337(2): Effect of a Vacuum Scalar Potential on the Frequency of an Electron Matter Wave

Consider the Einstein energy equation of the electron:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(1)$$

The effect of a scalar potential ϕ is to change this equation to

$$(E - e\phi)^2 = c^2 p^2 + m^2 c^4 \quad -(2)$$

where $-e$ is the charge on the electron. Now use:

$$E = \hbar\omega, \quad \underline{p} = \hbar \underline{\kappa} \quad -(3)$$

so
$$(\hbar\omega - e\phi)^2 = \hbar^2 \kappa^2 c^2 + m^2 c^4 \quad -(4)$$

The solution of eq. (4) is:

$$\boxed{\omega = \frac{e\phi}{\hbar} \pm c\left(\kappa^2 + \kappa_c^2\right)^{1/2}} \quad -(5)$$

where
$$\kappa_c = \frac{mc}{\hbar} \quad -(6)$$

When
$$\phi = 0 \quad -(7)$$

eq. (5) reduces to:

$$\omega^2 = c^2 \kappa^2 + \frac{m^2 c^4}{\hbar^2} \quad -(8)$$

Therefore the scalar potential is:

$$\phi = \frac{\hbar}{e}\left(\omega - c\left(\kappa^2 + \kappa_c^2\right)^{1/2}\right) JC^{-1} \quad -(9)$$

If ϕ is the scalar potential of the vacuum it always interacts with the electron, so the angular frequency being measured is $\omega - e\phi/\hbar$, and not ω. If the vacuum potential were zero, the relation between ω and κ of the electron would be eq. (8).

However, the true relation is always eq. (5) because ϕ is always non-zero and the electron is always in contact with the vacuum.

In an Aharonov Bohm type experiment, an extra potential ϕ is generated by an electric field $\underline{E}$ which is shielded from the electron beam. The wave function of the electron is changed, and its Young interferogram is changed. In order to find the wave function of the electron, quantize eq. (1) using:

$$E\psi = i\hbar\frac{\partial\psi}{\partial t}, \quad \underline{p}\psi = -i\hbar\underline{\nabla}\psi \quad -(10)$$

This means that:

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} = -c^2\hbar^2\nabla^2\psi + m^2c^4\psi \quad -(11)$$

i.e. $$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2\right)\phi+\left(\frac{mc}{\hbar}\right)^2\phi=0 \quad -(12)$$

or $$\left(\square+\left(\frac{mc}{\hbar}\right)^2\right)\phi=0 \quad -(13)$$

where the d'Alembertian is:
$$\square=\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2 \quad -(14)$$

Eq. (13) is a special case of the ECE wave equation. Its solution is
$$\phi=\exp\left(-i(\omega t-\kappa Z)\right) \quad -(15)$$

So $$\frac{\partial^2\phi}{\partial t^2}=-\omega^2,\ \frac{\partial^2\phi}{\partial Z^2}=-\kappa^2 \quad -(16)$$

and $$\left(-\frac{\omega^2}{c^2}+\kappa^2+\left(\frac{mc}{\hbar}\right)^2\right)\phi=0 \quad -(17)$$

a solution of which is
$$\omega^2=c^2\kappa^2+\frac{m^2c^4}{\hbar^2} \quad -(18)$$

which is eq. (8), QED.

So the scalar potential changes the wave function to
$$\boxed{\phi=\exp\left(-i\left(\left(\omega-\frac{e\phi}{\hbar}\right)t-\kappa Z\right)\right)} \quad -(19)$$

4) This produces a shift in a Young interferogram. In the absence of a potential the wave function is eq. (15). In the presence of the vacuum the measured frequency is $\omega - e\phi/\hbar$, so in eq. (5) there are two unknowns, the absolute frequency ω and ϕ.

Another method is needed to give another equation involving ω and ϕ in order to find the absolute value of ω and then ϕ. This method will be developed in the next note.
