -(1)$$

The rigorous relativistic hamiltonian is:

$$H = \gamma mc^2 + e\phi_W \quad -(2)$$

$$= \gamma mc^2 + U$$

so

$$H - e\phi_W = \gamma mc^2 \quad -(3)$$

and

$$(H - e\phi_W - mc^2)\phi = \frac{ie\hbar}{m}\,\underline{\sigma}\cdot\underline{\nabla}\left(\frac{\underline{\sigma}\cdot\underline{W}\phi}{1+\gamma}\right) + \dots \quad -(4)$$

From the de Broglie / Einstein equation:

$$\gamma mc^2 = \hbar\omega \quad -(5)$$

so

$$\gamma = \frac{\hbar\omega}{mc^2} \quad -(6)$$

where ω is the angular frequency of the electron's matter wave. It follows that:

$$(H - e\phi_W - mc^2)\phi = \frac{ie\hbar}{(\gamma+1)m}\,\underline{\sigma}\cdot\underline{\nabla}\,\underline{\sigma}\cdot\underline{W}\phi + \dots \quad -(7)$$

) By Pauli algebra:

$$\underline{\sigma} \cdot \underline{\nabla}\, \underline{\sigma} \cdot \underline{W} = \underline{\nabla} \cdot \underline{W} + i \underline{\sigma} \cdot \underline{\nabla} \times \underline{W} \quad -(8)$$

i.e. the real and physical part of eq. (7) is:

$$(H - e\phi_W - mc^2)\phi = -\frac{e\hbar}{(\gamma+1)m}\, \underline{\sigma} \cdot \underline{\nabla} \times \underline{W} \quad -(9)$$

It follows that the g factor of the electron is:

$$\boxed{g = 1 + \gamma} \quad -(10)$$

to any precision.

This is the best result of fundamental physics, both ECE2 and standard model.

Therefore:

$$g = 1 + \frac{\hbar\omega}{mc^2} \quad -(11)$$

For a rest electron:

$$g = 1 + \frac{\hbar\omega_0}{mc^2} = 2 \quad -(12)$$

because

$$\hbar\omega_0 = mc^2, \quad -(13)$$

the de Broglie equation.

) However, for a rest electron in contact with the vacuum:

$$\omega_0 \rightarrow \omega_0 + \omega(vac) \quad -(14)$$

so

$$\boxed{g = 2 + \frac{\hbar\omega(vac)}{mc^2}} \quad -(15)$$

This is true for any precision of g, to nine decimal places:

$$g = 2.00231931\!4 \quad -(16)$$

so

$$\boxed{\hbar\omega(vac) = 0.00231931\!4\,mc^2} \quad -(17)$$

This gives the vacuum energy per electron.

For a relativistic electron:

$$\boxed{g = 1 + \frac{\hbar(\omega + \omega(vac))}{mc^2}} \quad -(18)$$

Note carefully that the rigorous eq. (4) of relativistic quantum mechanics means that there is no spin orbit fine structure. So the Dirac approximation collapses entirely when it comes to fine structure.