338(3): Electron in Contact with the AB Vacuum, Calculation of the g Factor.

The relevant starting point is the equation: —(1)

$$(H - e\phi_W - mc^2)\psi = c^2 \underline{\sigma} \cdot (-i\hbar \underline{\nabla} - e\underline{W}) \left( \frac{1}{H - e\phi_W + mc^2} \right) \underline{\sigma} \cdot (\underline{p} - e\underline{W})\psi$$

$$= ie\hbar c^2 \underline{\sigma} \cdot \underline{\nabla} \left( \left( \frac{\underline{\sigma} \cdot \underline{W}}{H - e\phi_W + mc^2} \right) \psi \right) + \cdots$$

in which $\underline{\nabla}$ acts on all the terms inside the bracket, so the Leibniz theorem must be used. In general, eq. (1) contains many terms and many effects of an electron interacting with the AB vacuum, in which there is the potential:

$$W^\mu = \frac{\hbar}{e} \Omega^\mu \quad -(2)$$

as in previous notes.

The Dirac Approximation

In this approximation:

$$H = H_0 + mc^2 = mc^2 \quad -(3)$$

where $H$ is the relativistic hamiltonian and $H_0$ the non-relativistic hamiltonian, defined by:

$$H = \gamma mc^2 + U \quad -(4)$$

and

$$H_0 = \frac{p_0^2}{2m} + U \quad -(5)$$

The Dirac approximation means that:

$$H_0 = ?\, 0 \quad -(6)$$

2) if rigorously interpreted. In the loose interpretation it means that

$$E = \gamma mc^2 \rightarrow E_0 = mc^2 \quad -(7)$$

and that the Dirac theory is for a slow moving particle:

$$v << c. \quad -(8)$$

This is the interpretation given in Dirac's Nobel lecture. However, all of the successes usually attributed to the Dirac theory rely on:

$$H = mc^2 \quad -(9)$$

exactly. In this case, the g factor of the electron is

$$g = 2 \quad -(10)$$

and is calculated as follows. Using eqs. (1) and (9):

$$(H - e\phi_W - mc^2)\phi = ie\hbar\, \underline{\sigma} \cdot \underline{\nabla} \left( \left( \frac{c^2}{2mc^2 - e\phi_W} \right) \underline{\sigma} \cdot \underline{W}\, \phi \right) + \ldots$$

$$= \frac{ie\hbar}{2m}\, \underline{\sigma} \cdot \underline{\nabla} \left( \left( 1 - \frac{e\phi_W}{2mc^2} \right)^{-1} \underline{\sigma} \cdot \underline{W}\, \phi \right) + \ldots \quad -(11)$$

It is also assumed in the Dirac theory that:

$$e\phi_W << 2mc^2 \quad -(12)$$

which is a consequence of assuming that:

$$H = \gamma mc^2 + U \sim mc^2, \quad -(13)$$

where

$$U = e\phi_W \quad -(14)$$

3) Note carefully that there is an internal self-inconsistency in the Dirac theory, because eq. (9) is used, and eq. (9) implies that

$$U = 0. \quad -(15)$$

Accepting the usual Dirac theory for the sake of illustration only, eq. (12) means that:

$$\left(1 - \frac{e\phi_W}{2mc^2}\right)^{-1} \sim 1 + \frac{e\phi_W}{2mc^2} \quad -(16)$$

so eq. (11) becomes:

$$(H - e\phi_W - mc^2)\psi \sim \frac{ie\hbar}{2m} \underline{\sigma} \cdot \underline{\nabla}\left(\underline{\sigma} \cdot \underline{W}\left(1 + \frac{e\phi_W}{2mc^2}\right)\psi\right) + \ldots \quad -(17)$$

The first term on the right hand side of eq. (17) is the term that leads to eq. (10). The second term on the right hand side is the spin orbit term that leads to the Thomas factor and the fine structure of atomic spectra. The g factor term leads to ESR and NMR, but all these results are based on an internally self-contradictory approximation. Recent UFT papers have developed the subject.

The g factor hamiltonian is therefore:

$$(H - e\phi_W - mc^2)\psi = \frac{ie\hbar}{2m} \underline{\sigma} \cdot \underline{\nabla}\left(\underline{\sigma} \cdot \underline{W}\psi\right) + \ldots \quad -(18)$$

$$= \frac{ie\hbar}{2m}\left(\underline{\sigma}\cdot\underline{\nabla}\left(\underline{\sigma}\cdot\underline{W}\right)\phi + \underline{\sigma}\cdot\underline{\nabla}\phi\,\underline{\sigma}\cdot\underline{W}\right) \quad -(19)$$

using the Leibniz theorem. The first term is usually considered the theory, so:

$$(H - e\phi_W - mc^2)\phi = \frac{ie\hbar}{2m}\underline{\sigma}\cdot\underline{\nabla}\,\underline{\sigma}\cdot\underline{W}\,\phi + \dots \quad -(20)$$

where

$$\underline{\sigma}\cdot\underline{\nabla}\,\underline{\sigma}\cdot\underline{W} = i\,\underline{\sigma}\cdot\underline{\nabla}\times\underline{W} + \underline{\nabla}\cdot\underline{W} \quad -(21)$$

The real and physical part of eq. (20) is:

$$\mathrm{Re}(H - e\phi_W - mc^2)\phi = -\frac{e\hbar}{2m}\underline{\sigma}\cdot\underline{\nabla}\times\underline{W} \quad -(22)$$

The spin angular momentum of the electron is:

$$\underline{S} = \frac{\hbar}{2}\underline{\sigma} \quad -(23)$$

so:

$$\mathrm{Re}(H - e\phi_W - mc^2)\phi = -\frac{e}{m}\underline{S}\cdot\underline{\nabla}\times\underline{W} \quad -(24)$$

an equation which defines the spin magnetic dipole moment of the electron:

$$\boxed{\underline{m}_S = \frac{e}{m}\underline{S}} \quad -(25)$$

the quantity $e/m$ is the gyromagnetic ratio. Finally the Dirac g factor is defined

5) by writing eq. (25) as:

$$\underline{m}_S = 2 \frac{e}{2m} \underline{S} \quad -(26)$$

$$= g \frac{e}{2m} \underline{S}$$

This procedure is used because the orbital magnetic dipole moment of the electron is:

$$\underline{m}_L = \frac{e}{2m} \underline{L} \quad -(27)$$

where $\underline{L}$ is the orbital angular momentum of the electron. So the total magnetic dipole moment is:

$$\underline{m} = \underline{m}_L + \underline{m}_S = \frac{e}{2m} \left( \underline{L} + 2 \underline{S} \right) \quad -(28)$$

The factor:

$$g = 2 \quad -(29)$$

appears in Eq. (28).

The orbital magnetic dipole moment of the electron is obtained from:

$$(H - e\phi_W - mc^2)\psi = \frac{1}{2m} (\underline{p} - e\underline{W}) \cdot (\underline{p} - e\underline{W}) \psi + \ldots \quad -(29)$$

which is obtained by replacing $-i\hbar \underline{\nabla}$ by $\underline{p}$ in eq. (1). The interaction term in eq. (29) is:

$$(H - e\phi_W - mc^2)\psi = -\frac{e}{m} \underline{p} \cdot \underline{W} \psi + \ldots \quad -(30)$$

The vector potential is assumed to be:

$$\underline{W} = \frac{1}{2}\left(\underline{\nabla} \times \underline{W}\right) \times \underline{r} \quad -(31)$$

so:

$$\left(H - e\phi_W - mc^2\right)\phi = -\frac{e}{2m}\,\underline{p}\cdot\left(\underline{\nabla}\times\underline{W}\right)\times\underline{r}\,\phi + \dots$$

$$= -\frac{e}{2m}\,\underline{r}\times\underline{p}\cdot\underline{\nabla}\times\underline{W}\,\phi \quad -(32)$$

$$= -\frac{e}{2m}\,\underline{L}\cdot\underline{\nabla}\times\underline{W}\,\phi$$

Q.E.D. This gives the orbital magnetic dipole moment in Eq. (27).

Note carefully that in magnetic field applications, $\underline{\nabla}\times\underline{W}$ is developed in the usual theory as:

$$\underline{B} = \underline{\nabla}\times\underline{A} \quad -(33)$$

In ECE2 theory:

$$\underline{B} = \underline{\nabla}\times\underline{W} \quad -(34)$$

However, in the Aharonov Bohm vacuum, there is no magnetic field, but there is a potential. So $\underline{\nabla}\times\underline{W}$ is interpreted as a pure vacuum property, in which

$$\underline{W} = \frac{\hbar}{e}\,\underline{\Omega} \quad -(35)$$

where $\underline{\Omega}$ is the spin connection vector. So for an electron in contact with the vacuum, the Dirac g

7) factor is due fundamentally to the spin connection vector, more rigorously the curl $\underline{\nabla \times \Omega}$.

The g factor of the electron is therefore affected by the AB vacuum.

Experimentally, the g factor of the electron is

$$g = 2.002319314 \quad - (36)$$

(see UFT85), and in this new theory, this result is due to the development of eq. (1) without the Dirac approximation, and not to quantum electrodynamics.

This will be the subject of the next note.

---