279(1): Variable Frequency Theory of Diffraction and Reflection.

Consider Fig (1):

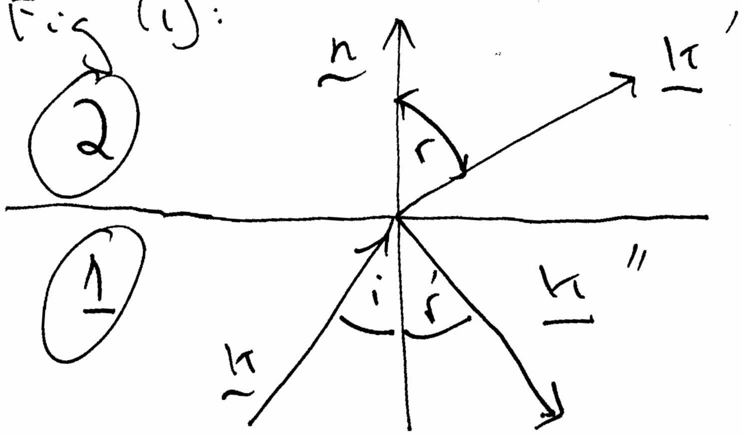

Fig (1)

In the usual theory:

$$\omega = \omega' = \omega'' \quad -(1)$$

and

$$\underset{\sim}{k} \cdot \underline{r} = \underset{\sim}{k}' \cdot \underline{r} = \underset{\sim}{k}'' \cdot \underline{r} \quad -(2).$$

i.e.

$$k_X X + k_Y Y = k_X' X + k_Y' Y = k_X'' X + k_Y'' Y \quad -(3)$$

where:

$$k_X = k \sin i \; ; \; k_X' = k' \sin r \; ; \; k_X'' = k'' \sin r'$$
$$k_Y = k \cos i \; ; \; k_Y' = k' \cos r ; \; k_Y'' = k'' \cos r' \quad -(4)$$

Here:

$$k = \frac{\omega}{c} \, , \; k'' = \frac{\omega''}{c} \, , \; k' = \frac{\omega'}{v} \quad -(5)$$

if it is assumed that medium 1 is air.

2) Fm eq. (1) it follows that:

$$k = k'' \quad - (6)$$

and

$$k' v = k c \quad - (7)$$

Now use:

$$\begin{aligned} k_x &= k \sin i \\ k_y &= k \cos i \\ k''_x &= k \sin r' \\ k''_y &= k \cos r' \end{aligned} \quad - (8)$$

It follows that: $\boxed{i = r'}$ $- (9)$

Fm eq. (7):

$$\frac{k'}{k} = \frac{c}{v} = n' \quad - (10)$$

where n' is the refractive index of medium 2. Finally use:

$$k_x = k \sin i \quad - (11)$$

$$k'_x = k' \sin r \quad - (12)$$

to obtain Snell's Law:

$$\frac{k' \sin r}{k \sin i} = 1 \quad - (13)$$

or

$$\boxed{\frac{\sin i}{\sin r} = \frac{k'}{k} = \frac{c}{v} = n'}$$

$- (14)$

3)

If it is assumed that the frequency changes from ω to ω' of the refracted beam then:

$$\omega t - \underset{\sim}{\kappa} \cdot \underset{\sim}{r} = \omega' t - \underset{\sim}{\kappa}' \cdot \underset{\sim}{r} \quad -(15)$$

at the interface. Here:

$$\underset{\sim}{\kappa} \cdot \underset{\sim}{r} = \kappa_X X + \kappa_Y Y \quad -(16)$$

$$\underset{\sim}{\kappa}' \cdot \underset{\sim}{r} = \kappa_X' X + \kappa_Y' Y \quad -(17)$$

where:

$$\kappa_X = \kappa \sin i, \quad \kappa_Y = \kappa \cos i \quad -(18)$$

$$\kappa_X' = \kappa' \sin r, \quad \kappa_Y' = \kappa' \sin r \quad -(19)$$

and

$$\kappa = \frac{\omega}{c}, \quad \kappa' = \frac{\omega'}{v} \quad -(20)$$

So:

$$\omega\left(t - \frac{1}{c}\left(X \sin i + Y \cos i\right)\right) = \omega'\left(t - \frac{1}{v}\left(X \sin r + Y \cos r\right)\right) \quad -(21)$$

Experimentally:

$$\frac{\sin i}{\sin r} = \frac{n'}{n} \quad -(22)$$

So

$$\boxed{\frac{\sin i}{\sin r} = \frac{\kappa'}{\kappa} = \frac{c}{v'}\frac{\omega'}{\omega} = \frac{n'}{n}} \quad -(23)$$

4) If the refractive index of air is:

$$n = 1 \quad -(24)$$

then:

$$\boxed{n' = \frac{c}{v'}\frac{\omega'}{\omega}} \quad -(25)$$

In the usual theory:

$$n' = \frac{c}{v'} \quad -(26)$$

Therefore the definition of the refractive index is changed from eq. (26) to eq. (25)

By conservation of energy the frequency of the reflected beam is:

$$\omega'' = \omega - \omega' \quad -(27)$$

and the wavenumber of the reflected beam is

$$\underline{\kappa}'' = \underline{\kappa} - \underline{\kappa}' \quad -(28)$$

The phases of the incoming and reflected beams must be the same, so:

5)

$$\omega''\left(t - \frac{1}{c}\left(X \sin r' + Y \cos r'\right)\right)$$
$$= \omega\left(t - \frac{1}{c}\left(X \sin i + Y \cos i\right)\right) \quad -(29)$$
$$= (\omega - \omega')\left(t - \frac{1}{c}\left(X \sin r' + Y \cos r'\right)\right)$$

where

$$i = r' \quad -(30)$$

experimentally. So:

$$\omega \tau = (\omega - \omega')\tau \quad -(31)$$

where

$$\tau = t - \frac{1}{c}\left(X \sin i + Y \cos i\right). \quad -(32)$$

This result is possible if a phase factor α is added to the right hand side of eq. (31):

$$\omega \tau = (\omega - \omega')\tau + \alpha \quad -(33)$$

so

$$\boxed{\alpha = \omega' \tau} \quad -(34)$$

It is always possible to add a phase factor in general, so:

$$\omega t - \underline{\kappa} \cdot \underline{r} = \omega' t - \underline{\kappa}' \cdot \underline{r} + \alpha_1$$

$$= \omega'' t - \underline{\kappa}'' \cdot \underline{r} + \alpha_2 \quad - (35)$$

with :

$$\omega = \omega' + \omega'' \quad - (36)$$

and :

$$\underline{\kappa} = \underline{\kappa}' + \underline{\kappa}'' \quad - (37)$$

So

$$\alpha_2 = \omega' t - \underline{\kappa}' \cdot \underline{r} \quad - (38)$$

and

$$\alpha_1 = \omega'' t - \underline{\kappa}'' \cdot \underline{r} \quad - (39)$$

The experimental result is :

$$i = r' \quad - (40)$$

and

$$\frac{\sin i}{\sin r} = n' \quad - (41)$$