322(2): Calculation of the Current Density

Use:

$$\underline{\nabla} \times \underline{\Omega} = \frac{4\pi G}{c^2} \underline{J}_m \quad -(1)$$

where

$$\underline{\Omega} = -\left(\frac{MGL}{E_0}\right) \frac{1}{r^3} \underline{k} \quad -(2)$$

In cylindrical polar coordinates:

$$\underline{\nabla} \times \underline{\Omega} = \left(\frac{1}{r}\frac{\partial \Omega_Z}{\partial \theta} - \frac{\partial \Omega_\theta}{\partial Z}\right)\underline{e}_r + \left(\frac{\partial \Omega_r}{\partial Z} - \frac{\partial \Omega_Z}{\partial r}\right)\underline{e}_\theta$$

$$+ \frac{1}{r}\left(\frac{\partial (r\Omega_\theta)}{\partial r} - \frac{\partial \Omega_r}{\partial \theta}\right)\underline{k} \quad -(3)$$

From eqs. (2) and (3):

$$\underline{\nabla} \times \underline{\Omega} = -\frac{\partial \Omega_Z}{\partial r}\underline{e}_\theta \quad -(4)$$

$$= \frac{3MGL}{E_0 r^4}\underline{e}_\theta$$

so

$$\underline{J}_m = \frac{c^2}{4\pi G} \cdot \frac{3MGL}{E_0 r^4}\underline{e}_\theta$$

$$= \frac{3Mc^2 L}{4\pi E_0 r^4}\underline{e}_\theta \quad -(5)$$

2) So:

$$\boxed{\underline{J}_m = \frac{3\underline{M}}{4\pi m} \frac{L}{r^4} \underline{e}_\theta} \quad -(6)$$

For a conic section orbit:

$$L^2 = m^2 \underline{M} G \alpha \quad -(7)$$

$$\frac{1}{r^4} = \left(\frac{1+\epsilon\cos\theta}{\alpha} \right)^4 \quad -(8)$$

The magnitude of the current is:

$$J_{m\theta} = \frac{3\underline{M}}{4\pi m} m \left(\underline{M} G \alpha\right)^{1/2} \left(\frac{1+\epsilon\cos\theta}{\alpha} \right)^4 \quad -(9)$$

$$= \frac{3\underline{M}^{3/2} G^{1/2}}{4\pi \alpha^{7/2}} \left(1+\epsilon\cos\theta\right)^4$$

$$J_{m\theta} = \frac{3}{4\pi} \left(\frac{G\underline{M}^3}{\alpha^7} \right)^{1/2} \left(1+\epsilon\cos\theta\right)^4 \quad -(10)$$

where

$$\alpha = a\left(1-\epsilon^2\right) \quad -(11)$$