329(5): Development of New Hamiltonians and Energy Levels

The new Hamiltonian is:

$$H_{01}\phi = -\frac{H_0}{4m^2c^2}\,\underline{\sigma}\cdot(\underline{p}-e\underline{A})\,\underline{\sigma}\cdot(\underline{p}-e\underline{A})\phi \quad -(1)$$

$$= -\frac{H_0}{4m^2c^2}\left(p^2+e^2A^2-e\underline{p}\cdot\underline{A}-e\underline{A}\cdot\underline{p}\right)\phi$$

Now quantize with:

$$-i\hbar\underline{\nabla}\phi = \underline{p}\,\phi \quad -(2)$$

and

$$-\hbar^2\nabla^2\phi = p^2\phi. \quad -(3)$$

It follows that:

$$H_{01}\phi = -\frac{H_0}{4m^2c^2}\left(-\hbar^2\nabla^2+e^2A^2+i\hbar e\underline{\nabla}\cdot\underline{A}+ie\hbar\underline{A}\cdot\underline{\nabla}\right)\phi$$

$$= \frac{\hbar^2H_0}{4m^2c^2}\nabla^2\phi - \frac{e^2A^2H_0}{4m^2c^2}\phi - \frac{ie\hbar H_0}{4m^2c^2}\left(\underline{\nabla}\cdot\underline{A}\phi+2\underline{A}\cdot\underline{\nabla}\phi\right) \quad -(4)$$

In this type of quantization two new type of expectation value appear:

2)

$$\langle H_{011} \rangle = \frac{\hbar^2 H_0}{4m^2c^2} \int \phi^* \nabla^2 \phi \, d\tau \quad -(5)$$

and

$$\langle H_{012} \rangle = -\frac{e^2 H_0 A^2}{4m^2c^2} \int \phi^* \phi \, d\tau \quad -(6)$$

$$= -\frac{e^2 H_0 A^2}{4m^2c^2}$$

It is also possible to consider the quantization scheme:

$$H_{01}\phi = -\frac{H_0}{4m^2c^2} \underline{\sigma} \cdot (-i\hbar \underline{\nabla} - e\underline{A}) \, \underline{\sigma} \cdot (\underline{p} - e\underline{A})\phi \quad -(7)$$

where

$$\underline{p} = \gamma m \underline{v}_0 \quad -(8)$$

In this type of quantization the first $\underline{p}$ is quantized through eq. (2) while the second $\underline{p}$ is the classical relativistic momentum, defined by eq. (8). This type of quantization will be considered in the next note.