278(4): Development of Complex Refractive Index Theory.

The new theory of reflection and refraction gives:

$$n^2 \omega_1^2 = \omega^2 + \omega_2^2 - 2\omega_1 \omega_2 \cos 2\theta \quad -(1)$$

where n is the refractive index of the layer above the interface, ω is the incoming frequency, ω_1 is the refracted frequency and ω_2 is the reflected frequency. The angle θ is the sum of the equal angles of incidence and reflection. By conservation of energy:

$$\omega = \omega_1 + \omega_2 \quad -(2)$$

The refractive index is:

$$n^2 = \mu_r \epsilon_r \quad -(3)$$

where μ_r is the relative permeability and ϵ_r the relative permittivity of the layer above the interface, the layer in which refraction occurs. In liquids such as water the relative permeability is close to one, so:

$$n^2 = \epsilon_r \quad -(4)$$

to an excellent approximation (see M. W. Evans, G. J. Evans, W. T. Coffey and P. Grigolini "Molecular Dynamics" (Wiley Interscience, New York, 1982), in the Omnia Opera section of www.aias.us).

In areas of absorption both n and ϵ_r are complex:

$$(n' + in'')^2 = \epsilon_r' + i\epsilon_r'' \quad -(5)$$

here ϵ_r'' is the dielectric loss, and ϵ_r' is the dielectric dispersion. Therefore:

$$n'^2 + 2in'n'' - n''^2 = \epsilon_r' + i\epsilon_r'' \quad -(6)$$

i.e.

$$2n'n'' = \epsilon_r'' \quad -(7)$$

and

$$n'^2 - n''^2 = \epsilon_r' \quad -(8)$$

The real and imaginary parts of the refractive index are therefore:

$$n'^2 = \left(\frac{\epsilon_r''}{2n'}\right)^2 + \epsilon_r' \quad -(9)$$

and

$$n'' = \frac{\epsilon_r''}{2n'} \quad -(10)$$

Eq. (9) is:

$$4n'^4 - 4n'^2\epsilon' - \epsilon''^2 = 0 \quad -(11)$$

so:

$$n'^2 = \frac{1}{2}\left(\epsilon' \pm \left(\epsilon'^2 + \epsilon''^2\right)^{1/2}\right) \quad -(12)$$

i.e.:

3)

$$n' = \frac{1}{\sqrt{2}} \left(\epsilon' \pm \left(\epsilon'^2 + \epsilon''^2 \right)^{1/2} \right)^{1/2} \quad -(13)$$

The physical n' is given by the positive root:

$$\boxed{n' = \frac{1}{\sqrt{2}} \left(\epsilon' + \left(\epsilon'^2 + \epsilon''^2 \right)^{1/2} \right)^{1/2}} \quad -(14)$$

and

$$n'' = \frac{\epsilon''}{2n'} \quad -(15)$$

The power absorption coefficient $\alpha(\omega)$ in neper cm^{-1} is:

$$\alpha(\omega) = \frac{\omega \epsilon''(\omega)}{n(\omega) c} \quad -(16)$$

So:

$$\boxed{\alpha(\omega) = \frac{\sqrt{2}\, \omega \epsilon''(\omega)}{c \left(\epsilon_r' + \left(\epsilon_r'^2 + \epsilon_r''^2 \right)^{1/2} \right)^{1/2}}} \quad -(17)$$

and is observed in the far infra-red to gamma ray regions.

From eqns (1) and (6):

4).

$$\left(n'^2 + 2in'n'' - n''^2\right)\omega_1^2 = \omega^2 + \omega_2^2 - 2\omega_1\omega_2\cos 2\theta \quad -(18)$$

Taking the real part of the left hand side:

$$\boxed{\left(n'^2 - n''^2\right)\omega_1^2 = \omega^2 + \omega_2^2 - 2\omega_1\omega_2\cos 2\theta} \quad -(19)$$

where:

$$n'^2 = \frac{1}{2}\left(\epsilon' + \left(\epsilon'^2 + \epsilon''^2\right)^{1/2}\right) \quad -(20)$$

and

$$n''^2 = \frac{\epsilon''^2}{4n'^2} \quad -(21)$$

The refracted angular frequency is therefore:

$$\omega_1 = \frac{1}{2A}\left(-B \pm \left(B^2 + 4AC\right)^{1/2}\right) \quad -(22)$$

where:

$$\left.\begin{aligned} A &= \left(n'^2 - n''^2\right) - 1 - 2\cos 2\theta \\ B &= 2\omega\left(1 + \cos 2\theta\right) \\ C &= 2\omega^2 \end{aligned}\right\} \quad -(23)$$

5) It is seen that ω_1 depends on the dielectric loss ϵ_r'' and dispersion ϵ_r' of the medium above the interface, on the angle of incidence $\theta/2$, and the initial frequency ω.

<u>Computer Algebra and Graphics</u>

These equations can be checked by computer algebra as usual and the refracted frequency graphed for models of ϵ' and ϵ''. The simplest model is the Debye theory of dielectric loss. In the far infra-red, resonance function theory can be used. In regions free of dielectric loss, eqn. (20) simplifies to:

$$n'^2 = \epsilon' \quad - (24)$$

and

$$n'' = 0. \quad - (25)$$
