279(3): Conservation of Total Energy in the Average Energy of a Beam of Photons and 180° Change of Phase on Reflection

Consider the classical energy density of an electromagnetic wave:

$$\frac{En}{V} = \epsilon E E^* + \frac{1}{\mu} B B^* \quad -(1)$$

Here En is the energy in joules, V is the volume of electromagnetic radiation, ϵ is the permittivity and μ is the permeability of the medium in which the radiation is propagating, $\underline{E}$ is the electric field strength and $\underline{B}$ the magnetic flux density of the beam.

It is immediately clear that the energy density does not depend on the phase of the wave. It follows that for a given volume V of electromagnetic radiation:

$$En(\text{incoming}) = En(\text{reflected}) + En(\text{refracted}) \quad -(2)$$

by conservation of total energy.

In the Planck theory the energy of one photon is:

2)

$$E_n = \hbar\omega \quad -(3)$$

where $\hbar$ is the reduced Planck constant and ω the angular frequency. The average energy of a photons in a monochromatic beam is:

$$\langle E_n \rangle = \left(\frac{x}{1-x}\right)\hbar\omega \quad -(4)$$

where:

$$x = \exp\left(-\frac{\hbar\omega}{kT}\right) \quad -(5)$$

and where

$$\hbar\omega \ll kT. \quad -(6)$$

Here k is Boltzmann's constant and T the temperature.

By quantum classical equivalence:

$$E_n\,(\text{classical}) = \langle E_n \rangle \quad -(7)$$

so:

$$\langle E_n(\text{incoming})\rangle = \langle E_n(\text{reflected})\rangle + \langle E_n(\text{refracted})\rangle \quad -(8)$$

At a given temperature T:

$$\boxed{\omega\left(\frac{x}{1-x}\right) = \omega_1\left(\frac{x_1}{1-x_1}\right) + \omega_2\left(\frac{x_2}{1-x_2}\right)} \quad -(9)$$

3) Here ω, ω_1 and ω_2 are the angular frequencies of the incoming, refracted and reflected waves and where:

$$x = \exp\left(-\frac{\hbar\omega}{kT}\right) \quad -(10)$$

$$x_1 = \exp\left(-\frac{\hbar\omega_1}{kT}\right) \quad -(11)$$

$$x_2 = \exp\left(-\frac{\hbar\omega_2}{kT}\right) \quad -(12)$$

Eq. (9) is conservation of total energy in the reflection and refraction of a monochromatic beam of n photons.

It is immediately clear that:

$$\boxed{\omega \neq \omega_1 \neq \omega_2} \quad -(13)$$

as observed by Garett Evans and Trevor Morris.

We have:

$$\hbar = 6.62618 \times 10^{-34} \text{ Js} \quad -(14)$$

$$k = 1.38066 \times 10^{-23} \text{ JK}^{-1} \quad -(15)$$

Assume

$$T = 300 \text{ K} \quad -(16)$$

and

$$\omega = 10^{15} \text{ rad s}^{-1} \quad -(17)$$

then

$$\frac{\hbar\omega}{kT} = \frac{6.62618}{1.38066 \times 300} \times 10^{4} \quad -(18)$$

4) so for shifts in the visible frequency, eq. (9) is well approximated by:

$$\boxed{\omega = \omega_1 + \omega_2} \quad - (19)$$

as used in UFT 278.

It is observed experimentally that there is a 180° change of phase on reflection. So:

$$\omega_2 t - \underset{\sim}{\kappa_2} \cdot \underline{r} \rightarrow \omega_2 t - \underline{\kappa_2} \cdot \underline{r} + \pi \quad - (20)$$

Assume that:

$$\underline{\kappa_{21}''} \cdot \underline{r} = \underline{\kappa_2''} \cdot \underline{r} - \pi \quad - (21)$$

then the phase after reflection is:

$$\phi = \omega_2 t - \underline{\kappa_{21}''} \cdot \underline{r} \quad - (22)$$

and as in note 279(2):

$$\boxed{n' = \frac{\kappa'}{\kappa - \kappa_{12}''}} \quad - (23)$$

Eq. (23) conserves total energy and momentum, and obeys Snell's Law.