315(6): Vector Format of the JCE Identity

Consider the JCE identity in the format:

$$D_\rho R^a{}_{\lambda\mu\nu} + D_\nu R^a{}_{\lambda\rho\mu} + D_\mu R^a{}_{\lambda\nu\rho} := R^a{}_{\lambda\rho\alpha} T^\alpha{}_{\mu\nu} + R^a{}_{\lambda\nu\alpha} T^\alpha{}_{\rho\mu} + R^a{}_{\lambda\mu\alpha} T^\alpha{}_{\nu\rho} \quad -(1)$$

This equation can be written as:

$$D_\mu \tilde{R}^a{}_\lambda{}^{\mu\nu} := R^a{}_{\lambda\mu\alpha} \tilde{T}^{\alpha\mu\nu} \quad -(2)$$

where the tilde denotes Hodge duality. Eq. (2) is:

$$\partial_\mu \tilde{R}^a{}_\lambda{}^{\mu\nu} + \omega^a{}_{\mu b} \tilde{R}^b{}_\lambda{}^{\mu\nu} := R^a{}_{\lambda\mu\alpha} \tilde{T}^{\alpha\mu\nu} \quad -(3)$$

Eq. (3) is:

$$\boxed{\partial_\mu \tilde{R}^a{}_\lambda{}^{\mu\nu} := j^a{}_\lambda{}^\nu} \quad -(4)$$

where:

$$j^a{}_\lambda{}^\nu = R^a{}_{\lambda\mu\alpha} \tilde{T}^{\alpha\mu\nu} - \omega^a{}_{\mu b} \tilde{R}^b{}_\lambda{}^{\mu\nu} \quad -(5)$$

Now assume that the Hodge dual curvature tensor can be written as:

2)

$$\tilde{R}^{a}{}_{\lambda}{}^{\mu\nu} = \begin{bmatrix} 0 & -\tilde{R}^{a}{}_{\lambda X}(\text{spin}) & -\tilde{R}^{a}{}_{\lambda Y}(\text{spin}) & -\tilde{R}^{a}{}_{\lambda Z}(\text{spin}) \\ \tilde{R}^{a}{}_{\lambda X}(\text{spin}) & 0 & \tilde{R}^{a}{}_{\lambda Z}(\text{orb}) & -\tilde{R}^{a}{}_{\lambda Y}(\text{orb}) \\ \tilde{R}^{a}{}_{\lambda Y}(\text{spin}) & -\tilde{R}^{a}{}_{\lambda Z}(\text{orb}) & 0 & \tilde{R}^{a}{}_{\lambda X}(\text{orb}) \\ \tilde{R}^{a}{}_{\lambda Z}(\text{spin}) & \tilde{R}^{a}{}_{\lambda Y}(\text{orb}) & -\tilde{R}^{a}{}_{\lambda X}(\text{orb}) & 0 \end{bmatrix} \quad -(6)$$

It follows that:

$$\underline{\nabla} \cdot \underline{\tilde{R}}^{a}{}_{\lambda}(\text{spin}) = j^{a}{}_{\lambda}{}^{0} \quad -(7)$$

$$\underline{\nabla} \times \underline{\tilde{R}}^{a}{}_{\lambda}(\text{orb}) + \frac{\partial \underline{\tilde{R}}^{a}{}_{\lambda}(\text{spin})}{\partial t} = \underline{j}^{a}{}_{\lambda} \quad -(8)$$

where:

$$\underline{j}^{a}{}_{\lambda} = j^{a}{}_{\lambda}{}^{1}\,\underline{i} + j^{a}{}_{\lambda}{}^{2}\,\underline{j} + j^{a}{}_{\lambda}{}^{3}\,\underline{k} \quad -(9)$$

Eqs. (7) to (9) give the required vector format of the Jacobi Cartan Evans identity.

Note the similarity of eqs. (7) and (8) to the homogeneous equations of electromagnetism in ECE theory. The latter can be written for polarization index a as:

3)

$$\underline{\nabla} \cdot \underline{B}^a = j^{a0} \quad -(10)$$

$$\underline{\nabla} \times \underline{E}^a + \frac{\partial \underline{B}^a}{\partial t} = \underline{j}^a \quad -(11)$$

where
$$j^{a\mu} = (j^{a0}, \underline{j}^a) \quad -(12)$$

is the magnetic charge current density. Here j^{a0} is the magnetic monopole.

Now introduce the concept of two index electric and magnetic fields.

The field tensor is defined as:

$$\boxed{\tilde{F}^a{}_\lambda{}^{\mu\nu} = \bar{W}^{(0)} \tilde{R}^a{}_\lambda{}^{\mu\nu}} \quad -(13)$$

where $\bar{W}^{(0)}$ has the units of weber, the units of magnetic flux:

$$\text{weber} = \text{volt sec} = \text{tesla m}^2 \quad -(14)$$

So
$$\bar{W}^{(0)} = m^2 B^{(0)} = m A^{(0)} \quad -(15)$$

In comparison the one index e/m tensor is:

$$\tilde{F}^{a\mu\nu} = A^{(0)} \tilde{T}^{a\mu\nu} \quad -(16)$$

4) Therefore eq. (13) is a new ECE Hypothesis.

Finally use the definitions:

$$\tilde{F}^{\mu\nu} := q^{\lambda}{}_{a}\tilde{F}^{a}{}_{\lambda}{}^{\mu\nu} \quad -(17)$$

$$\tilde{R}^{\mu\nu} := q^{\lambda}{}_{a}\tilde{R}^{a}{}_{\lambda}{}^{\mu\nu} \quad -(18)$$

So

$$\boxed{\tilde{F}^{\mu\nu} = W^{(0)}\tilde{R}^{\mu\nu}} \quad -(19)$$

In Eq. (19):

$$\tilde{F}^{\mu\nu} = \begin{bmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & E_z/c & -E_y/c \\ B_y & -E_z/c & 0 & E_x/c \\ B_z & E_y/c & -E_x/c & 0 \end{bmatrix} \quad -(20)$$

and

$$\tilde{R}^{\mu\nu} = \begin{bmatrix} 0 & -\tilde{R}_x(\text{spin}) & -\tilde{R}_y(\text{spin}) & -\tilde{R}_z(\text{spin}) \\ \tilde{R}_x(\text{spin}) & 0 & \tilde{R}_z(\text{orb}) & -\tilde{R}_y(\text{orb}) \\ \tilde{R}_y(\text{spin}) & -\tilde{R}_z(\text{orb}) & 0 & \tilde{R}_x(\text{orb}) \\ \tilde{R}_z(\text{spin}) & -\tilde{R}_y(\text{orb}) & -\tilde{R}_x(\text{orb}) & 0 \end{bmatrix} \quad -(21)$$

So

$$\boxed{\begin{aligned} \underline{B} &= W^{(0)}\underline{\tilde{R}}(\text{spin}) \quad -(22) \\ \frac{\underline{E}}{c} &= W^{(0)}\underline{\tilde{R}}(\text{orb}) \quad -(23) \end{aligned}}$$

5) With these definitions eqs. (7) to (9) become:

$$\underline{\nabla} \cdot \underline{B} = j^0 \quad -(24)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{j} \quad -(25)$$

where j^0 is the magnetic monopole and $\underline{j}$ the magnetic charge density.

The latter are defined geometrically from eq. (5):

$$j^{\nu} = q^{\lambda}_{\ a}\left(R^{a}_{\ \lambda\mu\alpha}\tilde{T}^{\alpha\mu\nu} - \omega^{a}_{\ \mu b}\tilde{R}^{b}_{\ \lambda}{}^{\mu\nu}\right)$$

$$= q^{\lambda}_{\ a}\, j^{a}_{\ \lambda}{}^{\nu}$$

$$= R_{\mu\alpha}\tilde{T}^{\alpha\mu\nu} - \omega^{\lambda}_{\ \mu b}\tilde{R}^{b}_{\ \lambda}{}^{\mu\nu}$$

$$-(26)$$

The magnetic monopole is:

$$j^0 = R_{\mu\alpha}\tilde{T}^{\alpha\mu 0} - \omega^{\lambda}_{\ \mu b}\tilde{R}^{b}_{\ \lambda}{}^{\mu 0}$$

$$-(27)$$

The Jacobi Cartan Evans identity leads directly to the field equations (24) and (25), which reduce to the MH homogeneous equations in the general space if there is no magnetic charge - current density.