## 312(2): Measurement of Photon Rest Mass Using A.C. Frequency and Flux Density

The expression for flux density of monochromatic radiation is:

$$\Phi = \frac{\hbar \omega}{3c^2\pi^2} \frac{(\omega^2 - \omega_0^2)^{3/2}}{e^y - 1} \quad -(1)$$

where $\omega_0$ is the photon rest frequency:

$$\omega_0 = m_0 c^2 / \hbar \quad -(2)$$

and $m_0$ is the rest mass of the photon. Here:

$$y = \frac{\hbar \omega}{kT} \quad -(3)$$

In the approximation:

$$\hbar \omega << kT \quad -(4)$$

then:

$$\Phi = \frac{kT}{3c^2\pi^2} (\omega^2 - \omega_0^2)^{3/2} \quad -(5)$$

so

$$\omega^2 - \omega_0^2 = \left( \frac{3c^2\pi^2 \Phi}{kT} \right)^{2/3} \quad -(6)$$

and

$$\boxed{\omega_0^2 = \omega^2 - \left( \frac{3c^2\pi^2 \Phi}{kT} \right)^{2/3}} \quad -(7)$$

so $\omega$ and $\Phi$ have to be measured.

2) ELF technology will have to be used in this experiment, where ELF is an abbreviation of "extremely low frequency". The most straightforward implementation of this technology is to use the 60 Hz mains frequency.

The flux density from the mains is:

$$\Phi = c\frac{E_n}{V} = c\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right) \quad - (8)$$

Here $E$ is the electric field strength in volts per metre and $B$ the magnetic flux density in tesla. Thus:

$$E = \text{volt}\ m^{-1} = J C^{-1} m^{-1} \quad - (9)$$

$$B = \text{tesla} = J C^{-1} m^{-2} s \quad - (10)$$

Here:

$$\epsilon_0 = 8.8542 \times 10^{-12}\ J^{-1} C^2 m^{-1}$$

$$\mu_0 = 4\pi \times 10^{-7}\ J s^2 C^{-2} m^{-1} \quad - (11)$$

Therefore using the mains frequency of 60 Hz then

$$\omega = 2\pi \times 60\ \text{rad s}^{-1} \quad - (12)$$

$$= 377.0\ \text{rad s}^{-1}$$

The flux density corresponding to this

) is found from $\underline{\Phi}$, which is watts per square metre. The power of the mains is known, and also the area of the wire carrying the electric current, so $\underline{\Phi}$ can be found. Knowing $\underline{\Phi}$ and $\omega$, eq. (5) gives:

$$\omega^2 - \omega_0^2 = \left( \frac{3c^2\pi^2 \underline{\Phi}}{kT} \right)^{2/3} \quad - (\overset{13}{\cancel{18}})$$

So:

$$\omega_0^2 = \omega^2 - \left( \frac{3c^2\pi^2 \underline{\Phi}}{kT} \right)^{2/3} \quad - (\overset{14}{\cancel{19}})$$

If there is no photon mass the following relation between the flux density $\underline{\Phi}$ and angular frequency must be obeyed:

$$\underline{\Phi} = \frac{\hbar\omega^4}{3c^2\pi^2(e^y - 1)} \quad - (\overset{15}{\cancel{20}})$$

For very low frequency radiation

$$\underline{\Phi} = \left( \frac{kT}{3c^2\pi^2} \right) \omega^3 \quad - (\overset{16}{\cancel{21}})$$

If the photon mass is zero, $\underline{\Phi}$ and $\omega$ of the mains should be related by eq. (16).

4) If the photon mass is non-zero, $\Phi$ and $\omega$ from the mains are related by eq. (7) in the approximation (4).

If a situation is considered where there are many frequencies generated, then as in eq. (12) of Note 310(3):

$$d\Phi = c\frac{dE}{V} = \frac{\hbar\omega}{3c^2\pi^2(e^y-1)}\left(\left((\omega+d\omega)^2-\omega_0^2\right)^{3/2} - \left(\omega^2-\omega_0^2\right)^{3/2}\right) \quad -(17)$$

and this equation will be developed in the next note.

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