332(3): Transition Rules in Spin Orbit Fine Structure

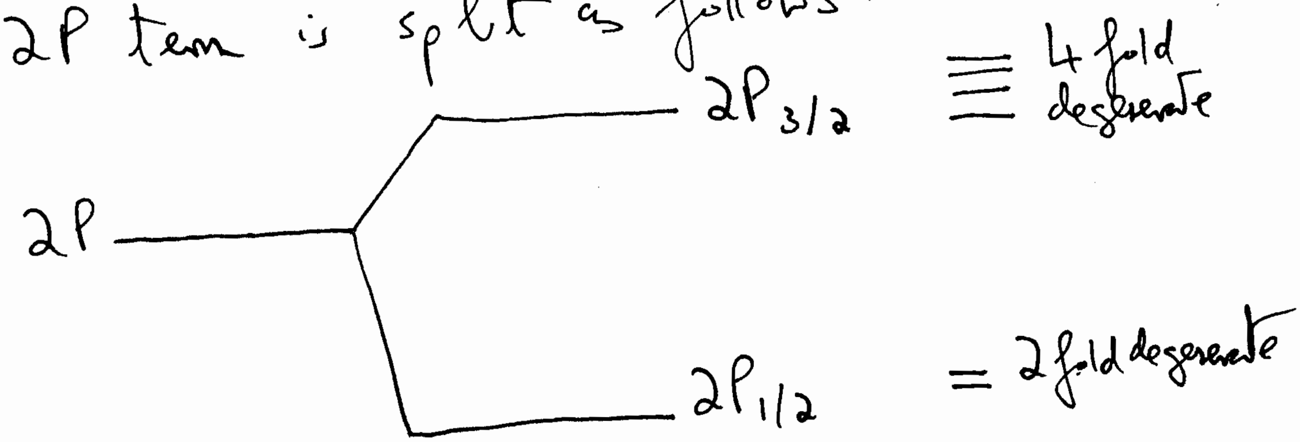
These are:

$$\Delta L = 1, \quad \Delta J = 0, \pm 1 \qquad -(1)$$

with

$$m_J = -J, \ldots, J \qquad -(2)$$

So a 2P term is split as follows:



These are defined as follows: $\qquad -(3)$

$$2P_{3/2}\left(n=2, L=1, J=3/2, m_J=-3/2, -1/2, 1/2, 3/2\right)$$

and

$$2P_{1/2}\left(n=2, L=1, J=1/2, m_J=-1/2, 1/2\right) \qquad -(4)$$

with

$$J = L + 1/2 \text{ and } L - 1/2 \qquad -(5)$$

The 3D term is split as follows: $\qquad -(6)$

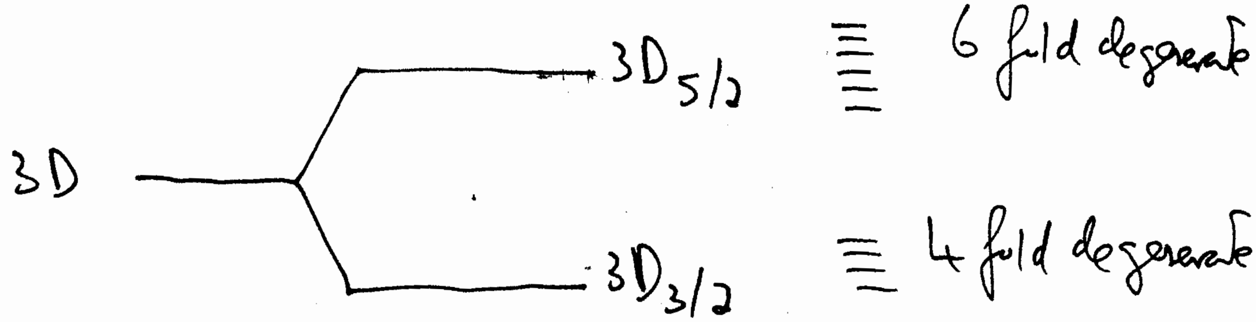
$$3D_{5/2}\left(n=3, L=2, J=5/2, m_J=-5/2, -3/2, -1/2, 1/2, 3/2, 5/2\right)$$

and

$$3D_{3/2}\left(n=3, L=2, J=3/2, m_J=-3/2, -1/2, 0, 1/2, 3/2\right) \qquad -(7)$$

Therefore:

2)



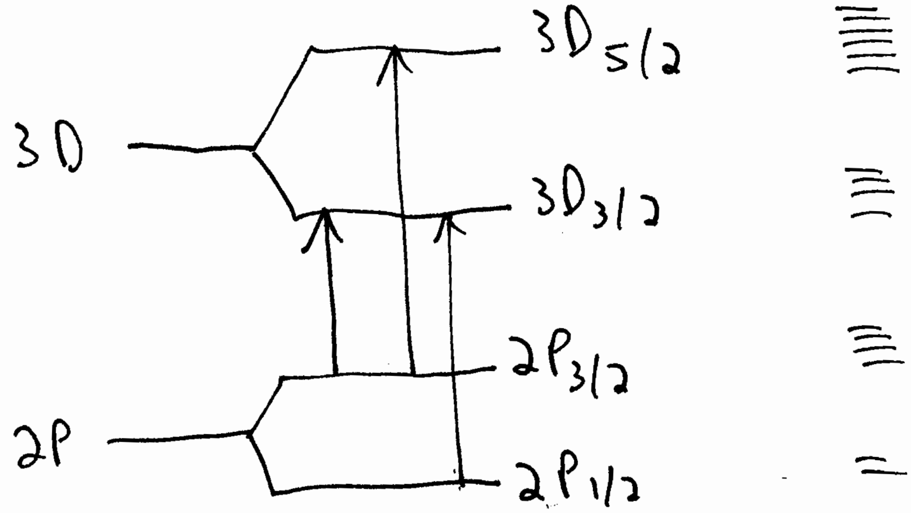
The allowed transitions are:

$$2P_{1/2} \longrightarrow 3D_{3/2} \quad (\Delta L = 1, \Delta J = 1)$$
$$2P_{3/2} \longrightarrow 3D_{5/2} \quad (\Delta L = 1, \Delta J = 1)$$
$$2P_{3/2} \longrightarrow 3D_{3/2} \quad (\Delta L = 1, \Delta J = 0)$$

In the conventional treatment the energy levels are given by:

$$E_{so} = -\frac{e^2}{16\pi\epsilon_0 m^2 c^2}\left(\frac{J(J+1) - L(L+1) - S(S+1)}{a_0^3 n^3 L(L+1/2)(L+1)}\right) \quad -(8)$$

So conventionally:



The relativistic approximation using the Dirac theory is:

$$E_{so1} = E_{so}\left(1 + \frac{2.662567\times10^{-5}}{n^2}\right) \quad -(9)$$

In a 2P to 3D transition:

$$n = 2 \rightarrow n = 3. \quad -(10)$$

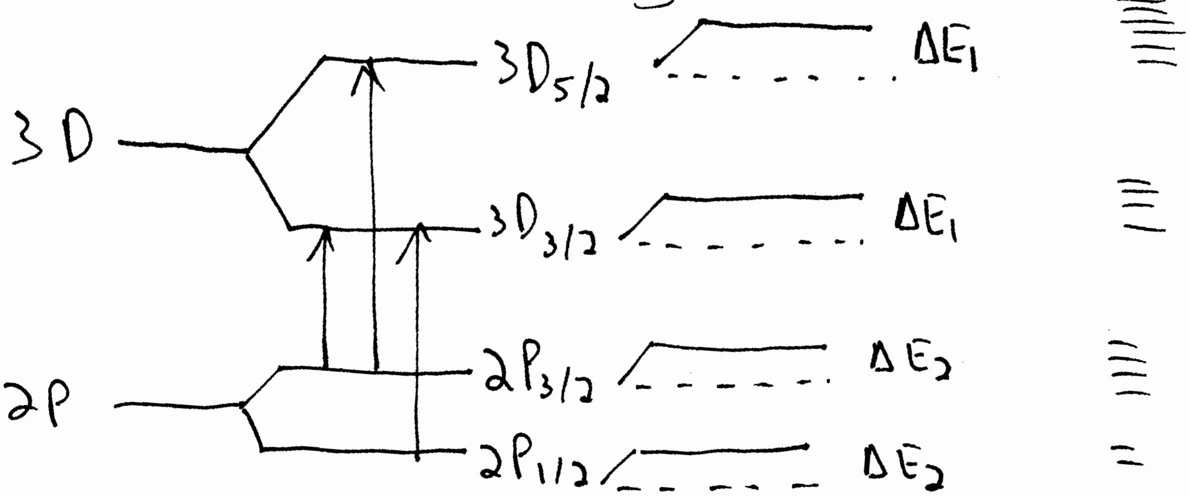
Therefore for $n = 2$:

$$E_{so12} = \left(1 + 6.66564\times10^{-6}\right)E_{so} \quad (11)$$

and for $n = 3$:

$$E_{so13} = \left(1 + 2.95841\times10^{-6}\right)E_{so} \quad -(12)$$

So in the relativistic theory:



$$\Delta E_1 = 2.95841\times10^{-6}E_{so} \quad -(13)$$

$$\Delta E_2 = 6.66564\times10^{-6}E_{so} \quad -(14)$$

Therefore $2P_{3/2}\rightarrow 3D_{3/2}$ is shifted by $\Delta E_1 - \Delta E_2$; $2P_{1/2}\rightarrow 3D_{3/2}$ is shifted by: $\Delta E_1 - \Delta E_2$ and $2P_{3/2}$ to $3D_{5/2}$ is also shifted by $\Delta E_1 - \Delta E_2$.

4) The Effect of a Magnetic Field

In the presence of a magnetic field eq. (9) becomes:

$$E_{so2} = E_{so}\left(1 + \frac{2.662567 \times 10^{-5}}{n^2}\right) - \frac{e\hbar}{2m} g_J m_J B \quad -(15)$$

where the Landé g factor is:

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \quad -(16)$$

where

$$m_J = -J, \ldots, J \quad -(17)$$

and

$$\Delta m_J = 0, \pm 1 \quad -(18)$$

Therefore a very rich new spectrum emerges because the degeneracy of $3D_{5/2}$, $3D_{3/2}$, $2P_{3/2}$ and $2P_{1/2}$ is lifted.