1) 337(4): Development of the ECE2 Minimal Prescription

This is
$$p^\mu \rightarrow p^\mu - eW^\mu \quad -(1)$$
so
$$E \rightarrow E - e\phi_W, \quad \underline{p} \rightarrow \underline{p} - e\underline{W} \quad -(2)$$

Here,
$$p^\mu = \left(\frac{E}{c}, \underline{p}\right); \quad W^\mu = \left(\frac{\phi_W}{c}, \underline{W}\right) \quad -(3)$$

The units of ϕ_W are JC^{-1} = volts. The units of $\underline{W}$ are $JC^{-1}sm^{-1}$ = tesla metres. In ECE2:
$$W^\mu = \left(\frac{\phi_W}{c}, \underline{W}\right) = \left(\omega^{(0)}, \underline{\omega}\right) W^{(0)}$$
$$= W^{(0)}\left(\omega^{(0)}, \underline{\omega}\right) \quad -(4)$$
where the spin connection four vector is:
$$\omega^\mu = \left(\omega^{(0)}, \underline{\omega}\right) \quad -(5)$$

It follows that:
$$\phi_W = W^{(0)} c\, \omega^{(0)} \quad -(6)$$
and
$$\underline{W} = W^{(0)} \underline{\omega} \quad -(7)$$

Therefore the units of $W^{(0)}$ are:
$$W^{(0)} = JC^{-1}s = \text{volt sec} \quad -(8)$$
$$= \text{weber}$$
$$= \text{magnetic flux}$$

Summary of Units

$$\phi_W = JC^{-1} = \text{volt} \quad -(9)$$

$$\underline{W} = JC^{-1}sm^{-1} = \text{tesla metres} \quad -(10)$$

$$\omega^{(0)} = \underline{\omega} = m^{-1} \quad -(11)$$

$$W^{(0)} = \text{weber} = \text{volt s} = JC^{-1}s \quad -(12)$$

In ECE2 the magnetic flux density $\underline{B}$ in tesla is:

$$\underline{B} = \underline{\nabla} \times \underline{W} = \underline{\nabla} \times \underline{A} + 2\underline{\omega} \times \underline{A} \quad -(13)$$

and the electric field strength $\underline{E}$ in volt m^{-1} is:

$$\underline{E} = -\underline{\nabla}\phi_W - \frac{\partial \underline{W}}{\partial t} \quad -(14)$$

$$= -\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t} + 2\left(c\omega_0 \underline{A} - \phi\underline{\omega}\right)$$

where

$$A^{\mu} = \left(\frac{\phi}{c}, \underline{A}\right) \quad -(15)$$

$$= A^{(0)} q^{\mu} \quad -(16)$$

here

$$q^{\mu} = \left(q^{(0)}, \underline{q}\right) \quad -(17)$$

is the tetrad four vector. The units of ϕ are the same as those of ϕ_W, and the units of $\underline{A}$ are the same as those of $\underline{W}$.

) In ECE2 the magnetic flux density $\underline{B}$ is defined as:

$$\underline{B} = W^{(0)} \underline{R}(\text{spin}) \quad -(18)$$

where $\underline{R}(\text{spin})$ is the spin curvature vector in units of m^{-2}. The electric field strength $\underline{E}$ is defined by:

$$\underline{E} = c W^{(0)} \underline{R}(\text{orbital}) \quad -(19)$$

where $\underline{R}(\text{orbital})$ is the orbital curvature vector in m^{-2}. From eqs. (12), (18) and the units of $\underline{R}(\text{spin})$:

$$B = J C^{-1} s m^{-2} = \text{tesla} \quad -(20)$$

Q.E.D. Similarly the units of $\underline{E}$ are

$$\underline{E} = m s^{-1} J C^{-1} s m^{-2} = J C^{-1} m^{-1}$$
$$= \text{volt } m^{-1} \quad -(21)$$

Q.E.D

Finally note that the elementary unit of weber is

$$W^{(0)} = \hbar / e \quad -(22)$$

where $\hbar$ is the reduced Planck constant. From eqs. (7) and (22):

$$\boxed{\underline{W} = \frac{\hbar}{e} \underline{\omega}} \quad -(23)$$

and from eqs. (6) and (22):

$$\boxed{\phi_W = \left(\frac{\hbar c}{e}\right) \omega^{(0)}} \quad -(24)$$

The Aharonov Bohm type of vacuum may

4) therefore be defined in terms of the vacuum potential:

$$\underline{W}^{\mu}(vac) = \left(\frac{\phi_W(vac)}{c}, \underline{W}(vac) \right) \quad -(25)$$

$$= W^{(0)} \left(\omega^{(0)}(vac), \underline{\omega}(vac) \right)$$

On the most elementary level:

$$\boxed{W^{\mu}(vac) = \frac{\hbar}{e} \omega^{\mu}(vac)} \quad -(26)$$

So the Aharonov Bohm vacuum is defined by the spin connection four vector within $\hbar / e$. The latter is an elementary or fundamental quantity.

In the absence of electric and magnetic fields the vacuum is defined by eq. (26). The fields $\underline{E}$ and $\underline{B}$ are defined by curvature, the vacuum potential by the spin connection.

The minimal prescription (1) can now be used in the Einstein energy equation as follows:

$$(p^{\mu} - eW^{\mu})(p_{\mu} - eW_{\mu}) = m^2c^2 \quad -(27)$$

and this equation developed in many different ways, both with W^{μ} and A^{μ}. This will be the subject of following notes.