334(3): Test of the Class One Hamiltonian by ESR in a Relativistic Electron Beam.

As in Note 330(7), conventional ESR is developed from:

$$H_{ESR} = \frac{1}{2m} \underline{\sigma} \cdot (\underline{p} - e\underline{A}) \underline{\sigma} \cdot (\underline{p} - e\underline{A}) \quad -(1)$$

which quantizes to:

$$H_{ESR}\phi = \frac{ie\hbar}{2m} \underline{\sigma} \cdot \underline{\nabla} \, \underline{\sigma} \cdot \underline{A} \phi + \dots \quad -(2)$$

so

$$\mathrm{Re} H_{ESR}\phi = -\frac{e\hbar}{2m} \underline{\sigma} \cdot \underline{\nabla} \times \underline{A} \phi + \dots$$

$$= -\frac{e\hbar}{2m} \underline{\sigma} \cdot \underline{B} \phi \quad -(3)$$

The spin angular momentum is:

$$\underline{S} = \frac{\hbar}{2} \underline{\sigma} \quad -(4)$$

so

$$\mathrm{Re} H_{ESR}\phi = -\frac{e}{m} \underline{S} \cdot \underline{B} \phi. \quad -(5)$$

If the magnetic field is aligned in the Z axis:

$$\mathrm{Re} H_{ESR}\phi = -\frac{e}{m} S_Z B_Z \phi \quad -(6)$$

where

$$S_Z \phi = m_s \hbar \phi \quad -(7)$$

and

$$m_s = -S, \dots + S = -\frac{1}{2} \text{ or } \frac{1}{2} \quad -(8)$$

d) Using the transition rule:

$$\Delta m_s = 1 \quad -(9)$$

for absorption of radiation, the resonance angular frequency is

$$\boxed{\omega = \frac{eB_Z}{m}} \quad -(10)$$

This is the well known electron spin resonance frequency for one free electron.

In the rigorous theory using the classical Hamiltonian, eq. (3) becomes:

$$ReH_{ESR}\phi = -\frac{e\hbar}{m}\left(\frac{\gamma^2}{1+\gamma}\right)\underline{\sigma}\cdot\underline{B}\phi \quad -(11).$$

$$= -2\frac{e\hbar}{m}\left(\frac{\gamma^2}{1+\gamma}\right)\underline{S}\cdot\underline{B}\phi$$

so the rigorous ESR resonance frequency becomes:

$$\boxed{\omega = 2\left(\frac{\gamma^2}{1+\gamma}\right)\frac{e}{m}B_Z} \quad -(12)$$

This can be measured experimentally in a relativistic electron beam.

In eq. (12):

$$\frac{\gamma^2}{1+\gamma} = \left(1 - \frac{p_0^2}{m^2c^2} + \left(1 - \frac{p_0^2}{m^2c^2}\right)^{1/2}\right)^{-1} \quad -(13)$$

3) The measured momentum of the electron is:

$$p = \gamma p_0 \quad -(14)$$

i.e. its relativistic momentum. So:

$$p_0^2 = \frac{p^2}{\gamma^2} = p^2\left(1 - \frac{p_0^2}{m^2c^2}\right) \quad -(15)$$

and p_0^2 can be found from the measured p^2:

$$p_0^2 = p^2 \Big/ \left(1 + \frac{p^2}{m^2c^2}\right) \quad -(16)$$

and used in eq. (12).

So this rigorous test of the Dirac equation uses esr in a relativistic electron beam.