306(6) General Rule for Number of Lines in the series $n' = n$ to $n = n + 1$, Left Circular Polarization

To establish this line we consider:

1) $n' = 2$ to $n = 3$, the $H\alpha$ line at 15,241.4 cm^{-1}
2) $n' = 3$ to $n = 4$, Infra red line at 5,334.4 cm^{-1}
3) $n' = 4$ to $n = 5$, Infra red line at 2,469.1 cm^{-1}.

For left circularly polarized probe radiation the selection rules are:

$$\text{Any } n, \quad \Delta l = \pm 1, \quad \Delta n = 1 \quad - (1)$$

In note 306(5) it was shown that for the $H\alpha$ line there are five transitions possible when the selection rule (1) is applied. For the infra-red line at 5,334.4 cm^{-1} there are ten transitions possible. (In note 306(5) there was a typo, eleven should have been ten). For the infra-red line at 2,469.1 cm^{-1} there are seventeen possible transitions as shown below. So:

n'	Number of Transitions
2	5
3	10
4	17

So the number of possible transitions in left circular polarization ($\Delta n = 1$) is $n'^2 + 1$.

In linear polarization ($\Delta n = 0$), the number of possible transitions is $2n' - 1$.

2) For the $n' = 4$ to $n = 5$ line the quantum numbers are as follows:

4s $n' = 4,\ l = 0,\ m = 0$
4p $n' = 4,\ l = 1,\ m = -1, 0, 1$
4d $n' = 4,\ l = 2,\ m = -2, -1, 0, 1, 2$
4f $n' = 4,\ l = 3,\ m = -3, -2, -1, 0, 1, 2, 3$

5s $n = 5,\ l = 0,\ m = 0$
5p $n = 5,\ l = 1,\ m = -1, 0, 1$
5d $n = 5,\ l = 2,\ m = -2, -1, 0, 1, 2$
5f $n = 5,\ l = 3,\ m = -3, -2, -1, 0, 1, 2, 3$
5g $n = 5,\ l = 4,\ m = -4, -3, -2, -1, 0, 1, 2, 3, 4$

Therefore for $\Delta m = 1$, the possible transitions are:

1) 4s → 5p ($n' = 4, l = 0, m = 0 \rightarrow n = 5, l = 1, m = 1$)
2) 4p → 5s ($n' = 4, l = 1, m = -1 \rightarrow n = 5, l = 0, m = 0$)

3
3) 4p → 5d ($n' = 4, l = 1, m = -1 \rightarrow n = 5, l = 2, m = 0$)

4)	"	0	"	1
5)	"	1	"	2

5
6) 4d → 5f ($n' = 4, l = 2, m = -2 \rightarrow n = 5, l = 3, m = -1$)

7)	"	-1	"	0
8)	"	0	"	1
9)	"	1	"	2
10)		2	"	3

11) 4f → 5g ($n' = 4, l = 3, m = -3 \rightarrow n = 5, l = 4, m = -2$)

12)	"	-2	"	-1
13)	"	-1	"	0
14)	"	0	"	1
15)	"	1	"	2
16)	"	2	"	3
	"	3	"	4

So the total number of lines is:

$$\boxed{N = 2 + \sum_{l=1}^{n'-1} (2l+1) = n'^2 + 1} \quad -(2)$$

Q.E.D.

Therefore if left circularly polarized probe radiation is used, the far infra-red line at 81.52 cm^{-1} is split into 170 lines ($n' = 13$ to $n = 14$) (the Planck distribution used in the Beer Lambert law is the result of basic quantum ~~theory~~). Therefore there are 170 Evans / Morris red shifts.

The microwave line at 1.704 cm^{-1} ($n' = 50$ to $n = 51$) is split into 2,501 lines. There are 2,501 Evans / Morris red shifts in the microwave.