306(1): Evans Morris Shifts from Debye Relaxation

The basic equation in this case is:

$$\left(\frac{\omega}{\omega_0}\right)^3\left(\frac{e^{y_0}-1}{e^{y}-1}\right) = \exp(-\alpha \ell) \quad -(1)$$

where ω_0 is the incident frequency, ω the shifted frequency, α the power absorption coefficient and ℓ the path length. Here:

$$y_0 = \exp\left(\frac{\hbar\omega_0}{kT}\right), \; y = \exp\left(\frac{\hbar\omega}{kT}\right) \quad -(2)$$

In the low frequency approximation:

$$\omega = \omega_0 \exp\left(-\frac{\alpha \ell}{2}\right) \quad -(3)$$

In the Debye theory:

$$\alpha = \frac{\omega \epsilon''(\omega)}{c n'(\omega)} = \frac{\sqrt{2}\,\omega\epsilon''}{\left(\epsilon' + \left(\epsilon'^2 + \epsilon''^2\right)^{1/2}\right)^{1/2}} \quad -(4)$$

$$\epsilon' = \epsilon_\infty + \frac{(\epsilon_0 - \epsilon_\infty)}{1+\omega^2\tau^2}, \quad -(5)$$

$$\epsilon'' = (\epsilon_0 - \epsilon_\infty)\frac{\omega\tau}{1+\omega^2\tau^2} \quad -(6)$$

where τ is the Debye relaxation time and

2) where ϵ_0 and ϵ_∞ are the static and infinite frequency relative permittivities of the sample.

So for a given and observed shifted frequency ω, the initial frequency ω_0 can be calculated:

$$\omega_0 = \omega \exp\left(\frac{\alpha \ell}{2}\right) \quad -(7)$$

in terms of τ and ℓ from eqs. (4) to (6)

The experimental information consists of the dielectric permittivity ϵ' and dielectric loss ϵ'' at the set of initial frequencies ω_0. These are frequencies generated by a klystron or a microwave generator. So the experimental data give:

$$\alpha(\omega_0) = \frac{\omega_0 \epsilon''(\omega_0)}{c n'(\omega_0)} \quad -(8)$$

$$= \frac{\sqrt{2}\, \omega_0 \epsilon''(\omega_0)}{\left(\epsilon'(\omega_0) + \left(\epsilon'(\omega_0)^2 + \epsilon''(\omega_0)^2\right)^{1/2}\right)^{1/2}}$$

where:

$$\epsilon'(\omega_0) = \epsilon_\infty + \frac{(\epsilon_0 - \epsilon_\infty)}{1 + \omega_0^2 \tau^2} \quad -(9)$$

3) and

$$\epsilon''(\omega_0) = (\epsilon_0 - \epsilon_\infty) \frac{\omega_0 \tau}{1 + \omega_0^2 \tau^2} \quad -(10)$$

The shifted set of frequencies can therefore be calculated directly from eq. (3):

$$\omega = \omega_0 \exp\left(- \frac{l \alpha(\omega_0)}{2} \right) \quad -(11)$$

for different l. Each set of shifted frequencies gives a Debye type dielectric loss and dispersion given by eqs. (4) to (6).

Therefore there exists an Evans / Morris shifted Debye theory.

The basic optical equation are:

$$\frac{1}{v} = \frac{1}{c} (n' - i n'') \quad -(12)$$

and

$$\epsilon' - i\epsilon'' = (n' - i n'')^2 \quad -(13)$$

$$= n'^2 - n''^2 - 2 i n' n''$$

where v is the complex valued velocity, n' and n'' are the real and imaginary parts of the

) complex refractive index. So:

$$\epsilon' = n'^2 - n''^2 \quad -(14)$$

$$\epsilon'' = 2n'n'' \quad -(15)$$

So

$$\epsilon' = n'^2 - \frac{\epsilon''^2}{4n'^2} \quad -(16)$$

so

$$n'^2 = \frac{1}{2}\left(\epsilon' + \left(\epsilon'^2 + \epsilon''^2\right)\right)^{1/2} \quad -(17)$$

and

$$n''^2 = \frac{4\epsilon''^2}{\left(\epsilon' + \left(\epsilon'^2 + \epsilon''^2\right)^{1/2}\right)} \quad -(18)$$

The complex velocity is defined by:

$$v = v' + iv'' = \frac{c}{n' - in''} \quad -(19)$$

$$= \frac{c\left(n' + in''\right)}{n'^2 + n''^2}$$

So

$$\frac{v'}{c} = \frac{n'}{n'^2 + n''^2} \quad -(20)$$

and

$$\frac{v''}{c} = \frac{n''}{n'^2 + n''^2} \quad -(21)$$

5) Therefore v' and v'' can be found from the Debye theory and plotted as a function of frequency. The experimentally observable velocities are $v'(\omega_0)$ and $v''(\omega_0)$, and the shifted velocities are $v'(\omega)$ and $v''(\omega)$.

The squared modulus of velocity is:

$$|vv^*|^2 := v_m^2 \quad -(21)$$

where

$$v = v' + iv'' \quad -(22)$$

$$v^* = v' - iv'' \quad -(23)$$

so

$$v_m^2 = v'^2 + v''^2 \quad -(24)$$

Define the photon mass m by the Einstein / de Broglie theory:

$$\hbar\omega = \gamma mc^2 \quad -(25)$$

where the Lorentz factor is defined by:

$$\gamma = \left(1 - \frac{v_m^2}{c^2}\right)^{-1/2} \quad -(26)$$

Therefore the photon mass can be found from the Debye theory of relaxation. The unshifted photon mass is $m(\omega_0)$ and the shifted mass is $m(\omega)$