322(6): Gravitomagnetic Field for the Precession of the Perihelion of Mercury

This is defined by:

$$\Omega_Z = \frac{MGL}{mc^2}\left(-\frac{x^2}{r^3} + \frac{(x^2-1)\alpha}{r^4}\right) \quad -(1)$$

in previous notation, where:

$$L^2 = m^2 MG\alpha \quad -(2)$$

Here

$$\alpha = b(1-\epsilon^2)^{1/2} \quad -(3)$$

where b is the perihelion, and ϵ the eccentricity. If the precession at the perihelion is $\Delta\theta$, then:

$$\Delta\theta = (x-1)\frac{\pi}{2} \quad -(4)$$

Therefore at the perihelion ($r=b$):

$$\Omega_Z = -\frac{(MG)^{3/2}(1-\epsilon^2)^{1/2}x^2}{c^2 b^{5/2}} \quad -(5)$$

to an excellent approximation.

Here

$$\Delta\theta = 43.11'' \text{ per century} \quad -(6)$$
$$= 7.9673 \times 10^{-7} \text{ radians per year}$$

2) so

$$x = 1 + 1.268 \times 10^{-7} \quad -(7)$$

and

$$x^2 - 1 \sim 0 \quad -(8)$$

to an excellent approximation, thus justifying eq. (5). The mass of Mercury is:

$$M = 3.285 \times 10^{23} \text{ kgm}, \quad -(9)$$

the perihelion of Mercury is:

$$b = 4.60012 \times 10^{10} \text{ m}, \quad -(10)$$

the eccentricity of Mercury is:

$$\epsilon = 0.205630 \quad -(11)$$

We have:

$$G = 6.67428 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \quad -(12)$$

$$c = 2.9979 \times 10^{8} \text{ ms}^{-1} \quad -(13)$$

so for the precession of the perihelion of Mercury:

$$\boxed{\Omega_Z = -2.462 \times 10^{-28} \text{ rad s}^{-1}} \quad -(14)$$

in ECE2, using the gravitomagnetic Ampère equation.