338(2): Development of the Vacuum W^μ Potential

Consider the Einstein energy equation for a free particle:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(1)$$

where E is the total relativistic energy:

$$E = \gamma m c^2 = \hbar \omega \quad -(2)$$

and where $\underline{p}$ is the relativistic momentum:

$$\underline{p} = \hbar \underline{\kappa} = \gamma m \underline{v_0} . \quad -(3)$$

The Lorentz factor is:

$$\gamma = \left(1 - \frac{v_0^2}{c^2} \right)^{-1/2} \quad -(4)$$

Here $\underline{v}_0$ is the velocity in the observer frame, related to the momentum $\underline{p}_0$ in the observer frame by:

$$p_0 = m \underline{v}_0 . \quad -(5)$$

The presence of a four potential:

$$W^\mu = \left(\frac{\phi_W}{c}, \underline{W} \right) \quad -(6)$$

is represented by the minimal prescription:

$$p^\mu \rightarrow p^\mu - e W^\mu \quad -(7)$$

in electromagnetism, but an equivalent expression in gravitation. So

$$E \rightarrow E - e\phi_W \quad -(8)$$

and

$$\underline{p} \rightarrow \underline{p} - e \underline{W} \quad -(9)$$

For a free particle:

$$H = E \quad -(10)$$

where H is the hamiltonian. The potential energy is:

$$U = e\phi_W \quad (11)$$

So eq. (8) is equivalent to:

$$E \rightarrow H - U \quad (12)$$

i.e

$$H = E + U \quad (13)$$

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Therefore the minimal prescription is very fundamental and transforms eq. (1) into the hamiltonian of ECE2 special relativity:

$$H = (c^2p^2 + m^2c^4)^{1/2} + U \quad (14)$$

This is the same, mathematically, as the hamiltonian of the Sommerfeld atom. The lagrangian of special relativity is

$$\mathcal{L} = -\frac{mc^2}{\gamma} - U \quad (15)$$

and in UFT325 it was shown that simultaneous solution of eqns (14) and (15) leads to orbital precession from special relativity.

By definition:

$$W^\mu = \frac{\hbar}{e}\Omega^\mu \quad (16)$$

where Ω^μ is the spin connection four vector:

$$\Omega^\mu = (\Omega^0, \underline{\Omega}) \quad (17)$$

3) Therefore the minimal prescription means that:

$$E \rightarrow E - e\phi_W = E - \hbar\omega_1 \quad -(18)$$

and

$$\underline{p} \rightarrow \underline{p} - e\underline{W} = \underline{p} - \hbar\underline{\kappa}_1 \quad -(19)$$

From eq. (18):

$$\boxed{H = E + U = \hbar(\omega + \omega_1)} \quad -(20)$$

and from eq. (19):

$$\boxed{\underline{p} = \hbar(\underline{\kappa} + \underline{\kappa}_1)} \quad -(21)$$

Eqs. (20) and (21) introduce the concept of quantization of the potential energy. Usually in quantum mechanics the potential energy is not quantized, for example in the Sommerfeld atom:

$$U = -\frac{e^2}{4\pi\epsilon_0 r}, \quad -(22)$$

which is the potential energy between the electron and proton, assumed to be the Coulomb potential. From eqs (20) and (22):

$$\langle \hbar\omega_1 \rangle = -\frac{e^2}{4\pi\epsilon_0 r} \quad -(23)$$

where $\langle \; \rangle$ denotes "expectation value". In relativistic quantum mechanics the relativistic momentum / energy is quantized:

4)

$$p^\mu = i\hbar \partial^\mu \quad -(24)$$

where

$$p^\mu = \left(\frac{E}{c}, \underline{p} \right), \quad \partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\underline{\nabla} \right) \quad -(25)$$

so

$$E\phi = i\hbar \frac{\partial \phi}{\partial t} \quad -(26)$$

and

$$\underline{p}\,\phi = -i\hbar \underline{\nabla}\phi \quad -(27)$$

From eqs. (1), (26) and (27):

$$\left(\Box + \left(\frac{mc}{\hbar}\right)^2 \right)\phi = 0 \quad -(28)$$

which is a limit of the ECE wave equation:

$$\Box\phi = R\phi \quad -(29)$$

The solution of eq. (28) is:

$$\phi = \exp\left(-i\left(\omega t - \underline{\kappa}\cdot\underline{r}\right)\right) \quad -(30)$$

at instant t and position $\underline{r}$.

In the presence of inversion, the W^μ theory of ECE2 means that the Planck and de Broglie postulates are changed to:

$$E = \hbar(\omega + \omega_1) \quad -(31)$$

$$\underline{p} = \hbar(\underline{\kappa} + \underline{\kappa}_1) \quad -(32)$$

) Eq. (30) can be written as:

$$\phi = \exp(-i x^{\mu} \kappa_{\mu}) \quad -(33)$$

where the position four vector is:

$$x^{\mu} = (ct, \underline{r}) \quad -(34)$$

and the wave four-vector is:

$$\kappa_{\mu} = \left(\frac{\omega}{c}, -\underline{\kappa}\right) \quad -(35)$$

Therefore the contravariant interaction four-vector is:

$$\kappa_1^{\mu} = \left(\frac{\omega_1}{c}, \underline{\kappa_1}\right) \quad -(36)$$

The interaction energy momentum is therefore:

$$\boxed{p_1^{\mu} = eW^{\mu} = \hbar \kappa_1^{\mu}} \quad -(37)$$

The Aharonov Bohm spacetime or "vacuum" (used for convenience) contains energy momentum:

$$p^{\mu}(vac) = \hbar \kappa^{\mu}(vac) \quad -(38)$$

and is described by eq. (28) with:

$$\phi = \phi(vac) \quad -(39)$$

and

$$m = m(vac) \quad -(40)$$

Here $\phi(vac)$ is the vacuum wavefunction

) and $m(vac)$ is the mass of the vacuum particle.

These equations describe the radiative corrections, notably the anomalous g factor of the electron, the Lamb shift and Casimir effect, in an entirely new way.

In this theory, the vacuum has all the properties of material matter. The hamiltonian and lagrangian of the vacuum are:

$$H(vac) = E(vac) + U(vac) \quad -(41)$$

and the lagrangian of the vacuum is:

$$\mathcal{L}(vac) = -\frac{mc^2}{\gamma(vac)} + U(vac) \quad -(42)$$

The interaction of material matter with the vacuum is described by a shift in the matter four vector:

$$\kappa^\mu(\text{matter}) \rightarrow \kappa^\mu(\text{matter}) + \kappa^\mu(\text{vacuum}) \quad -(43)$$

i.e

$$\omega(\text{matter}) \rightarrow \omega(\text{matter}) + \omega(\text{vacuum}) \quad -(44)$$

and

$$\underline{\kappa}(\text{matter}) \rightarrow \underline{\kappa}(\text{matter}) + \underline{\kappa}(\text{vacuum}) \quad -(45)$$

In the above defined notation:

$$\boxed{\begin{aligned} \omega &\longrightarrow \omega + \omega_1 \quad -(46) \\ \underline{\kappa} &\longrightarrow \underline{\kappa} + \underline{\kappa}_1 \quad -(47) \end{aligned}}$$

7) The vacuum can never be absent, i.e. so what is always observed is $\omega + \omega_1$ and $\underline{\kappa} + \underline{\kappa}_1$. It follows that vacuum energy momentum is always present in any experimental measurement of frequency and wave number. What is observed is:

$$\omega \rightarrow \omega + \omega(vac) \quad -(48)$$

$$\underline{\kappa} \rightarrow \underline{\kappa} + \underline{\kappa}(vac) \quad -(49)$$

and

From eqs. (1), (48) and (49):

$$\boxed{(\omega + \omega(vac))^2 = c^2(\underline{\kappa} + \underline{\kappa}(vac))^2 + \left(\frac{mc^2}{\hbar}\right)^2} \quad -(51)$$

This is the fundamental equation of the interaction of the vacuum with a matter wave such as an electron wave. Here:

$$(\underline{\kappa} + \underline{\kappa}(vac))^2 = \kappa^2 + \kappa^2(vac) + 2\underline{\kappa}\cdot\underline{\kappa}(vac) \quad -(52)$$

$$= \kappa^2 + \kappa^2(vac) + 2\kappa\kappa(vac)\cos\theta$$

where θ is the angle between $\underline{\kappa}$ and $\underline{\kappa}(vac)$. Therefore vacuum wave/particles can scatter off matter wave particles.

In order to test this ~~theory~~ experimentally

8) it is necessary to have two equations in the two unknowns, $\omega(\text{vac})$ and $\underline{\kappa}(\text{vac})$. This is possible by measuring ω and $\underline{\kappa}$ at two different frequencies,

so:

$$\left(\omega_A + \omega(\text{vac})\right)^2 = c^2\left(\underline{\kappa}_A + \underline{\kappa}(\text{vac})\right)^2 + \left(\frac{mc^2}{\hbar}\right)^2 \quad -(53)$$

and

$$\left(\omega_B + \omega(\text{vac})\right)^2 = c^2\left(\underline{\kappa}_B + \underline{\kappa}(\text{vac})\right)^2 + \left(\frac{mc^2}{\hbar}\right)^2 \quad -(54)$$

Note carefully that the mass of all elementary particles is changed by the vacuum:

$$\boxed{m \longrightarrow m + m(\text{vac})} \quad -(55)$$

and the rest energy is changed to:

$$\boxed{E_0 = \left(m + m(\text{vac})\right)c^2} \quad -(56)$$

The vacuum rest energy is:

$$\boxed{E_0(\text{vac}) = m(\text{vac})c^2} \quad -(57)$$

where $E_0 = \hbar\omega_0(\text{vac}) = m(\text{vac})c^2 \quad -(58)$

where $\omega_0(\text{vac})$ is the vacuum rest frequency.

9) Here E_0 is the vacuum zero point or rest energy. This can become considerable if $n(vac)$ is large enough. It could well be responsible for low energy nuclear reactions.

Finally in this note, two vacuum particles can annihilate, and produce elementary particle pairs:

$$\hbar\omega(vac) + \hbar\omega(vac) = e^- + e^+ \quad (59)$$

This is an example where an electron positron pair is produced from the annihilation of two vacuum particles of energy:

$$E = \hbar\omega(vac) = \gamma m(vac)c^2 \quad (60)$$

In the next note it is shown that the g factor of the electron can be described by this theory.