222(5): Calculation of Orbital Precession and Deflection of Light due to Gravity in Terms of the Gravitomagnetic Field.

In the notation of previous papers of the UFT series, the Binet equation is:

$$F(r) = -\frac{L^2}{mr^2}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)+\frac{1}{r}\right) \quad -(1)$$

where $F(r)$ is the force of gravitational attraction between $m$ orbiting $M$, $r$ is the distance between $m$ and $M$, $L$ is the conserved angular momentum and $\theta$ the angle of the plane polar coordinate system $(r, \theta)$. For a precessing orbit:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(2)$$

where the advance of a point such as the perihelion is:

$$\Delta\theta = (x-1)\theta \quad -(3)$$

and $\alpha$ is the half right latitude. In the solar system, $x$ is very close to unity so to an excellent approximation:

$$L^2 = Mm^2G\alpha \quad -(4)$$

From eqs. (1) and (2), as in previous work, the force of attraction between $m$ and $M$ for a precessing orbit is:

2) $$\underline{F} = m\underline{g} = mMG\left(-\frac{x^2}{r^2} + (x^2-1)\frac{\alpha}{r^3}\right)\underline{e}_r \quad -(5)$$

So the magnitude of $g$ is:

$$g = MG\left(-\frac{x^2}{r^2} + (x^2-1)\frac{\alpha}{r^3}\right) \quad -(6)$$

and

$$\underline{g} = g\,\underline{e}_r \quad -(7)$$

For light grazing the sun, the orbit is a hyperbola,

so:

$$\alpha = a(\epsilon^2 - 1) \quad -(8)$$

and the angle of deflection is

$$\Delta\xi = \frac{2}{\epsilon} \quad -(9)$$

The velocity of the orbit is defined by:

$$\underline{v} = \frac{dr}{dt}\underline{e}_r + \omega r\,\underline{e}_\theta \quad -(10)$$

$$= \frac{L}{mr}\underline{e}_\theta + \frac{L}{mr^2}\frac{dr}{d\theta}\underline{e}_r$$

The gravitomagnetic field of the orbit is:

$$\underline{\Omega} = -\frac{1}{c^2}\underline{v}\times\underline{g} \quad -(11)$$

3)

So:

$$\underline{\Omega} = \frac{MGL}{mc^2}\left(-\frac{x^2}{r^3} + \frac{(x^2-1)\alpha}{r^4}\right)\underline{k} \quad -(12)$$

In the solar system:

$$x \sim 1 \quad -(13)$$

to an excellent approximation, so:

$$\Omega_Z = -\frac{MGL}{mc^2 r^3} \quad -(14)$$

and

$$\Omega_Z^2 = \left(\frac{MG}{mc^2 r^3}\right)^2 L^2 \quad -(15)$$

where

$$L^2 = m^2 MG\alpha = m^2 MG\, a\left(\epsilon^2 - 1\right) \quad -(16)$$

Therefore

$$\boxed{\Omega_Z^2 = \frac{(MG)^3}{c^4 r^6}\, a\left(\epsilon^2 - 1\right)} \quad -(17)$$

At closest approach:

$$r = a = R_0 \quad -(18)$$

So:

4)

$$\Omega_Z^2 = \frac{(MG)^3}{c^4 R_0^5}\left(\epsilon^2 - 1\right) \quad -(19)$$

In light deflection by the sun $\epsilon^2$ is very large, so to an excellent approximation:

$$\boxed{\Omega_Z^2 = \frac{(MG)^3 \epsilon^2}{c^4 R_0^5}} \quad -(20)$$

So

$$\Omega_Z = \frac{\epsilon}{c^2} \frac{(MG)^{3/2}}{R_0^{5/2}} \quad -(21)$$

$$\Omega_Z = \frac{\epsilon}{c^2}\left(\frac{(MG)^3}{R_0^5}\right)^{1/2}.$$

The angle of deflection is therefore:

$$\Delta\phi = \frac{2}{\epsilon} = \frac{2}{\Omega_Z c^2}\left(\frac{(MG)^3}{R_0^5}\right)^{1/2} \quad -(22)$$

Therefore light deflection due to gravitation is expressed in terms of known quantities, and is due to the gravitomagnetic field. Similarly the precession of the perihelion is due to the gravitomagnetic field from eq. (12).

5) The Gravitomagnetic Field for Light Deflection

From Eq. (22):

$$\Omega_Z = \frac{2}{\Delta\phi\, c^2}\left(\frac{(MG)^3}{R_0^{\,5}}\right)^{1/2} \quad -(23)$$

For light deflection by the sun, as in UFT 150:

$$\Delta\phi\,(\text{experimental}) = 8.4848 \times 10^{-6} \text{ radians}$$

$$R_0 = 6.955 \times 10^{8} \text{ m}$$

$$M = 1.9891 \times 10^{30} \text{ kg}$$

$$G = 6.67428 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

$$c = 2.9979 \times 10^{8} \text{ m s}^{-1}$$

So

$$\boxed{\Omega_Z = 0.0314 \text{ radians per second}} \quad -(24)$$

In the next note eq. (18) will be used to calculate the precession of the perihelion for a typical planet, showing that the precession is again due to the gravitomagnetic field.