312(4): Photon Rest Mass from Half Wave Antenna Theory.

The total power in watts emitted by an antenna is:

$$P = \frac{1}{2\mu_0 c k^2} \sum_{l \text{ odd}} |a_E(l, 0)|^2 \quad -(1)$$

If the source dimensions are much less than the wavelength the multipole expansion is completely dominated by the lowest value of l. Therefore:

$$P = \frac{1}{2\mu_0 c k^2} |a_E(1, 0)|^2 \quad -(2)$$

For a half-wave antenna:

$$kd = \pi \quad -(3)$$

where d is the length of the antenna. In this case:

$$a_E(1, 0) = 4(6\pi)^{1/2} \left(\frac{\mu_0 c I}{4\pi d} \right) \quad -(4)$$

So

$$\boxed{\begin{aligned} P &= \frac{3\mu_0 c}{\pi} I^2 \\ &= 359.75 I^2 \end{aligned}} \quad -(5)$$

The relation between power in watts and flux density Φ in watts per square metre is:

2)
$$P = \int_0^{2\pi} d\phi \int_0^{\pi} \Phi r^2 \sin\theta \, d\theta \quad -(6)$$

in spherical polar coordinates. In these equations I is the current in amperes or Cs^{-1}, μ_0 is the vacuum permeability:

$$\mu_0 = 4\pi \times 10^{-7} \, Js^2 C^{-2} m^{-1} \quad -(7)$$

and
$$c = 2.9979 \times 10^8 \, ms^{-1} \quad -(8)$$

From eq. (6):
$$P = 2\pi \Phi r^2 \int_0^{\pi} \sin\theta \, d\theta \quad -(9)$$

$$= 4\pi^2 r^2 \Phi$$

Therefore:
$$\boxed{P = 4\pi r^2 \Phi \text{ in watts}} \quad -(10)$$

From the Planck / Rayleigh theory with rest photon mass m_0:

$$\Phi = \left(\frac{(\omega^2 - \omega_0^2)^{3/2}}{3c^2\pi^2} \right) \langle \hbar\omega \rangle \quad -(11)$$

for monochromatic radiation of angular freq.

$$\omega = 2\pi f \quad -(12)$$

The rest frequency of de Broglie is:

$$\omega_0 = \frac{m_0 c^2}{\hbar} \quad -(13)$$

In eq. (11):

$$\langle \hbar\omega \rangle = \frac{\hbar\omega}{e^{y} - 1} \quad -(14)$$

where:

$$y = \frac{\hbar\omega}{kT} \quad -(15)$$

Therefore the total power in the quantum theory is:

$$P = r^2 \Phi \quad -(16)$$

i.e.

$$P = \frac{r^2 (\omega^2 - \omega_0^2)^{3/2}}{3c^2\pi^2} \left(\frac{\hbar\omega}{e^{y} - 1} \right) \quad -(17)$$

From eqns. (5) and (17) the power in watts emitted by a half wave antenna can be expressed in terms of ω_0:

$$\boxed{P = \frac{3\mu_0 c}{\pi} I^2 = \frac{r^2(\omega^2-\omega_0^2)^{3/2}}{3c^2\pi^2}\left(\frac{\hbar\omega}{e^y-1}\right)} \quad -(18)$$

The half wave antenna is a small section of a power line, with a current I in amps at an alternating frequency of $\omega = 2\pi f$, where f is about 60 Hz. The source dimension is much less than the wavelength so eq. (2) is valid.

It is advantageous to maximize the distance r between the transmitter and receiver so as to increase the power P of the radiation from the Planck law:

$$P = \frac{r^2(\omega^2-\omega_0^2)^{3/2}}{3c^2\pi^2}\left(\frac{\hbar\omega}{e^y-1}\right)$$

in watts. $-(19)$

At room temperature:

$$T = 293\,K \quad -(20)$$

and at

$$f = 60\,Hz \quad -(21)$$

) it follows that:

$$\hbar\omega \ll kT \quad -(22)$$

so

$$e^{y} \sim 1 + y \,. \quad -(23)$$

Therefore:

$$P = \frac{r^2\left(\omega^2-\omega_0^2\right)^{3/2} kT}{3c^2\pi^2} \quad -(24)$$

where

$$k = 1.38066 \times 10^{-23}\ \mathrm{J\,K^{-1}} \quad -(25)$$

If the transmitted signal is received a long distance r from the source, the power in watts can be increased to a measurable value. In the approximation (22):

$$\frac{3\mu_0 c}{\pi} I^2 = r^2\left(\omega^2-\omega_0^2\right)^{3/2} \frac{kT}{3c^2\pi^2} \quad -(26)$$

Therefore:

$$\boxed{\omega^2-\omega_0^2 = \left(\frac{9\mu_0 c^3 \pi I^2}{kT r^2}\right)^{2/3}} \quad -(27)$$

Units Check

$$\mathrm{RHS} = \left(s^2 J\, C^{-2} m^{-1} m^3 C^2 s^{-3} s^{-2} / (J m^2)\right)^{2/3}$$

$$= s^{-2}$$

6) At the point $\omega = \omega_0$ — (28)

the signal drops to zero.

The lowest detectable power of contemporary receiver technology is 10^{-15} watts or less. This would have to detect a power of:

$$P = r^2 (\omega^2 - \omega_0^2)^{3/2} \frac{kT}{3c^2\pi^2} \quad -(29)$$

$$= \frac{3\mu_0 c}{\pi} I^2$$

$$= 359.75\, I^2$$

from a half wave antenna. If two different current inputs are used for the same r, then:

$$\frac{P_1}{P_2} = \frac{I_1^2}{I_2^2} = \left(\frac{(\omega_1^2 - \omega_0^2)}{(\omega_2^2 - \omega_0^2)} \right)^{3/2} \quad - (30)$$

Therefore the experiment consists of using one source with I_1 and ω_1, and another source with I_2 and ω_2 and working out ω_0 from eq. (30).
