1) 312(1): A Simple Calculation of $m = \gamma m_0$ from The Monochromatic Planck Distribution

From eq. (17) of Notes 310 and UFT310, the flux density in watts per square metre for a monochromatic beam is:

$$\Phi = \frac{\hbar\omega}{3c^2\pi^2} \cdot \frac{(\omega^2 - \omega_0^2)^{3/2}}{e^y - 1} \quad -(1)$$

where the rest frequency ω_0 is defined by:

$$\hbar\omega_0 = m_0 c^2 \quad -(2)$$

where m_0 is the rest mass of the photon. Using the lowest observable frequency of

$$f_0 = 0.01 \text{ Hz} \quad -(3)$$

it is found from eq. (2) that:

$$m_0 = 7.39 \times 10^{-53} \text{ kg} \quad -(4)$$

Therefore at visible frequencies ω_0 has an entirely negligible effect on Φ of eq. (1). So to an excellent approximation:

$$\Phi = \frac{\hbar\omega^4}{3c^2\pi^2(e^y - 1)} \quad -(5)$$

where

$$y = \frac{\hbar\omega}{kT} \quad -(6)$$

2)

Now assume that:

$$\hbar\omega \ll kT \quad -(7)$$

So

$$\Phi \sim \frac{\hbar\omega^4}{3c^2\pi^2\left(1+\frac{\hbar\omega}{kT}-1\right)}$$

$$= \frac{kT\omega^3}{3c^2\pi^2} \quad -(8)$$

Now use the de Broglie equation:

$$E = \hbar\omega = \gamma mc^2 \quad -(9)$$

Then:

$$\Phi = \frac{kT}{\hbar^3 c^2\pi^2}\left(\gamma m_0 c^2\right)^3 \quad -(10)$$

so

$$\Phi = \left(\frac{kTc^4}{3\hbar^3\pi^2}\right)\left(\gamma m_0\right)^3 \quad -(11)$$

For the sake of argument use:

$$\Phi(\mathrm{exp}) = 1.0 \text{ watt m}^{-2} \quad -(12)$$

Then the moving mass of the photon is:

$$m = \gamma m_0 = \frac{\hbar}{c}\left(\frac{3\pi^2}{ckT}\right)^{1/3} \quad -(13)$$

3) Assume:

$$T = 296\,K,\; k = 1.3806 \times 10^{-23}\,JK^{-1},\; c = 3 \times 10^{8}\,ms^{-1} \quad -(14)$$

then:

$$\boxed{m = \gamma m_0 = 1.02 \times 10^{-38}\,kg} \quad -(15)$$

This means that:

$$\gamma = \frac{1.02 \times 10^{-38}}{7.39 \times 10^{-53}} = 1.38 \times 10^{14} \quad -(16)$$

so that v is very close to c because:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(17)$$

This calculation illustrates the fact that the photon mass m_0 produces a speed of light v in the ensemble of free photons that is very close to c. In the writings of the de Broglie / Vigier group this is described as v being c for all practical purposes (FAPP) for free photons.

4)

However when there is absorption of light by an atom or molecule:

$$\frac{\Phi}{\Phi_0} = \exp\left(-\alpha \ell\right) \quad -(18)$$

where the integrated power absorption coefficient is:

$$\alpha = \frac{N}{V} \frac{|\mu_{if}|^2}{6\epsilon_0 \hbar v} \quad -(19)$$

Here N is the number of atoms in a volume V, μ_{if} is the transition dipole moment, ϵ_0 the vacuum permittivity, $\hbar$ the reduced Planck constant, ℓ the sample length and v the velocity of light in a gas of atoms, or a material made up of atoms and molecules.

Then as in UFT306:

$$v = v' + iv'' = \frac{c}{n' - in''} \quad -(20)$$

so:

$$\frac{v'}{c} = \frac{n'}{n'^2 + n''^2} \quad -(21)$$

$$\frac{v''}{c} = \frac{n''}{n'^2 + n''^2} \quad -(22)$$

and the velocity depends on the dielectric properties of the material through:

$$n'^2 = \frac{1}{2}\left(\epsilon' + \left(\epsilon'^2 + \epsilon''^2\right)^{1/2}\right) \quad -(23)$$

$$n''^2 = \frac{4\epsilon''^2}{\left(\epsilon' + \left(\epsilon'^2 + \epsilon''^2\right)^{1/2}\right)} \quad -(24)$$

The velocity of the Lorentz factor is defined as

$$V_m^2 = |vv^*|^2 = v'^2 + v''^2$$

$$= \frac{c^2}{n'^2 + n''^2} \quad -(25)$$

If the dielectric loss is very small, as in a glass, then:

$$n'^2 \sim \epsilon' \quad -(26)$$

$$n''^2 \sim 0 \quad -(27)$$

and

$$V_m = \frac{c}{n'} \quad -(28)$$

Eq. (28) is the usual equation defining the refractive index:

$$n' = \frac{c}{V_m} \quad -(29)$$

So if:

$$n' = 1.5 \quad -(30)$$

for the sake of argument, then:

6)

$$v_m = 1.5 \times 10^{8}\ ms^{-1} \quad -(31)$$

In this case the moving mass is:

$$m = \left(1 - \frac{1}{4}\right)^{-1/2} m_0$$

$$= \frac{2}{\sqrt{3}} m_0 = 1.155 m_0 \quad -(32)$$

$$= 8.53 \times 10^{-53}\ kg$$

The material has slowed the photon velocity to $1.5 \times 10^{8}\ ms^{-1}$ so the moving mass m is very close to the rest mass m_0.

In order to obtain a fully self-consistent result use:

$$\Phi \sim \left(\frac{kTc^4}{3\hbar^3 \pi^2}\right)(\gamma m_0)^3 = \Phi_0 \exp(-\alpha l) \quad -(33)$$

where α is given by eq. (19).

This will be the subject of the next note