315(4): Development of Eqs. (7) to (10) of Note 315(3)

Consider the first Evans identity:

$$T^{b}_{\mu a} \tilde{T}^{a\mu\nu} := 0 \quad -(1)$$

where:

$$T^{b}_{\mu a} = q^{\nu}_{a} T^{b}_{\mu\nu}. \quad -(2)$$

It follows that:

$$\left(q^{\nu}_{a} T^{b}_{\mu\nu}\right) \tilde{T}^{a\mu\nu} := 0 \quad -(3)$$

In eq. (3), the index summation convention applies inside the brackets by definition. So:

$$\left(q^{0}_{a} T^{b}_{\mu 0} + q^{1}_{a} T^{b}_{\mu 1} + q^{2}_{a} T^{b}_{\mu 2} + q^{3}_{a} T^{b}_{\mu 3}\right) \tilde{T}^{a\mu\nu} = 0 \quad -(4)$$

for

$$\nu = 0, 1, 2, 3 \quad -(5)$$

This gives four equations: $\quad -(6)$

$$\left(q^{0}_{a} T^{b}_{\mu 0} + q^{1}_{a} T^{b}_{\mu 1} + q^{2}_{a} T^{b}_{\mu 2} + q^{3}_{a} T^{b}_{\mu 3}\right) \tilde{T}^{a\mu 0} = 0 \quad -(7)$$

$$\left(q^{0}_{a} T^{b}_{\mu 0} + q^{1}_{a} T^{b}_{\mu 1} + q^{2}_{a} T^{b}_{\mu 2} + q^{3}_{a} T^{b}_{\mu 3}\right) \tilde{T}^{a\mu 1} = 0$$

$$\left(q^{0}_{a} T^{b}_{\mu 0} + q^{1}_{a} T^{b}_{\mu 1} + q^{2}_{a} T^{b}_{\mu 2} + q^{3}_{a} T^{b}_{\mu 3}\right) \tilde{T}^{a\mu 2} = 0 \quad -(8)$$

$$\left(q^{0}_{a} T^{b}_{\mu 0} + q^{1}_{a} T^{b}_{\mu 1} + q^{2}_{a} T^{b}_{\mu 2} + q^{3}_{a} T^{b}_{\mu 3}\right) \tilde{T}^{a\mu 3} = 0 \quad -(9)$$

2) Now use:

$$\tilde{T}^{0\mu 0} = q^{0}_{a}\tilde{T}^{a\mu 0} \quad -(10)$$

$$\vdots$$

$$\tilde{T}^{0\mu 3} = q^{0}_{a}\tilde{T}^{a\mu 3} \quad -(11)$$

So eqs. (6) to (9) become:

$$\tilde{T}^{0\mu 0}T^{b}_{\mu 0} + \tilde{T}^{1\mu 0}T^{b}_{\mu 1} + \tilde{T}^{2\mu 0}T^{b}_{\mu 2} + \tilde{T}^{3\mu 0}T^{b}_{\mu 3} = 0 \quad -(12)$$

$$\tilde{T}^{0\mu 1}T^{b}_{\mu 0} + \tilde{T}^{1\mu 1}T^{b}_{\mu 1} + \tilde{T}^{2\mu 1}T^{b}_{\mu 2} + \tilde{T}^{3\mu 1}T^{b}_{\mu 3} = 0 \quad -(13)$$

$$\tilde{T}^{0\mu 2}T^{b}_{\mu 0} + \tilde{T}^{1\mu 2}T^{b}_{\mu 1} + \tilde{T}^{2\mu 2}T^{b}_{\mu 2} + \tilde{T}^{3\mu 2}T^{b}_{\mu 3} = 0 \quad -(14)$$

$$\tilde{T}^{0\mu 3}T^{b}_{\mu 0} + \tilde{T}^{1\mu 3}T^{b}_{\mu 1} + \tilde{T}^{2\mu 3}T^{b}_{\mu 2} + \tilde{T}^{3\mu 3}T^{b}_{\mu 3} = 0 \quad -(15)$$

Eqs. (12) to (15) are each of the type:

$$T^{b}_{\mu 0}\tilde{T}^{a\mu 0} + T^{b}_{\mu 1}\tilde{T}^{a\mu 1} + T^{b}_{\mu 2}\tilde{T}^{a\mu 2} + T^{b}_{\mu 3}\tilde{T}^{a\mu 3} = 0 \quad -(16)$$

for

$$a = 0, 1, 2, 3 \quad -(17)$$

noting that:

$$\tilde{T}^{a\mu 0} = \tilde{T}^{\mu 0 a} \quad -(18)$$

and so on.

Therefore eq. (16) is:

3)

$$T^{b}_{\mu\nu} \tilde{T}^{a\mu\nu} = 0 \quad -(19)$$

Eq. (12) is Eq. (19) for $a = 0$, Eq. (13) is Eq. (19) for $a = 1$, Eq. (14) is Eq. (19) for $a = 2$, Eq. (15) is Eq. (19) for $a = 3$.

It follows that Eq. (3) implies Eq. (19), which was the conclusion of UFT 314. Given eq. (19) it follows that:

$$\partial^{\nu}_{a} \left(T^{b}_{\mu\nu} \tilde{T}^{a\mu\nu} \right) = 0 \quad -(20)$$

Finally:

$$\left(\partial^{\nu}_{a} T^{b}_{\mu\nu} \right) \tilde{T}^{a\mu\nu} = \partial^{\nu}_{a} \left(T^{b}_{\mu\nu} \tilde{T}^{a\mu\nu} \right) \quad -(21)$$

Q.E.D.

Eq. (21) is an example of the associative law of matrices:

$$A(BC) = (AB)C \quad -(22).$$

Finally, the vector format of eq. (19) is:

$$\underline{E}^{(1)} \cdot \underline{B}^{(2)} + \underline{B}^{(1)} \cdot \underline{E}^{(2)} = 0 \quad -(23)$$

as shown in UFT 314