315(7): The Complete Set of Electromagnetic Field Equations from the First and Second JCE Identities in Four Dimensions.

Consider the Jacobi Cartan Evans identity in a space of any dimensionality: $\quad -(1)$

$$D_\rho R^a{}_{\lambda\mu\nu} + D_\nu R^a{}_{\lambda\rho\mu} + D_\mu R^a{}_{\lambda\nu\rho}$$
$$:= R^a{}_{\lambda\rho\alpha} T^\alpha{}_{\mu\nu} + R^a{}_{\lambda\nu\alpha} T^\alpha{}_{\rho\mu} + R^a{}_{\lambda\mu\alpha} T^\alpha{}_{\nu\rho}$$

In four dimensions, the second JCE identity is: $\quad -(2)$

$$D_\rho \tilde{R}^a{}_{\lambda\mu\nu} + D_\nu \tilde{R}^a{}_{\lambda\rho\mu} + D_\mu \tilde{R}^a{}_{\lambda\nu\rho}$$
$$:= R^a{}_{\lambda\rho\alpha} \tilde{T}^\alpha{}_{\mu\nu} + R^a{}_{\lambda\nu\alpha} \tilde{T}^\alpha{}_{\rho\mu} + R^a{}_{\lambda\mu\alpha} \tilde{T}^\alpha{}_{\nu\rho}$$

Eqs. (1) and (2) are respectively:

$$D_\mu \tilde{R}^a{}_\lambda{}^{\mu\nu} := R^a{}_{\lambda\mu\alpha} \tilde{T}^{\alpha\mu\nu} \quad -(3)$$

and

$$D_\mu R^a{}_\lambda{}^{\mu\nu} := R^a{}_{\lambda\mu\alpha} T^{\alpha\mu\nu} \quad -(4)$$

Now define:

$$\tilde{R}^a{}_\lambda{}^{\mu\nu} := q^a{}_\lambda \tilde{R}^{\mu\nu} \quad -(5)$$

and

$$R^a{}_\lambda{}^{\mu\nu} := q^a{}_\lambda R^{\mu\nu} \quad -(6)$$

Using the tetrad postulate it follows

2) that:

$$D_\mu \tilde{R}^a{}_\lambda{}^{\mu\nu} = D_\mu \left(q^a{}_\lambda \tilde{R}^{\mu\nu} \right)$$

$$= \left(D_\mu q^a{}_\lambda \right) \tilde{R}^{\mu\nu} + q^a{}_\lambda D_\mu \tilde{R}^{\mu\nu} \quad -(7)$$

$$= q^a{}_\lambda D_\mu \tilde{R}^{\mu\nu}$$

because: $$D_\mu q^a{}_\lambda = 0 \quad -(8)$$

by the tetrad postulate.

So eqs. (3) and (4) become:

$$q^a{}_\lambda D_\mu \tilde{R}^{\mu\nu} = R^a{}_{\lambda\mu\alpha} \tilde{T}^{\alpha\mu\nu} \quad -(9)$$

and $$q^a{}_\lambda D_\mu R^{\mu\nu} = R^a{}_{\lambda\mu\alpha} T^{\alpha\mu\nu} \quad -(10)$$

i.e. $$D_\mu \tilde{R}^{\mu\nu} = q^\lambda{}_a R^a{}_{\lambda\mu\alpha} \tilde{T}^{\alpha\mu\nu} \quad -(11)$$

$$= R_{\mu\alpha} \tilde{T}^{\alpha\mu\nu}$$

and $$D_\mu R^{\mu\nu} = R_{\mu\alpha} T^{\alpha\mu\nu} \quad -(12)$$

So the first and second JCE identities in four dimensions are:

$$\boxed{\begin{aligned} D_\mu \tilde{R}^{\mu\nu} &= R_{\mu\alpha} \tilde{T}^{\alpha\mu\nu} \quad -(13) \\ D_\mu R^{\mu\nu} &= R_{\mu\alpha} T^{\alpha\mu\nu} \quad -(14) \end{aligned}}$$

3) Now use the definition of the covariant derivative of a rank two tensor:

$$D_\sigma T^{\mu_1 \mu_2} = \partial_\sigma T^{\mu_1 \mu_2} + \Gamma^{\mu_1}{}_{\sigma\lambda} T^{\lambda \mu_2} + \Gamma^{\mu_2}{}_{\sigma\lambda} T^{\mu_1 \lambda} \quad -(15)$$

(S.M. Carroll, online notes, chapter 3).

So:

$$D_\mu \tilde{R}^{\mu\nu} = \partial_\mu \tilde{R}^{\mu\nu} + \Gamma^{\mu}{}_{\mu\lambda} \tilde{R}^{\lambda\nu} + \Gamma^{\nu}{}_{\mu\lambda} \tilde{R}^{\mu\lambda} \quad -(16)$$

$$D_\mu R^{\mu\nu} = \partial_\mu R^{\mu\nu} + \Gamma^{\mu}{}_{\mu\lambda} R^{\lambda\nu} + \Gamma^{\nu}{}_{\mu\lambda} R^{\mu\lambda} \quad -(17)$$

It follows that:

$$\boxed{\begin{aligned} \partial_\mu \tilde{R}^{\mu\nu} &= j^\nu \quad -(18) \\ \partial_\mu R^{\mu\nu} &= J^\nu \quad -(19) \end{aligned}}$$

where:

$$j^\nu := R_{\mu\alpha} \tilde{T}^{\alpha\mu\nu} - \Gamma^{\mu}{}_{\mu\lambda} \tilde{R}^{\lambda\nu} - \Gamma^{\nu}{}_{\mu\lambda} \tilde{R}^{\mu\lambda} \quad -(20)$$

and

$$J^\nu = R_{\mu\alpha} T^{\alpha\mu\nu} - \Gamma^{\mu}{}_{\mu\lambda} R^{\lambda\nu} - \Gamma^{\nu}{}_{\mu\lambda} R^{\mu\lambda} \quad -(21)$$

Now define the electromagnetic field tensor $F^{\mu\nu}$ and its Hodge dual $\tilde{F}^{\mu\nu}$ as:

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$$F^{\mu\nu} := W^{(0)} R^{\mu\nu} \quad -(22)$$

and

$$\tilde{F}^{\mu\nu} := W^{(0)} \tilde{R}^{\mu\nu} \quad -(23)$$

Eqs. (22) and (23) are new ECE hypotheses.

It follows that:

$$\partial_\mu \tilde{F}^{\mu\nu} = W^{(0)} j^{\nu} := j_m^{\nu} \quad -(24)$$

and

$$\partial_\mu F^{\mu\nu} = W^{(0)} J^{\nu} := J_E^{\nu}, \quad -(25)$$

where j_m^{ν} and J_E^{ν} are the magnetic and electric charge current densities.

Define the field tensors as:

$$\tilde{F}^{\mu\nu} := \begin{bmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & E_z/c & -E_y/c \\ B_y & -E_z/c & 0 & E_x/c \\ B_z & E_y/c & -E_x/c & 0 \end{bmatrix} \quad -(26)$$

and

$$F^{\mu\nu} := \begin{bmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{bmatrix} \quad -(27)$$

to obtain:

5)

$$\underline{\nabla} \cdot \underline{B} = W^{(0)} j^0 = j_M^0 \quad -(28)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = c W^{(0)} \underline{j} = \underline{j}_m \quad -(29)$$

$$\underline{\nabla} \cdot \underline{E} = c W^{(0)} J^0 = J_E^0 \quad -(30)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = W^{(0)} \underline{J} = \underline{J}_E \quad -(31)$$

In these equations:

$$\underline{j} = j^1 \underline{i} + j^2 \underline{j} + j^3 \underline{k} = j_x \underline{i} + j_y \underline{j} + j_z \underline{k} \quad -(32)$$

and

$$\underline{J} = J^1 \underline{i} + J^2 \underline{j} + J^3 \underline{k} = J_x \underline{i} + J_y \underline{j} + J_z \underline{k} \quad -(33)$$

For:

$$\mu = 0, 1, 2, 3 \quad -(34)$$

Eq. (20) defines j^0, j^1, j^2 and j^3; and
Eq. (21) defines J^0, J^1, J^2 and J^3.

Eq. (22) has the correct units because $F^{\mu\nu}$ is in tesla, $W^{(0)}$ is in weber or m^2 tesla, and $R^{\mu\nu}$ is in inverse metres squared. The units of j^μ and J^μ are inverse metres cubed, so $W^{(0)} j^{(0)}$ is correctly in units of tesla per metre.

6) Eqs. (28) to (31) have the same vector structure as the Maxwell Heaviside equations, but in a space with non-zero torsion and curvature.

The Magnetic Monopole or magnetic charge density

This is:

$$j_m^0 = \overline{W}^{(0)} j^0,$$

$$j^0 = R_{\mu\alpha}\tilde{T}^{\alpha\mu 0} - \Gamma^{\mu}_{\mu\lambda}\tilde{R}^{\lambda 0} - \Gamma^{0}_{\mu\lambda}\tilde{R}^{\mu\lambda} \quad -(34)$$

The Magnetic Current Density

This is

$$\underline{j}_m = c\overline{W}^{(0)}\underline{j} \quad -(35)$$

where $\underline{j}$ is given by eq. (32) and the components of $\underline{j}$ are: $\quad -(36)$

$$j^{\sim} = R_{\mu\alpha}\tilde{T}^{\alpha\mu\sim} - \Gamma^{\mu}_{\mu\lambda}\tilde{R}^{\lambda\sim} - \Gamma^{\sim}_{\mu\lambda}\tilde{R}^{\mu\lambda}$$

with

$$\sim = 1, 2, 3 \quad -(37)$$

The Electric Charge Density

This is: $$J_E^0 = c\overline{W}^{(0)} J^0 \quad -(38)$$

where:

7) $$J^0 = R_{\mu\alpha} T^{\alpha\mu 0} - \Gamma^\mu_{\mu\lambda} R^{\lambda 0} + \Gamma^0_{\mu\lambda} R^{\mu\lambda} \quad - (39)$$

The Electric Current Density

This is:

$$\underline{J}_E = W^{(0)} \underline{J} \quad - (40)$$

where:

$$\underline{J} = J^1 \underline{i} + J^2 \underline{j} + J^3 \underline{k} \quad - (41)$$

and for $\nu = 1, 2, 3$: – (42)

$$J^\nu = R_{\mu\alpha} T^{\alpha\mu\nu} - \Gamma^\mu_{\mu\lambda} R^{\lambda\nu} - \Gamma^\nu_{\mu\lambda} R^{\mu\lambda} \quad - (43)$$

Equations with the same structure can be derived for the gravitational field, and weak and strong nuclear fields. They provide an internal geometrical structure for the magnetic and electric charge current densities. There is exact symmetry between eqs. (24) and (25). Obviously, eqs (28) to (31) have been tested many times in many laboratories.

So the JCE identities give the correct experimental results.

) It is possible to define the electromagnetic field as:

$$F^a{}_{\lambda\mu\nu} := W^{(0)} R^a{}_{\lambda\mu\nu} . \quad -(44)$$

The original ECE hypothesis is:

$$F^a{}_{\mu\nu} = A^{(0)} T^a{}_{\mu\nu} . \quad -(45)$$

The Cartan identity is:

$$D_\mu T^a{}_{\nu\rho} + D_\rho T^a{}_{\mu\nu} + D_\nu T^a{}_{\rho\mu} := R^a{}_{\mu\nu\rho} + R^a{}_{\rho\mu\nu} + R^a{}_{\nu\rho\mu} . \quad -(46)$$

i.e.

$$D_\mu \tilde{T}^{a\mu\nu} := \tilde{R}^a{}_\mu{}^{\mu\nu} \quad -(47)$$

in four dimensions. Eq. (46) is true in all dimensions.

Therefore:

$$D_\mu F^a{}_{\nu\rho} + D_\rho F^a{}_{\mu\nu} + D_\nu F^a{}_{\rho\mu} := \frac{A^{(0)}}{W^{(0)}} \left(F^a{}_{\mu\nu\rho} + F^a{}_{\rho\mu\nu} + F^a{}_{\nu\rho\mu} \right) \quad -(47)$$

where

$$W^{(0)} = B^{(0)} m^2 = A^{(0)} m . \quad -(48)$$