334(1): The Anomalous Zeeman Effect in the Class One Hamiltonian.

The hamiltonian is:

$$H = \frac{1}{m} \underline{\sigma} \cdot \underline{p}_0 \frac{\gamma^2}{1+\gamma} \underline{\sigma} \cdot \underline{p}_0 \quad -(1)$$

In the presence of a magnetic field:

$$\underline{p}_0 \rightarrow \underline{p}_0 - e\underline{A} \quad -(2)$$

so

$$H = \frac{1}{m} \underline{\sigma} \cdot (\underline{p}_0 - e\underline{A}) \frac{\gamma^2}{1+\gamma} \underline{\sigma} \cdot (\underline{p}_0 - e\underline{A}) \quad -(3)$$

First consider this as a classical hamiltonian, then:

$$H = \frac{1}{m} \left(\frac{\gamma^2}{1+\gamma}\right) (\underline{p}_0 - e\underline{A}) \cdot (\underline{p}_0 - e\underline{A}) \quad -(4)$$

$$= \frac{1}{m} \left(\frac{\gamma^2}{1+\gamma}\right) \left(p_0^2 - 2e\underline{p}_0 \cdot \underline{A} + e^2 A^2\right).$$

Now use:

$$\underline{A} = \frac{1}{2} \underline{B} \times \underline{r} \quad -(5)$$

and

$$\underline{B} \times \underline{r} \cdot \underline{p}_0 = \underline{B} \cdot \underline{r} \times \underline{p}_0 = \underline{B} \cdot \underline{L} \quad -(6)$$

where the orbital angular momentum is:

$$\underline{L} = \underline{r} \times \underline{p}_0 \quad -(7)$$

then

$$H = \frac{1}{m} \left(\frac{\gamma^2}{1+\gamma}\right) \left(p_0^2 - e\underline{L} \cdot \underline{B} + e^2 A^2\right) -(8)$$

This hamiltonian consists of:

2) a) the kinetic energy:

$$H_1 = \frac{1}{m}\left(\frac{\gamma^2}{1+\gamma}\right) p_0^2 \quad -(9)$$

b) the orbital angular momentum term:

$$H_2 = -\frac{e}{m}\,\underline{L}\cdot\underline{B}\left(\frac{\gamma^2}{1+\gamma}\right) \quad (10)$$

c) the quadratic term:

$$H_3 = \frac{1}{m}\left(\frac{\gamma^2}{1+\gamma}\right) e^2 A^2 \quad -(11)$$

So these well known terms are all multiplied by:

$$\frac{\gamma^2}{1+\gamma} \xrightarrow[\gamma \to 1]{} \frac{1}{2} \; ; \; \frac{\gamma^2}{1+\gamma} \xrightarrow[\gamma \to \infty]{} 1 \quad -(12)$$

The limit $\gamma \to 1$ is the non-relativistic limit, and $\gamma \to \infty$ is the hyperrelativistic limit as v_0 approaches c.

The Zeeman effect is modified to:

$$H_2 \phi = -\frac{e}{m}\,\frac{\gamma^2}{1+\gamma}\,\underline{B}\cdot\underline{L}\,\phi \quad -(13)$$

The operator: $L_z \phi = m_L \hbar \phi \quad -(14)$

is modified to $\left(\left(\frac{\gamma^2}{1+\gamma}\right) L_z\right)\phi = A\phi \quad -(15)$

where $\langle A \rangle = \int \phi^* \frac{\gamma^2}{1+\gamma} L_z \phi \, d\tau \quad -(16)$

3)

Thus far, the development is rigorous, but a subjective choice must now be made of how to develop eq. (15).

By definition:

$$\gamma = \left(1 - \frac{p_0^2}{m^2c^2}\right)^{-1/2} \quad -(17)$$

so p_0^2 can be regarded as a classical function and used as an input parameter. It follows that the energy levels of the Zeeman effect are, in this case:

$$E = -\left(\frac{\gamma^2}{1+\gamma}\right)\frac{e\hbar}{m} B_Z m_L \quad -(18)$$

where

$$m_L = -L, \dots, L \quad -(19)$$

and

$$\Delta L = 1 \quad -(20)$$

The energy levels of the H atom are then:

$$\boxed{E_H = -\frac{1}{2}mc^2\left(\frac{\alpha}{n}\right)^2 - \left(\frac{\gamma^2}{1+\gamma}\right)\frac{e\hbar}{m} m_L B_Z} \quad -(21)$$

In this equation:

$$\frac{\gamma^2}{1+\gamma} = \left(\frac{1}{\gamma^2} + \frac{1}{\gamma}\right)^{-1} \quad -(22)$$

) where:

$$\frac{1}{\gamma^2} = 1 - \frac{p_0^2}{m^2c^2} \; ; \; \frac{1}{\gamma} = \left(1 - \frac{p_0^2}{m^2c^2}\right)^{1/2} \quad -(23)$$

The transition rule in eq. (21) are:

$$\Delta n \text{ (any)} \; ; \; \Delta L = 1 \; ; \; \Delta m_L = 0, \pm 1 \quad -(24)$$

with

$$m_L = -L, \ldots, L \quad -(25)$$

In eq. (21) α is the fine structure constant and n the principal quantum number. Therefore if p_0^2 is regarded as a classical function, the usual Zeeman effect is shifted.

The second subjective choice is to define the expectation value:

$$\frac{p_0^2}{m^2c^2} = \left\langle \frac{p_0^2}{m^2c^2} \right\rangle = \frac{\alpha^2}{n^2} \quad -(26)$$

$$= 5.3144 \times 10^{-5} / n^2$$

where we have used:

$$\left\langle \frac{p^2}{2m} \right\rangle = -E = \frac{1}{2} mc^2 \left(\frac{\alpha}{n}\right)^2 \quad -(27)$$

in the H atom. So in this case:

$$\frac{\gamma^2}{1+\gamma} = \left(\left(1 - \left(\frac{\alpha}{n}\right)^2\right)^{1/2} + \left(1 - \left(\frac{\alpha}{n}\right)^2\right)\right)^{-1} \quad -(28)$$

and:

5)

$$E_H = -\frac{1}{2}mc^2\left(\frac{\alpha}{n}\right)^2 - \left(\left(1-\left(\frac{\alpha}{n}\right)^2\right)^{1/2} + 1 - \left(\frac{\alpha}{n}\right)^2\right)^{-1}\frac{e\hbar}{m}m_L B_Z \qquad -(29)$$

and the Zeeman spectrum is split into hyperfine structure as in previous work.

In order to describe the anomalous Zeeman effect a subjective choice of quantization is needed for eq. (3).

The usual choice is:

$$H = \frac{1}{m}\underline{\sigma}\cdot\left(-i\hbar\underline{\nabla} - e\underline{A}\right)\frac{\gamma^2}{1+\gamma}\underline{\sigma}\cdot\left(\underline{p}_0 - e\underline{A}\right). \qquad -(30)$$

which introduces the spin quantum number. This will be described in the next note.