325(6): Calculation of the Perihelion Precession of the Earth

This is observed to be 11.45" a year, which is 5.551×10^{-5} radians per year. The ellipticity of the earth's orbit is:

$$\epsilon = 0.0167123 \quad -(1)$$

Assume that:

$$\Delta\theta = 5.551 \times 10^{-5} = 2\pi(1-x) \quad -(2)$$

then

$$x = 1 - \frac{5.551}{2\pi} \times 10^{-5} \quad -(3)$$

$$= 1 - 8.835 \times 10^{-6}$$

It is assumed that this precession is produced by

$$\underline{F} = m\left(\gamma\left(\frac{d^2r}{dt^2}\underline{e}_r + \underline{v}\times\underline{\Omega}\right) - \frac{\gamma^2}{1+\gamma}\frac{\underline{v}}{c}\left(\frac{\underline{v}}{c}\cdot\underline{g}\right)\right) \quad -(4)$$

where

$$\underline{v} = \underline{\omega}\times\underline{r} = \omega r\,\underline{e}_\theta. \quad -(5)$$

It follows that:

$$\underline{F} = m\left(\gamma\left(\frac{d^2r}{dt^2}\underline{e}_r - \omega\Omega r\underline{e}_r\right)\right) \quad -(6)$$

$$= m\gamma\left(\frac{d^2r}{dt^2}\underline{e}_r - \underline{\Omega}\times(\underline{\omega}\times\underline{r})\right)$$

2) In the non-relativistic theory:

$$\underline{F} = m \left(\frac{d^2 r}{dt^2} - \omega^2 r \right) \underline{e}_r \quad -(7)$$

Therefore in the relativistic theory it is assumed that

$$\omega^2 r \rightarrow \omega \Omega r \quad -(8)$$

i.e

$$\omega \rightarrow \Omega \quad -(9)$$

the orbital velocity is effectively changed from $\omega r \underline{e}_\theta$ to $\Omega r \underline{e}_\theta$. So the velocity $\underline{v}$ of the Lorentz transform is changed to:

$$\underline{v}_\Omega = \underline{\Omega} \times \underline{r} \quad -(10)$$

Therefore in eq. (68) of Note 323(5):

$$x^2 + (x^2 - 1)(1 + \epsilon) = \left(1 - \frac{v_\Omega^2}{c^2} \right)^{1/2} \quad -(11)$$

From eq. (3):

$$x^2 \sim 1 - 1.767 \times 10^{-6} \quad -(12)$$

and since

$$v_\Omega << c \quad -(13)$$

$$\left(1 - \frac{v_\Omega^2}{c^2} \right)^{1/2} \sim 1 - \frac{v_\Omega^2}{2c^2} \quad -(14)$$

3) This gives: $$\frac{v_\Omega}{c} = 2.257 \times 10^{-3} \quad -(15)$$

The velocity of the Lorentz transform is therefore $$v_\Omega = 6.7685 \times 10^{5}\ ms^{-1} \quad -(16)$$

This compares with the orbital velocity of the earth about the sun in the usual non-relativistic theory of

$$v = 3 \times 10^{4}\ ms^{-1} \quad -(17)$$

This is a plausible first theory. A more accurate method would be to derive the Binet equation from eq. (4).