306(2): Red Shifts of the Far Infra Red Absorption of Molecular Liquids

As in note 306(1) the red shifts are given by

$$\frac{\omega}{\omega_0} = \exp\left(-\frac{\alpha(\omega_0)\,\ell}{2}\right) \quad -(1)$$

in the low frequency approximation:

$$\hbar\omega_0 << kT \quad -(2)$$

$$\hbar\omega << kT \quad -(3)$$

In the far infra-red, the memory function theory produces the dielectric loss and dispersion in terms of a continued fraction in the space of the Laplace variable:

$$p = -i\omega_0 . \quad -(4)$$

~~The~~ initial frequency ω_0 is used to define the Laplace variable p. The continued fraction is:

$$\tilde{C}(p) = \cfrac{C(0)}{p + \cfrac{K_0(0)}{p + \cfrac{K_1(0)}{\cdots}}} \quad -(5)$$

The dielectric permittivity is:

$$\epsilon' = \epsilon_\infty + \frac{(\epsilon_0 - \epsilon_\infty)}{\omega_0}\, \mathrm{Im}\, \tilde{C}(p) \quad -(6)$$

and the dielectric loss is:

2)

$$\epsilon'' = (\epsilon_0 - \epsilon_\infty)\,\omega_0\,\mathrm{Re}\,\tilde{C}(p) \quad -(7)$$

The far infra-red is adequately described by using the memory function:

$$K_1(t) = K_1(0)\exp(-\gamma t) \quad -(8)$$

whose Laplace transform is:

$$K_1(p) = \frac{K_1(0)}{p+\gamma} \quad -(9)$$

Therefore:

$$\tilde{C}(p) = \cfrac{C(0)}{p + \cfrac{K_0(0)}{p + \cfrac{K_1(0)}{p+\gamma}}} \quad -(10)$$

The far infra-red power absorption coefficient is:

$$\alpha(\omega_0) = \frac{\omega_0\,\epsilon''(\omega_0)}{n'(\omega_0)\,c} \quad -(11)$$

where:

$$n'(\omega_0) = \frac{1}{\sqrt{2}}\left(\epsilon'(\omega_0) + \left(\epsilon'(\omega_0)^2 + \epsilon''(\omega_0)^2\right)^{1/2}\right)^{1/2} \quad -(12)$$

The imaginary part of the refractive index is defined by:

3)
$$n''(\omega_0) = \frac{2\epsilon''(\omega_0)}{\left(\epsilon'(\omega_0) + \left(\epsilon'(\omega_0)^2 + \epsilon''(\omega_0)^2\right)^{1/2}\right)^{1/2}} \quad -(13)$$

The real part of the velocity is:
$$v'(\omega_0) = c\left(\frac{n'(\omega_0)}{n'(\omega_0)^2 + n''(\omega_0)^2}\right) \quad -(14)$$

and the imaginary part of velocity is:
$$v''(\omega_0) = c\left(\frac{n''(\omega_0)}{n'(\omega_0)^2 + n''(\omega_0)^2}\right) \quad -(15)$$

the square modulus of velocity is:
$$v_m^2(\omega_0) = v'(\omega_0)^2 + v''(\omega_0)^2 \quad -(16)$$

The Lorentz factor is:
$$\gamma(\omega_0) = \left(1 - \frac{v_m^2(\omega_0)}{c^2}\right)^{-1/2} \quad -(17)$$

and the photon mass is:
$$m(\omega_0) = \frac{\hbar\omega_0}{\gamma(\omega_0)c^2} \quad -(18)$$

4) Therefore using these equations the photon mass can be evaluated from the far infra-red spectrum in terms of the initial frequency ω_0. Finally the complete set of equations can be recomputed in terms of the red shifted frequency:

$$\omega = \omega_0 \exp\left(-\frac{\ell \alpha(\omega_0)}{2}\right) \quad -(19)$$

In other words, wherever ω_0 occurs in the set of equations (4) to (18), it is replaced by ω.

The red shift ω / ω_0 is defined by the power absorption coefficient $\alpha(\omega_0)$ calculated or observed at the initial frequency ω_0, or set of frequencies ω_0.

So the entire initial spectrum $\alpha(\omega_0)$ is shifted to $\alpha(\omega)$.