15(3): Development of New Field Equations from the Evans Identity.

Consider the first Evans identity in the format:

$$T^{b}_{\ \mu a} \tilde{T}^{a\mu\nu} = 0 \qquad -(1)$$

where:

$$T^{b}_{\ \mu a} = q^{\nu}_{\ a} T^{b}_{\ \mu\nu} \qquad -(2)$$

There is summation over repeated ν indices in Eq. (2)

so:

$$T^{b}_{\ \mu a} = q^{0}_{\ a} T^{b}_{\ \mu 0} + q^{1}_{\ a} T^{b}_{\ \mu 1} + q^{2}_{\ a} T^{b}_{\ \mu 2} + q^{3}_{\ a} T^{b}_{\ \mu 3} \qquad -(3)$$

From eqs (2) and (3): $\qquad -(4)$

$$\left(q^{0}_{\ a} T^{b}_{\ \mu 0} + q^{1}_{\ a} T^{b}_{\ \mu 1} + q^{2}_{\ a} T^{b}_{\ \mu 2} + q^{3}_{\ a} T^{b}_{\ \mu 3} \right) \tilde{T}^{a\mu\nu} = 0$$

Eq. (1) splits into four equations for:

$$\nu = 0, 1, 2, 3, \qquad -(5)$$

$$\left(q^{0}_{\ a} T^{b}_{\ \mu 0} + q^{1}_{\ a} T^{b}_{\ \mu 1} + q^{2}_{\ a} T^{b}_{\ \mu 2} + q^{3}_{\ a} T^{b}_{\ \mu 3} \right) \tilde{T}^{a\mu\nu} = 0, \qquad -(6)$$

$$\left(q^{0}_{\ a} T^{b}_{\ \mu 0} + q^{1}_{\ a} T^{b}_{\ \mu 1} + q^{2}_{\ a} T^{b}_{\ \mu 2} + q^{3}_{\ a} T^{b}_{\ \mu 3} \right) \tilde{T}^{a\mu 0} = 0 \qquad -(7)$$

$$\left(q^{0}_{\ a} T^{b}_{\ \mu 0} + q^{1}_{\ a} T^{b}_{\ \mu 1} + q^{2}_{\ a} T^{b}_{\ \mu 2} + q^{3}_{\ a} T^{b}_{\ \mu 3} \right) \tilde{T}^{a\mu 1} = 0 \qquad -(8)$$

$$\left(q^{0}_{\ a} T^{b}_{\ \mu 0} + q^{1}_{\ a} T^{b}_{\ \mu 1} + q^{2}_{\ a} T^{b}_{\ \mu 2} + q^{3}_{\ a} T^{b}_{\ \mu 3} \right) \tilde{T}^{a\mu 2} = 0 \qquad -(9)$$

$$\left(q^{0}_{\ a} T^{b}_{\ \mu 0} + q^{1}_{\ a} T^{b}_{\ \mu 1} + q^{2}_{\ a} T^{b}_{\ \mu 2} + q^{3}_{\ a} T^{b}_{\ \mu 3} \right) \tilde{T}^{a\mu 3} = 0 \qquad -(10)$$

Adding eqs. (7) to (10):

$$q^0_a\left(T^b_{\mu 0}\tilde{T}^{a\mu 0} + T^b_{\mu 0}\tilde{T}^{a\mu 1} + T^b_{\mu 0}\tilde{T}^{a\mu 2} + T^b_{\mu 0}\tilde{T}^{a\mu 3}\right)$$
$$+ q^1_a\left(T^b_{\mu 1}\tilde{T}^{a\mu 0} + T^b_{\mu 1}\tilde{T}^{a\mu 1} + T^b_{\mu 1}\tilde{T}^{a\mu 2} + T^b_{\mu 1}\tilde{T}^{a\mu 3}\right)$$
$$+ q^2_a\left(T^b_{\mu 2}\tilde{T}^{a\mu 0} + T^b_{\mu 2}\tilde{T}^{a\mu 1} + T^b_{\mu 2}\tilde{T}^{a\mu 2} + T^b_{\mu 2}\tilde{T}^{a\mu 3}\right)$$
$$+ q^3_a\left(T^b_{\mu 3}\tilde{T}^{a\mu 0} + T^b_{\mu 3}\tilde{T}^{a\mu 1} + T^b_{\mu 3}\tilde{T}^{a\mu 2} + T^b_{\mu 3}\tilde{T}^{a\mu 3}\right)$$
$$= 0 \quad -(11)$$

It is clear that eq. (11) has many possible solutions, each giving new field relations in electrodynamics and gravitation. The fundamental structure of these solutions is:

$$T^b_{\mu 0}\tilde{T}^{a\mu 0} + T^b_{\mu 0}\tilde{T}^{a\mu 1} + T^b_{\mu 0}\tilde{T}^{a\mu 2} + T^b_{\mu 0}\tilde{T}^{a\mu 3} = 0 \quad -(12)$$

$$T^b_{\mu 1}\tilde{T}^{a\mu 0} + T^b_{\mu 1}\tilde{T}^{a\mu 1} + T^b_{\mu 1}\tilde{T}^{a\mu 2} + T^b_{\mu 1}\tilde{T}^{a\mu 3} = 0 \quad -(13)$$

$$T^b_{\mu 2}\tilde{T}^{a\mu 0} + T^b_{\mu 2}\tilde{T}^{a\mu 1} + T^b_{\mu 2}\tilde{T}^{a\mu 2} + T^b_{\mu 2}\tilde{T}^{a\mu 3} = 0 \quad -(14)$$

$$T^b_{\mu 3}\tilde{T}^{a\mu 0} + T^b_{\mu 3}\tilde{T}^{a\mu 1} + T^b_{\mu 3}\tilde{T}^{a\mu 2} + T^b_{\mu 3}\tilde{T}^{a\mu 3} = 0 \quad -(15)$$

Eqs. (12) to (15) are possible solutions of Eq. (11).
Adding eqs. (12) to (15) gives another possible solution:

3)

$$T^b_{\mu 0}\tilde{T}^{0\mu 0} + T^b_{\mu 0}\tilde{T}^{0\mu 1} + T^b_{\mu 0}\tilde{T}^{0\mu 2} + T^b_{\mu 0}\tilde{T}^{0\mu 3}$$
$$+ T^b_{\mu 1}\tilde{T}^{1\mu 0} + T^b_{\mu 1}\tilde{T}^{1\mu 1} + T^b_{\mu 1}\tilde{T}^{1\mu 2} + T^b_{\mu 1}\tilde{T}^{1\mu 3}$$
$$+ T^b_{\mu 2}\tilde{T}^{2\mu 0} + T^b_{\mu 2}\tilde{T}^{2\mu 1} + T^b_{\mu 2}\tilde{T}^{2\mu 2} + T^b_{\mu 2}\tilde{T}^{2\mu 3}$$
$$+ T^b_{\mu 3}\tilde{T}^{3\mu 0} + T^b_{\mu 3}\tilde{T}^{3\mu 1} + T^b_{\mu 3}\tilde{T}^{3\mu 2} + T^b_{\mu 3}\tilde{T}^{3\mu 3} \quad -(16)$$
$$= 0$$

In UFT314 it was assumed that the sum of the diagonals in Eq. (16) is zero:

$$T^b_{\mu 0}\tilde{T}^{a\mu 0} + T^b_{\mu 1}\tilde{T}^{a\mu 1} + T^b_{\mu 2}\tilde{T}^{a\mu 2} + T^b_{\mu 3}\tilde{T}^{a\mu 3} = 0 \quad -(17)$$

For electrodynamics and in vector notation this leads to the result:

$$\underline{E}^{(1)} \cdot \underline{B}^{(2)} + \underline{E}^{(2)} \cdot \underline{B}^{(1)} = 0. \quad -(18)$$

Eq. (18) was verified for plane waves.

The assumption leading to Eq. (17) means that:

$$T^b_{\mu 0}\left(\tilde{T}^{a\mu 1} + \tilde{T}^{a\mu 2} + \tilde{T}^{a\mu 3}\right)$$
$$+ T^b_{\mu 1}\left(\tilde{T}^{a\mu 0} + \tilde{T}^{a\mu 2} + \tilde{T}^{a\mu 3}\right) = 0$$
$$+ T^b_{\mu 2}\left(\tilde{T}^{a\mu 0} + \tilde{T}^{a\mu 1} + \tilde{T}^{a\mu 3}\right)$$
$$+ T^b_{\mu 3}\left(\tilde{T}^{a\mu 0} + \tilde{T}^{a\mu 1} + \tilde{T}^{a\mu 2}\right) \quad -(19)$$

4) Eq. (19) is implied by Eqs. (16) and (17) and can be used to derive more field relations in electrodynamics and gravitation. In electrodynamics:

$$
\begin{aligned}
& F^{b}_{\mu 0}\left(\tilde{F}^{a\mu 1}+\tilde{F}^{a\mu 2}+\tilde{F}^{a\mu 3}\right) \\
+\ & F^{b}_{\mu 1}\left(\tilde{F}^{a\mu 0}+\tilde{F}^{a\mu 2}+\tilde{F}^{a\mu 3}\right) \\
+\ & F^{b}_{\mu 2}\left(\tilde{F}^{a\mu 0}+\tilde{F}^{a\mu 1}+\tilde{F}^{a\mu 3}\right) \\
+\ & F^{b}_{\mu 3}\left(\tilde{F}^{a\mu 0}+\tilde{F}^{a\mu 1}+\tilde{F}^{a\mu 2}\right)
\end{aligned}
\quad = 0 \qquad -(20)
$$

where:

$$
F^{b}_{\mu\nu} = \begin{bmatrix} 0 & E^{b}_{X} & E^{b}_{Y} & E^{b}_{Z} \\ -E^{b}_{X} & 0 & -cB^{b}_{Z} & cB^{b}_{Y} \\ -E^{b}_{Y} & cB^{b}_{Z} & 0 & -cB^{b}_{X} \\ -E^{b}_{Z} & -cB^{b}_{Y} & cB^{b}_{X} & 0 \end{bmatrix} \qquad -(21)
$$

$$
\tilde{F}^{a\mu\nu} = \begin{bmatrix} 0 & -cB^{a}_{X} & -cB^{a}_{Y} & -cB^{a}_{Z} \\ cB^{a}_{X} & 0 & E_{Z} & -E^{a}_{Y} \\ cB^{a}_{Y} & -E^{a}_{Z} & 0 & E^{a}_{X} \\ cB^{a}_{Z} & E^{a}_{Y} & -E^{a}_{X} & 0 \end{bmatrix} \qquad -(22)
$$

Eq. (20) is evaluated by summing over repeated μ indices using eqs. (21) and (22). It is proven as follows that eq. (20) is true given eqs. (21) and (22).

5)

<u>Proof</u>

Eq. (20) is:

$$F^{b}_{10}\left(\tilde{F}^{a12}+\tilde{F}^{a13}\right)+F^{b}_{20}\left(\tilde{F}^{a21}+\tilde{F}^{23}\right)+F^{b}_{30}\left(\tilde{F}^{a31}+\tilde{F}^{a32}\right)$$
$$+F^{b}_{01}\left(\tilde{F}^{a02}+\tilde{F}^{a03}\right)+F^{b}_{21}\left(\tilde{F}^{a20}+\tilde{F}^{a23}\right)+F^{b}_{31}\left(\tilde{F}^{a30}+\tilde{F}^{a32}\right)$$
$$+F^{b}_{02}\left(\tilde{F}^{a01}+\tilde{F}^{a03}\right)+F^{b}_{12}\left(\tilde{F}^{a10}+\tilde{F}^{13}\right)+F^{b}_{32}\left(\tilde{F}^{a30}+\tilde{F}^{a31}\right)$$
$$+F^{b}_{03}\left(\tilde{F}^{a01}+\tilde{F}^{a02}\right)+F^{b}_{13}\left(\tilde{F}^{a10}+\tilde{F}^{a12}\right)+F^{b}_{23}\left(\tilde{F}^{20}+\tilde{F}^{a21}\right)$$
$$=0 \quad -(23)$$

Using eqs. (21) and (22), eq. (23) is:

$$-E^{b}_{X}\left(E^{a}_{Z}-E^{a}_{Y}\right)-E^{b}_{Y}\left(-E^{a}_{Z}+E^{a}_{X}\right)-E^{b}_{Z}\left(E^{a}_{Y}-E^{a}_{X}\right)$$
$$+E^{b}_{X}\left(-cB^{a}_{Y}-cB^{a}_{Z}\right)+cB^{b}_{Z}\left(cB^{a}_{Y}+E^{a}_{X}\right)-cB^{b}_{Y}\left(cB^{a}_{Z}-E^{a}_{X}\right)$$
$$-E^{b}_{Y}\left(cB^{a}_{X}+cB^{a}_{Z}\right)-cB^{b}_{Z}\left(cB^{a}_{X}-E^{a}_{Y}\right)+cB^{b}_{X}\left(cB^{a}_{Z}+E^{a}_{Y}\right)$$
$$+E^{b}_{Z}\left(-cB^{a}_{X}-cB^{a}_{Y}\right)+cB^{b}_{Y}\left(cB^{a}_{X}+E^{a}_{Z}\right)-cB^{b}_{X}\left(cB^{a}_{Y}-E^{a}_{Z}\right)$$
$$=0 \quad -(24)$$

<u>Q.E.D.</u>, <u>if $a=b$.</u>

Therefore eqs. (17) and (18) are also true, <u>Q.E.D.</u>

The following four identities are also true:

6)

$$F^{b}_{\mu 0}\left(\tilde{F}^{a\mu 1}+\tilde{F}^{a\mu 2}+\tilde{F}^{a\mu 3}\right)=0 \quad -(25)$$

$$F^{b}_{\mu 1}\left(\tilde{F}^{a\mu 0}+\tilde{F}^{a\mu 2}+\tilde{F}^{a\mu 3}\right)=0 \quad -(26)$$

$$F^{b}_{\mu 2}\left(\tilde{F}^{a\mu 0}+\tilde{F}^{a\mu 1}+\tilde{F}^{a\mu 3}\right)=0 \quad -(27)$$

$$F^{b}_{\mu 3}\left(\tilde{F}^{a\mu 0}+\tilde{F}^{a\mu 1}+\tilde{F}^{a\mu 2}\right)=0 \quad -(28)$$

with summation over the μ indices in each case.

Eqs. (25) to (28) are true if:

$$a = b \quad -(29)$$

In this case, eq. (18) reduces for each sense of polarization to:

$$\boxed{\underline{E} \cdot \underline{B} = 0} \quad -(30)$$

This is the well known result that electric and magnetic fields are perpendicular in free space. For example, if:

$$\underline{E} = \frac{E^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)e^{i\phi} \quad -(31)$$

then it follows from:

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(32)$$

7) that:

$$\underline{B} = \frac{B^{(0)}}{\sqrt{2}} (i\underline{i} + \underline{j}) e^{i\phi} \quad -(33)$$

From eqs. (31) and (33) eq. (30) follows, Q.E.D.

Note that if:

$$\underline{E}^{(1)} = \frac{E^{(0)}}{\sqrt{2}} (\underline{i} - i\underline{j}) e^{i\phi} \quad -(34)$$

then

$$\underline{B}^{(2)} = \frac{B^{(0)}}{\sqrt{2}} (-i\underline{i} + \underline{j}) e^{-i\phi} \quad -(35)$$

Also:

$$\underline{E}^{(2)} = \frac{E^{(0)}}{\sqrt{2}} (\underline{i} + i\underline{j}) e^{-i\phi} \quad -(36)$$

and

$$\underline{B}^{(1)} = \frac{B^{(0)}}{\sqrt{2}} (i\underline{i} + \underline{j}) e^{i\phi} \quad -(37)$$

So

$$\underline{E}^{(1)} \cdot \underline{B}^{(2)} + \underline{E}^{(2)} \cdot \underline{B}^{(1)} = 0 \quad -(38)$$

Therefore

$$\boxed{\begin{array}{c} (\underline{E} \cdot \underline{B})^a = (\underline{E} \cdot \underline{B})^b = 0 \quad -(39) \\ \underline{E}^a \cdot \underline{B}^b + \underline{E}^b \cdot \underline{B}^a = 0 \quad -(40) \end{array}}$$

and