# 322(1): Detailed Calculation of the Gravitomagnetic Field and Mass Current for Planar Orbits.

For planar orbits without precession:

$$\underline{v} = \frac{dr}{dt}\underline{e}_r + \omega r \underline{e}_\theta . \quad -(1)$$

Now use:

$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = \omega \frac{dr}{d\theta} . \quad -(2)$$

The orbit is

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(3)$$

So

$$\frac{dr}{d\theta} = \frac{\epsilon}{\alpha} r^2 \quad -(4)$$

and

$$\frac{dr}{dt} = \frac{\epsilon}{\alpha}\omega r^2 . \quad -(5)$$

Therefore:

$$\underline{v} = \omega r \left( \frac{\epsilon r}{\alpha}\underline{e}_r + \underline{e}_\theta \right) \quad -(6)$$

and

$$\underline{g} = -\frac{MG}{r^2}\underline{e}_r \quad -(7)$$

The gravitomagnetic field of the orbit is:

$$\underline{\Omega} = -\frac{1}{c^2}\underline{v}\times\underline{g} \quad -(8)$$

where:

2)

$$\underline{v} \times \underline{g} = \begin{vmatrix} \underline{e}_r & \underline{e}_\theta & \underline{k} \\ \frac{\epsilon \omega r^2}{\alpha} & \omega r & 0 \\ -\frac{MG}{r^2} & 0 & 0 \end{vmatrix} \quad -(9)$$

So

$$\underline{\Omega} = -\frac{\omega}{c^2} \frac{MG}{r} \underline{k} \quad -(10)$$

where

$$\omega = \frac{L}{mr^2} \quad -(11)$$

by Lagrangian dynamics. Here L is the angular momentum, a constant of motion. So:

$$\underline{\Omega} = -\frac{MGL}{mc^2 r^3} \underline{k} = -\frac{MG}{r^3}\left(\frac{L}{E_0}\right) \underline{k} \quad -(12)$$

where the rest energy:

$$E_0 = mc^2 \quad -(13)$$

is another constant of motion.

Therefore $\underline{\Omega}$ is a relativistic gravity, part of a generally covariant unified field theory, ECE2.

3). In eq. (13):

$$L^2 = m^2 \underline{M} G \alpha \quad -(14)$$

and

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(15)$$

Therefore

$$\underline{\Omega} = -\frac{\underline{M}G}{mc^2}\left(\frac{1+\epsilon\cos\theta}{\alpha}\right)^3 m\left(\underline{M}G\alpha\right)^{1/2}\underline{k}$$

$$= -\frac{(\underline{M}G)^{3/2}}{c^2\alpha^{1/2}}\left(1+\epsilon\cos\theta\right)^3\underline{k}. \quad -(16)$$

The half right latitude is defined by:

$$\alpha = a(1-\epsilon^2) \quad -(17)$$

for an ellipse. Here a is the semi major axis, an observable. Therefore:

$$\boxed{\underline{\Omega} = -\frac{1}{c^2}\left(\frac{(\underline{M}G)^3}{a(1-\epsilon^2)}\right)^{1/2}\left(1+\epsilon\cos\theta\right)^3\underline{k}}$$

$-(18)$

As usual in astronomy $a$ and $\epsilon$ can be observed and are tabulated in ephemera.

## Graphics

We have:

$$\Omega_Z = -\frac{1}{c^2}\left(\frac{(MG)^3}{a(1-\epsilon^2)}\right)^{1/2}\left(1+\epsilon\cos\theta\right)^3 \quad -(19)$$

so the plot of $\Omega_Z$ against $\theta$ can be made for a given $\epsilon$ and $a$. We have:

$$\frac{1}{\Omega_Z^{1/3}} = -\left(c^2\frac{(a(1-\epsilon^2))}{(MG)^3}\right)^{-1/6}\cdot\left(\frac{1}{1+\epsilon\cos\theta}\right) \quad -(20)$$

so the plot of $1/\Omega_Z^{1/3}$ against $\theta$ is an ellipse, or more generally a conical section.

This is a fundamentally new discovery in astronomy.

## Calculation of the Mass Current $\underline{J}_m$

The mass current is defined by:

$$\underline{J}_m = -\frac{1}{4\pi G}\,\underline{\nabla}\times(\underline{v}\times\underline{g}) \quad -(21)$$

In Cartesian coordinates:

$$\underline{J}_m = -\frac{1}{4\pi G}\left(\underline{v}(\underline{\nabla}\cdot\underline{g}) - (\underline{\nabla}\cdot\underline{v})\underline{g} + (\underline{g}\cdot\underline{\nabla})\underline{v} - (\underline{v}\cdot\underline{\nabla})\underline{g}\right) \quad -(22)$$

Now transform $\underline{v}$ and $\underline{g}$ from cylindrical to Cartesian coordinates using:

$$\underline{e}_r = \underline{i}\cos\theta + \underline{j}\sin\theta \quad -(23)$$

$$\underline{e}_\theta = -\underline{i}\sin\theta + \underline{j}\cos\theta \quad -(24)$$

So

$$\underline{g} = -\frac{mG}{r^2}\left(\underline{i}\cos\theta + \underline{j}\sin\theta\right) \quad -(25)$$

$$\underline{v} = \omega r\left(\frac{\epsilon r}{\alpha}\left(\underline{i}\cos\theta + \underline{j}\sin\theta\right) - \underline{i}\sin\theta + \underline{j}\cos\theta\right) \quad -(26)$$

where:

$$\cos\theta = \frac{X}{r}, \quad \sin\theta = \frac{Y}{r} \quad -(27)$$

So:

$$\underline{g} = -\frac{mG}{r^2}\left(X\underline{i} + Y\underline{j}\right) \quad -(28)$$

$$\underline{v} = \omega r\left(\frac{\epsilon r}{\alpha}\left(\underline{i}\cos\theta + \underline{j}\sin\theta\right) - \underline{i}\sin\theta + \underline{j}\cos\theta\right)$$

$$= \omega r\left(\frac{\epsilon}{\alpha}\left(X\underline{i} + Y\underline{j}\right) - \frac{Y}{r}\underline{i} + \frac{X}{r}\underline{j}\right)$$

6) $$= \omega r \left( \left( \frac{\epsilon X}{\alpha} - \frac{Y}{r} \right) \underline{i} + \left( \frac{\epsilon Y}{\alpha} + \frac{X}{r} \right) \underline{j} \right) \quad -(29)$$

Finally evaluate eq. (22) by computer algebra and graph the results. We have:

$$\underline{\nabla} \cdot \underline{g} = \frac{\partial g_X}{\partial X} + \frac{\partial g_Y}{\partial Y} \quad -(30)$$

$$\underline{\nabla} \cdot \underline{v} = \frac{\partial v_X}{\partial X} + \frac{\partial v_Y}{\partial Y} \quad -(31)$$

$$\underline{g} \cdot \underline{\nabla} = g_X \frac{\partial}{\partial X} + g_Y \frac{\partial}{\partial Y} \quad -(32)$$

$$\underline{v} \cdot \underline{\nabla} = v_X \frac{\partial}{\partial X} + v_Y \frac{\partial}{\partial Y} \quad -(33)$$

and

$$r^2 = X^2 + Y^2 \quad -(34)$$

The final result $\underline{J}_m(X, Y)$ can be back transformed into $\underline{J}_m(r, \theta)$.