319(5): Solutions of the ECE2 Equivalence Principle

In ECE2 theory the force is defined by:

$$\underline{F} = m\underline{g} = -\underline{\nabla}U - \frac{\partial \underline{p}}{\partial t} - 2U\underline{\omega} + 2c\omega_0\underline{p} \quad -(1)$$

as part of a generally covariant unified field theory. Here U is the potential energy:

$$U = m\Phi \quad -(2)$$

where m is the mass of an object attracted by a mass M. The spin connection four vector is:

$$\omega^\mu = (\omega_0, \underline{\omega}), \quad -(3)$$

the momentum $\underline{p}$ is defined by:

$$\underline{p} = m\underline{Q}. \quad -(4)$$

The gravitational four potential is:

$$\Phi^\mu = \left(\frac{\Phi}{c}, \underline{Q}\right). \quad -(5)$$

If it is assumed that the equivalent of the magnetic monopole is zero the field equations are:

$$\underline{\nabla} \cdot \underline{\Omega} = 0 \quad -(6)$$

$$\underline{\nabla} \times \underline{g} + \frac{\partial \underline{\Omega}}{\partial t} = \underline{0} \quad -(7)$$

$$\underline{\nabla} \cdot \underline{g} = \underline{\kappa} \cdot \underline{g} = 4\pi G \rho_m \quad -(8)$$

$$\underline{\nabla} \times \underline{\Omega} - \frac{1}{c^2}\frac{\partial \underline{g}}{\partial t} = \frac{4\pi G}{c^2}\underline{J}_m = \underline{\kappa} \times \underline{\Omega} \quad -(9)$$

2) These have the same mathematical structure as the Maxwell Heaviside (MH) equations of electrodynamics but are the gravitational equations of ECE2 in a space with non-zero torsion and curvature, part of a generally covariant unified field theory, a theory of relativity. The general covariance of Eqs. (6) to (9) takes the form of a quasi Lorentz covariance. The MH theory is Lorentz covariant and is a theory of special relativity.

By the ECE antisymmetry law:

$$-\underline{\nabla}\Omega - \frac{\partial \underline{p}}{\partial t} = -2\Omega\underline{\omega} + 2c\omega_0\underline{p} \quad -(10)$$

Eq. (10) is the ECE2 principle of equivalence. It generalizes the Newton and Einstein principles of equivalence.

From eqs. (1) and (10):

$$\underline{F} = m\underline{g} = 2\left(-\underline{\nabla}\Omega - \frac{\partial \underline{p}}{\partial t}\right) = 4\left(c\omega_0\underline{p} - \Omega\underline{\omega}\right) \quad -(11)$$

These equations are much more richly structured than the Newtonian theory of universal gravitation, which is defined by:

$$\underline{F} = m\underline{g} = -m\underline{\nabla}\phi = -\frac{mMG}{r^2}\underline{e}_r \quad (12)$$

 where ϕ is the Newtonian potential. Eq. (12) is the Newtonian principle of equivalence.

In this note a complete solution of eq. (11) is given in the Newtonian limit (12). In other words conditions are found under which eq. (11) reduces to eq. (12), and the tetrad and spin connection vectors found for this solution.

In the Newtonian limit the acceleration due to gravity is:

$$\underline{g} = -\frac{MG}{r^2}\underline{e}_r \quad -(13)$$

where M is the mass of an object such as the earth that attracts m, and G is the Newton constant. From eq. (8):

$$\underline{\nabla}\cdot\underline{g} = \underline{\kappa}\cdot\underline{g} \quad -(14)$$

where

$$\underline{\kappa} = \frac{1}{r^{(0)}}\underline{q} - \underline{\omega} \quad -(15)$$

and where the tetrad four vector is defined by:

$$q^\mu = \left(q^{(0)}, \underline{q}\right) \quad -(16)$$

From eqs. (13) and (14):

$$\underline{\nabla}\cdot\underline{g} = \frac{2MG}{r^3} \quad -(17)$$

and

$$\underline{\kappa}\cdot\underline{g} = -\frac{MG}{r^2}\kappa_r \quad -(18)$$

4) where
$$\underline{\kappa} = \kappa_r \, \underline{e}_r \quad -(19)$$

Therefore
$$\kappa_r = -\frac{2}{r} \quad -(20)$$

and
$$\boxed{\frac{1}{r^{(0)}} q_r - \omega_r = -\frac{2}{r}} \quad -(21)$$

Eqs. (20) and (21) can be interpreted as the geometrical explanation of universal gravitation or as "the reason for gravity". Newton did not attempt to explain the reason for gravity, he was informed of the inverse square law (13) by Robert Hooke. Newton's contribution was to derive the elliptical orbit:

$$r = \frac{\alpha}{1 + \epsilon \cos\theta} \quad -(21a)$$

from the inverse square law (12).

ECE2 theory goes much further than this.

In order to reduce eq. (11) to eq. (12) use:

$$\underline{\nabla} U = \frac{\partial p}{\partial t} \quad -(22)$$

i.e.

$$\underline{\nabla} \Phi = \frac{\partial Q}{\partial t} \quad -(23)$$

and

$$c\omega_0 \underline{p} = -\Phi \underline{\omega} \quad -(24)$$

Eqs. (22) and (24) are the Newtonian equivalence principles.

Using eqs. (22) and (24) in eq. (11):

$$\underline{F} = m\underline{g} = -4\underline{\nabla}\Phi = -8\Phi\underline{\omega}$$
$$= -\frac{mMG}{r^2}\,\underline{e}_r \quad -(25)$$

So

$$\boxed{\Phi = -\frac{mMG}{4r}} \quad -(26)$$

and

$$\underline{\nabla}\Phi = \frac{mMG}{4r^2}\underline{e}_r \quad -(27)$$

From eq. (25):

$$\underline{\nabla}\Phi = -2\Phi\underline{\omega} \quad -(28)$$

so

$$\frac{mMG}{4r^2}\underline{e}_r = \frac{mMG}{2r}\underline{\omega} \quad -(29)$$

i.e.

$$\boxed{\underline{\omega} = \frac{1}{2r}\underline{e}_r} \quad -(30)$$

Using eq. (15) and (21):

$$\underline{\kappa} = \frac{1}{r^{(0)}}\underline{q} - \underline{\omega} = -\frac{2}{r}\underline{e}_r \quad -(31)$$

) From eqs. (30) and (31):

$$\boxed{\underline{q} = -\frac{5r^{(0)}}{2r}\underline{e}_r} \quad -(32)$$

Eqs. (30) and (32) are the geometrical reason for universal gravitation:

$$\underline{\omega} = \frac{1}{2r}\underline{e}_r, \quad \underline{q} = -\frac{5}{2}\frac{r^{(0)}}{r}\underline{e}_r \quad -(33)$$

From eq. (24):

$$\omega_0 c \underline{p} = -U\underline{\omega} = \frac{mMG}{8r^2}\underline{e}_r \quad -(34)$$

and self consistently:

$$\underline{g} = -8c\omega_0\underline{p} = -\frac{MG}{r^2}\underline{e}_r \quad -(35)$$

Finally:

$$\frac{d\underline{p}}{dt} = \underline{\nabla}U = -\frac{mMG}{8r^2}\underline{e}_r \quad -(36)$$

so

$$\underline{p} = -\int_0^\tau \frac{mMG}{8r^2}\underline{e}_r\, dt \quad -(37)$$

If

$$\underline{p} = p_r\,\underline{e}_r \quad -(38)$$

then from eqs. (35) and (37):

$$-8c\omega_0 p_r = -\frac{mMG}{r^2} \quad -(39)$$

where

$$p_r = -\int_0^{\tau} \frac{mMG}{8r^2}\, dt \quad -(40)$$

Therefore:

$$\boxed{\omega_0 c = \frac{1}{r^2}\left(\int_0^{\tau} \frac{1}{r^2}\, dt\right)^{-1}} \quad -(41)$$

In these calculations:

$$\boxed{\phi = 4\Phi} \quad -(42)$$

From eqs. (1), (22) and (24):

$$\boxed{\begin{aligned} \underline{F} = m\underline{g} &= -4\underline{\nabla}\,\Phi = -8\,\Phi\,\underline{\omega} \\ &= -4\frac{d\underline{p}}{dt} = 8c\omega_0\,\underline{p} \end{aligned}} \quad -(43)$$

Under these conditions the ECE2 theory reduces to the Newtonian theory of universal gravitation. However, the Newtonian theory is classical, and non-relativistic. The ECE2 theory is a theory of general relativity, part of a generally covariant unified field theory. The following is a summary of the Newtonian solution of ECE2.

8) SUMMARY OF THE NEWTONIAN SOLUTION

$$\phi = 4\Phi \quad -(44)$$

$$\omega^{\mu} = (\omega_0, \underline{\omega}) = \left(\frac{1}{cr^2}\left(\int_0^{\tau}\frac{1}{r^2}\,dt\right)^{-1}, \frac{1}{2r}\underline{e}_r\right) \quad -(45)$$

$$\underline{\kappa} = \frac{1}{r^{(0)}}\underline{v} - \underline{\omega} = -\frac{2}{r}\underline{e}_r \quad -(46)$$

$$\underline{v} = -\frac{5r^{(0)}}{2r}\underline{e}_r \quad -(47)$$

$$\underline{\nabla}U = \frac{d\underline{p}}{dt} \quad -(48)$$

$$c\omega_0\underline{p} = -U\underline{\omega} \quad -(49)$$

$$\underline{F} = m\underline{g} = -4\underline{\nabla}U = -8U\underline{\omega} = -4\frac{d\underline{p}}{dt} = 8c\omega_0\underline{p} \quad -(50)$$

$$\underline{\omega} = \frac{1}{2r}\underline{e}_r \quad -(51)$$

$$v_0 = r^{(0)}\omega_0 \quad -(52)$$

$$= \frac{r^{(0)}}{cr^2}\left(\int_0^{\tau}\frac{1}{r^2}\,dt\right)^{-1} \quad -(53)$$