279(8): Comparison of Experiment and Theory.

The experimental data are the following shifts:

1) 404 nm to 680 nm; 7.42060×10^{14} Hz to 4.40871×10^{14} Hz.

2) 532 nm to 680 nm; 5.63519×10^{14} Hz to 4.40871×10^{14} Hz.

3) 630 nm to 680 nm; 4.75861×10^{14} Hz to 4.40871×10^{14} Hz

The conversion from wavelength (λ) to frequency (f) has been done by assuming:

$$f = \frac{c}{\lambda} \quad -(1)$$

So it has been assumed that the velocity of the light ~~. . .~~ is c. So the above figures refer to the vacuum relation between f and λ, eq. (1). The static permittivity of olive oil is:

$$\epsilon_0' = 3.252 \quad -(2)$$

and the static refractive index is:

$$n_0' = 1.4615 \quad -(3)$$

The angle of refraction is not given, neither is the angle of incidence, but these can be used as parameters.

The n photon theory proceeds by

2) solving the equations of conservation of energy and momentum as in note 279(5):

$$\left(\frac{x}{1-x}\right)\omega = \left(\frac{x_1}{1-x_1}\right)\omega_1 + \left(\frac{x_2}{1-x_2}\right)\omega_2 \quad -(4)$$

and

$$\left(\frac{x_2}{1-x_2}\right)^2\omega_2^2 = \left(\frac{x}{1-x}\right)^2\omega^2 + n_1^2\left(\frac{x_1}{1-x_1}\right)^2\omega_1^2$$
$$-2\left(\frac{x}{1-x}\right)\left(\frac{x_1}{1-x_1}\right)n_1\,\omega\omega_1\cos\theta_3 \quad -(5)$$

where

$$\theta_3 = \theta - \theta_1 \quad -(6)$$

Here θ is the angle of incidence and θ_1 is the angle of refraction.

The reflected frequency ω_2 is eliminated between equations (4) and (5) to give the refracted frequency ω_1 in terms of the incident frequency ω. Experimentally:

$$\omega_1 = 4.40871 \times 10^{14}\ Hz \quad -(7)$$

and

$$\omega = \left.\begin{matrix} 7.42060 \times 10^{14} \\ 5.63519 \times 10^{14} \\ 4.75861 \times 10^{14} \end{matrix}\right\} Hz \quad -(8)$$

In general the refractive index is that of the medium in which refraction occurs,

3) which is olive oil. In general the refractive index is complex:

$$n_1 = n_1' + i n_1'' \quad -(9)$$

where:

$$n_1'^2 = \frac{1}{2}\left(\epsilon_{1r}' + \left(\epsilon_{1r}'^2 + \epsilon_{1r}''^2\right)^{1/2}\right) \quad -(10)$$

and

$$n_1'' = \frac{\epsilon_{1r}''}{2 n_1'} \quad -(11)$$

where the relative permittivity is:

$$\epsilon_{1r} = \epsilon_{1r}' + i \epsilon_{1r}''. \quad -(12)$$

Here:

$$\epsilon_{1r}' = n_1'^2 - n_1''^2 \quad -(13)$$

The power absorption coefficient is:

$$\alpha_1 = \frac{\epsilon_{1r}'' \omega}{n_{1r}' c} \quad -(14)$$

In the simplest theory:

$$n_1 = n_1' = 1.4665 \quad -(15)$$

Therefore the first attempt would be to solve eqs. (4) and (5) with eq. (15) to see if this is sufficient to produce the experimental data.

4) A mainframe computer is ~~needed~~ to solve eqs. (4) and (5) in general. However they can be approximated roughly as in note 279(6) by:

$$1 - x \sim 1 \quad -(16)$$

and

$$x = \exp\left(-\frac{\hbar\omega}{kT}\right) \sim 1 - \frac{\hbar\omega}{kT} \quad -(17)$$

for

$$\hbar\omega << kT \quad -(18)$$

Denoting:

$$A = 1 - \frac{\hbar\omega}{kT} \quad -(19)$$

$$A_1 = 1 - \frac{\hbar\omega_1}{kT} \quad -(20)$$

$$A_2 = 1 - \frac{\hbar\omega_2}{kT} \quad -(21)$$

eqs. (4) and (5) become:

$$A\omega = A_1\omega_1 + A_2\omega_2 \quad -(22)$$

and

$$A_2^2\omega_2^2 = A^2\omega^2 + n_1^2 A_1^2\omega_1^2 - 2AA_1 n_1 \omega\omega_1 \cos\theta_3 \quad -(23)$$

From eq. (22):

$$A_2\omega_2 = (A\omega - A_1\omega_1) \quad -(24)$$

5) Eqs. (23) and (24) can be solved by Maxima as shown by Dr. Horst Eckardt. Therefore ω_1 can be expressed analytically in terms of ω, n_1 and θ_3 for a given temperature T.

In the first instance:

$$n_1 = n_1' = 1.4665 \quad -(25)$$

and eq. (8) can be used for ω. The angle:

$$\theta_3 = \theta - \theta_1 \quad -(26)$$

can be used as a parameter. As shown by computer algebra there are four possible solutions for ω_1 in terms of ω.

Finally if the refracting medium absorbs then eqs. (10) and (11) must be used for n_1
