337(5): Energy from the Vacuum from the W^μ Potential

Consider the W^μ potential of ECE2 theory:

$$W^\mu = \left(\frac{\phi_W}{c}, \underline{W} \right) \quad -(1)$$

$$= W^{(0)} \left(\omega^{(0)}, \underline{\omega} \right)$$

where the elementary unit of quantized magnetic flux in weber is

$$W^{(0)} = \frac{\hbar}{e} \quad -(2)$$

This potential exists in the vacuum, in regions where there are no electric and magnetic fields. The energy / momentum from this vacuum is given by the minimal prescription:

$$p^\mu \rightarrow p^\mu + eW^\mu \quad -(3)$$

and the Hamilton Jacobi equation:

$$(p^\mu + eW^\mu)(p_\mu + eW_\mu) = m^2c^2 \quad -(4)$$

which can be solved in various ways. For example:

$$p^\mu = \frac{m^2c^2}{p_\mu + eW_\mu} - eW^\mu \quad -(5)$$

The energy term is given by:

$$\mu = 0 \quad -(6)$$

so

$$p^0 = \frac{E}{c} = \frac{m^2c^2}{p_0 + eW_0} - eW^0 \quad -(7)$$

) where E is the total relativistic energy:

$$E = \gamma mc^2 \quad -(8)$$

In eq. (7):

$$W_0 = \frac{\phi_W}{c} = \frac{\hbar}{e}\omega^{(0)}, \quad -(9)$$

So

$$\frac{E}{c} = \frac{m^2c^2}{\frac{E}{c} + \hbar\omega^{(0)}} - \hbar\omega^{(0)} \quad -(10)$$

i.e.

$$E = \frac{m^2c^4}{E + \hbar\omega^{(0)}c} - \hbar\omega^{(0)}c \quad -(11)$$

The units of $\omega^{(0)}$ are m^{-1}, or wave number, so:

$$f = \omega^{(0)}c \quad -(12)$$

where f has the units of frequency, s^{-1}. If the scalar spin connection $\omega^{(0)}$ is defined as 2π multiplied by inverse metres then:

$$\omega = \omega^{(0)}c = 2\pi\bar{\nu}c \quad -(13)$$

Therefore the vacuum wavenumber is

$$\bar{\nu}(\text{vac}) = \omega^{(0)} \quad -(14)$$

and the vacuum wavenumber is the scalar spin connection. This means that the vacuum is automatically quantized.

) Therefore eq. (11) is:

$$E = \frac{m^2 c^4}{E + \hbar\omega} - \hbar\omega \quad -(15)$$

where ω is the vacuum angular frequency. Eq. (15) is:

$$(E + \hbar\omega)(E + \hbar\omega) = m^2 c^4 \quad -(16)$$

i.e.

the vacuum energy quantum $\hbar\omega$ is added to the energy E through the minimal prescription:

$$E \rightarrow E + \hbar\omega \quad -(17)$$

where

$$\omega = c\omega^{(0)} = 2\pi c \; (\text{vac}) \quad -(18)$$

where $\bar{\nu}(\text{vac})$ is the vacuum wavenumber.

So

$$\boxed{\omega^{(0)} = 2\pi\bar{\nu}(\text{vac})} \quad -(19)$$

and

$$\omega = 2\pi c\bar{\nu}(\text{vac}) \quad -(20)$$

is the vacuum angular frequency. It is clear that the vacuum energy $\hbar\omega$ is added to the total energy:

$$E = \gamma mc^2 \rightarrow \gamma mc^2 + \hbar\omega \quad -(21)$$

Finally, using the de Broglie / Einstein equation:

$$\gamma m c^2 = \hbar \omega \quad -(22)$$

where ω is the angular frequency of a matter wave, it becomes clear that the Aharonov Bohm vacuum induces a frequency shift. Denoting the vacuum angular frequency by $\omega(vac)$, then:

$$\boxed{E = \hbar \omega \rightarrow \hbar(\omega + \omega(vac))} \quad -(23)$$

The concept of vacuum frequency is well known in quantum field theory and quantum optics. So

$$E(vac) = \hbar \omega(vac) \quad -(24)$$

is the zero point energy of harmonic oscillator theory in quantum mechanics.

The spin connection is the origin of zero point energy in ECE2.