329(4): New Spin Orbit Coupling Terms

As in Note 329(3) the fundamental Hamiltonian of special relativity is developed as:

$$H_0 = H - mc^2 = \frac{p^2}{(1+\gamma)m} + U \quad -(1)$$

here

$$\gamma = \left(1 - \frac{p_0^2}{m^2 c^2}\right)^{-1/2} \quad -(2)$$

and

$$p_0 = m \underline{V_0} \quad -(3)$$

The Hamiltonian (1) was approximated by:

$$H_0 \sim \frac{p^2}{2m}\left(1 - \frac{H_0 - U}{2mc^2}\right) + U \quad -(4)$$

where

$$\underline{p} = \gamma m \underline{V_0} \quad -(5)$$

Eq. (4) can be written as:

$$H_0 = \frac{p^2}{2m}\left(1 + \frac{U}{2mc^2}\right) - \frac{p^2}{4m^2 c^2} H_0 + U \quad -(6)$$

In the $SU(2)$ basis:

$$H_0 = \frac{1}{2m} \underline{\sigma} \cdot \underline{p}\left(1 + \frac{U}{2mc^2}\right)\underline{\sigma} \cdot \underline{p} + U$$

$$-\frac{1}{4m^2 c^2}\underline{\sigma} \cdot \underline{p} \, H_0 \, \underline{\sigma} \cdot \underline{p} \quad -(7)$$

a)

As in UFT 250 and UFT 252, the usual Dirac approximation leads to:

$$H_0 = \frac{1}{2m}\, \underline{\sigma} \cdot \underline{p}\left(1 + \frac{U}{2mc^2}\right)\underline{\sigma}\cdot\underline{p} + U \quad -(8)$$

So there is a new spin orbit term in the rigorous development:

$$\boxed{H_{01} = -\frac{1}{4m^2c^2}\, \underline{\sigma}\cdot\underline{p}\, H_0\, \underline{\sigma}\cdot\underline{p}} \quad -(9)$$

Using the quantization:

$$\underline{p}\,\phi = -i\hbar\, \underline{\nabla}\phi \quad -(10)$$

The new term gives:

$$\boxed{H_{01}\phi = \frac{\hbar^2}{4m^2c^2}\, \underline{\sigma}\cdot\underline{\nabla}\, H_0\, \underline{\sigma}\cdot\underline{\nabla}\,\phi} \quad -(11)$$

where H_0 is the classical hamiltonian, a constant of motion. Therefore:

$$H_{01}\phi = \frac{\hbar^2 H_0}{4m^2c^2}\, \nabla^2\phi \quad -(12)$$

where we have used:

3)

$$\underline{\sigma} \cdot \underline{\nabla} \, \underline{\sigma} \cdot \underline{\nabla} = \nabla^2 \dots \quad (13)$$

and the fact that Ho is a constant.

In the first approximation the hydrogenic wave functions can be used for ψ and:

$$H_0 = - \frac{me^4}{32\pi^2 \epsilon_0^2 \hbar^2 n^2} \quad (14)$$

where n is the principal quantum number of the H atom.

So

$$H_{01} \psi = - \frac{e^4}{128\pi^2 \epsilon_0^2 mc^2 n^2} \nabla^2 \psi \quad (15)$$

and

$$\langle H_{01} \rangle = - \frac{e^4}{128\pi^2 \epsilon_0^2 mc^2 n^2} \int \psi^* \nabla^2 \psi \, d\tau \quad (16)$$

are shifts in the energy levels of the H atom.

These shifts occur in the absence of a vector potential and appear to be new to science.

Conventionally, the vector potential $\underline{A}$ is introduced by:

$$\underline{p} \rightarrow \underline{p} - e\underline{A} \quad (17)$$

So as in 4FT250 and 4FT252 the new

? Hamiltonian (9) can be developed as:

$$H_{01} = -\frac{1}{4m^2c^2} \, \underline{\sigma} \cdot (\underline{p} - e\underline{A}) H_0 \, \underline{\sigma} \cdot (\underline{p} - e\underline{A}) \quad -(18)$$

So

$$\boxed{H_{01}\psi = -\frac{H_0}{4m^2c^2} \, \underline{\sigma} \cdot (\underline{p} - e\underline{A}) \, \underline{\sigma} \cdot (\underline{p} - e\underline{A}) \psi}$$

$$-(19)$$

In ECE2 theory $\underline{A}$ is replaced by a new type of vector potential and the scalar potential is also developed. As in notes for UFT 250, eq. (19) can be developed in several new ways, leading to several new spin orbit effects.

If we denote:

$$\underline{\pi} := \underline{p} - e\underline{A} \quad -(20)$$

then:

$$\underline{\sigma} \cdot \underline{\pi} \, \underline{\sigma} \cdot \underline{\pi} = \underline{\pi} \cdot \underline{\pi} + i \underline{\sigma} \cdot \underline{\pi} \times \underline{\pi} \quad -(21)$$

The term $\underline{\pi} \times \underline{\pi}$ is an operator on ψ, leading to new types of ESR and NMR, MRI and so on.