279(2): A Simple Theory of Snell's Law with Varying Frequency.

In the notation of note 279(1):

$$\omega = \omega' + \omega'' \quad -(1)$$

and

$$\underline{\kappa} = \underline{\kappa}' + \underline{\kappa}''. \quad -(2)$$

Therefore:

$$\omega t - \underline{\kappa} \cdot \underline{r} = (\omega' + \omega'') t - (\underline{\kappa}' + \underline{\kappa}'') \cdot \underline{r} \quad -(3)$$

From eq. (2):

$$\kappa_X = \kappa_X' + \kappa_X'' \quad -(4)$$

and

$$\kappa_Y = \kappa_Y' + \kappa_Y'' \quad -(5)$$

Eq. (3) is:

$$\underline{\kappa} \cdot \underline{r} = (\underline{\kappa}' + \underline{\kappa}'') \cdot \underline{r} \quad -(6)$$

as a result of eq. (1). Eq. (6) is:

$$\kappa_X X + \kappa_Y Y = \kappa_X' X + \kappa_Y' Y + \kappa_X'' X + \kappa_Y'' Y \quad -(7)$$

Experimentally, Snell's Law is:

$$i = r' \quad -(8)$$

), and

$$\sin i = n' \sin r \quad -(9)$$

Here:

$$\kappa_x = \kappa \sin i, \quad \kappa_x' = \kappa' \sin r, \quad \kappa_x'' = \kappa'' \sin r' \quad -(10)$$

So:

$$\kappa \sin i = \kappa' \sin r + \kappa'' \sin r' \quad -(11)$$

from eq. (4). Therefore:

$$(\kappa - \kappa'') \sin i = \kappa' \sin r \quad -(12)$$

i.e.

$$n' (\kappa - \kappa'') = \kappa' \quad -(13)$$

and

$$\boxed{n' = \frac{\kappa'}{\kappa - \kappa''}} \quad -(14)$$

The refractive index is defined by

$$n' = \frac{c}{v'} = \frac{\kappa'}{\kappa - \kappa''} \quad -(15)$$

with:

$$\omega = \omega' + \omega'' \quad -(16)$$

and

$$\underline{\kappa} = \underline{\kappa}' + \underline{\kappa}'' \quad -(17)$$

In order to avoid confusion of notation

3) eq. (14) is:

$$n = \frac{\kappa_1}{\kappa - \kappa_2} \quad (18)$$

where κ_1 is the magnitude of the wavenumber in the refracted wave, κ_2 is the magnitude of the wavenumber in the reflected wave, and κ is the magnitude of the wavenumber in the incident wave.

The theory can now be developed as in UFT 278 with self consistent conservation of total energy and momentum.

The refractive index in the old theory was defined by:

$$n = \frac{\kappa_1}{\kappa} \quad (19)$$

with:

$$\omega = ? \; \omega_1 = ? \; \omega_2 \quad (20)$$

The correct conservation of energy is:

$$\omega = \omega_1 + \omega_2 \quad (21)$$

and the correct definition of the refractive index is eq. (18).

As in UFT 278:

$$n^2 = \epsilon_r \mu_r = \frac{\epsilon \mu}{\epsilon_0 \mu_0} \quad (22)$$