

329(b): Development of a New ESR Hamiltonian

Consider Eq. (7) of Note 329(b):

$$H_0 \psi = -\frac{H_0}{4m^2 c^2} \underline{\sigma} \cdot (-i\hbar \underline{\nabla} - e\underline{A}) \underline{\sigma} \cdot (\underline{p} - e\underline{A}) \psi \quad (1)$$

with:

$$\begin{aligned} & \underline{\sigma} \cdot (-i\hbar \underline{\nabla} - e\underline{A}) \underline{\sigma} \cdot (\underline{p} - e\underline{A}) \\ &= -i\hbar \underline{\sigma} \cdot \underline{\nabla} \underline{\sigma} \cdot \underline{p} + e^2 \underline{\sigma} \cdot \underline{A} \underline{\sigma} \cdot \underline{A} - e \underline{\sigma} \cdot \underline{A} \underline{\sigma} \cdot \underline{p} \\ & \quad + i e \hbar \underline{\sigma} \cdot \underline{\nabla} \underline{\sigma} \cdot \underline{A} \end{aligned} \quad (2)$$

The new ESR Hamiltonian is:

$$H_{\text{ESR}} \psi = -\frac{i e \hbar H_0}{4m^2 c^2} \underline{\sigma} \cdot \underline{\nabla} \underline{\sigma} \cdot \underline{A} \psi \quad (3)$$

Now use: $\underline{\sigma} \cdot \underline{\nabla} \underline{\sigma} \cdot \underline{A} = \underline{\nabla} \cdot \underline{A} + i \underline{\sigma} \cdot \underline{\nabla} \times \underline{A}$ (4)

so $\text{Re}(H_{\text{ESR}}) \psi = \frac{e \hbar H_0}{4m^2 c^2} \underline{\sigma} \cdot \underline{\nabla} \times \underline{A} \psi \quad (5)$

$$= \frac{e \hbar H_0}{4m^2 c^2} \underline{\sigma} \cdot \underline{B} \psi$$

def $\underline{B} = \underline{\nabla} \times \underline{A} \quad (6)$

2) Eq. (5) can be tested experimentally using EPR, NMR and MRI technologies, now very highly developed. The conventional eq. (6) has been used, but in ECE-2 theory this is replaced by an equation involving the spin conversion and telrad.

Firstly we, in the first approximation:

$$H_0 = \frac{-me^4}{32\pi^2 \epsilon_0^2 \hbar^2 n^2} \quad -(7)$$

so

$$\langle \text{Re}(H_{\text{ESR}}) \rangle = \frac{-e^5 \underline{\sigma} \cdot \underline{B}}{128\pi^2 \epsilon_0^2 \hbar mc^2 n^2} \quad -(8)$$

This has been worked out for the H atom but there are many developments possible for all atoms and molecules, giving a new type of EPR, NMR and MRI.