278(3) : A Theory of Reflection and Refraction Based on Conservation of Energy and Momentum.

The reflection and refraction are shown in Fig (1).

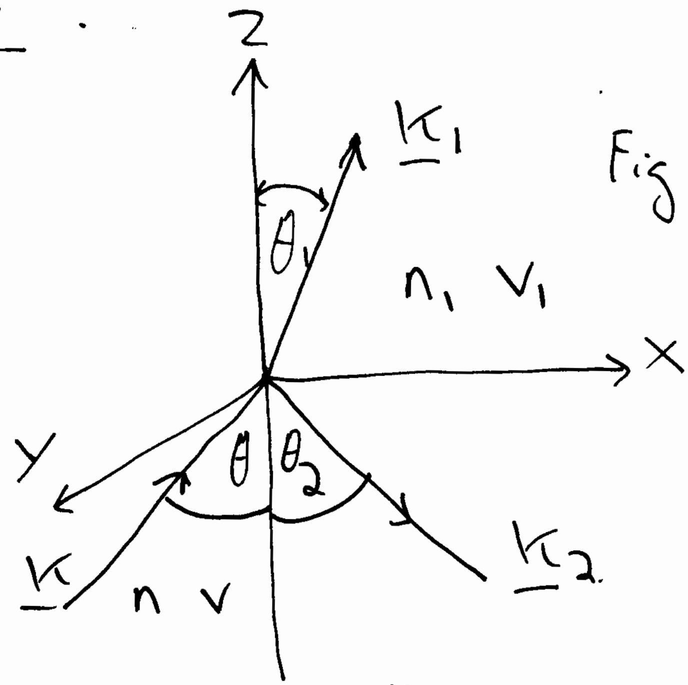

Fig (1)

Experimentally, the angle of reflection is equal to the angle of incidence

$$\theta = \theta_2 \quad - (1)$$

and Snell's Law of refraction:

$$\frac{\sin\theta}{\sin\theta_1} = \frac{v}{v_1} = \frac{n_1}{n} \quad - (2)$$

Here n_1 and n are the refractive indices of the medium above and below the interface respectively and where v_1 and v are the respective phase velocities.

By conservation of energy:

$$\hbar\omega = \hbar\omega_1 + \hbar\omega_2 \quad - (3)$$

and by conservation of momentum:

2)

$$\underline{p} = \underline{p_1} + \underline{p_2} \quad - (4)$$

i.e.

$$\underline{k} = \underline{k_1} + \underline{k_2} \quad - (5)$$

and

$$\omega = \omega_1 + \omega_2 \quad - (6)$$

The total energy before the wave / photon meets the interface is the same as the total energy of the reflected and refracted waves. Similarly for the total momentum before and after the interface.

Therefore :

$$\underline{k_1} = \underline{k} - \underline{k_2} \quad - (7)$$

and :

$$k_1^2 = k^2 + k_2^2 - 2 k_1 k_2 \cos(2\theta) - (8)$$

where

$$2\theta = \theta + \theta_2 - (9)$$

experimentally.

Now use :

$$k_1 = \frac{\omega_1}{V_1} \quad - (10)$$

and

$$k = \frac{\omega}{c}, \quad k_2 = \frac{\omega_2}{c} \quad - (11)$$

) assuming that the medium before reflection is air.

So:

$$\omega_1^2 = \left(\frac{v_1}{c}\right)^2 \left(\omega^2 + \omega_2^2 - 2\omega_1\omega_2 \cos 2\theta\right) - (12)$$

$$= \left(\frac{\epsilon_0\mu_0}{\epsilon_1\mu_1}\right)\left(\omega^2 + \omega_2^2 - 2\omega_1\omega_2 \cos 2\theta\right)$$

Here

$$\frac{\epsilon_0\mu_0}{\epsilon_1\mu_1} = \frac{1}{\epsilon_r\mu_r} - (13)$$

where

$$n^2 = \epsilon_r\mu_r - (14)$$

is the refractive index of the medium above the boundary.

So:

$$n^2\omega_1^2 = \omega^2 + \omega_2^2 - 2\omega_1\omega_2 \cos 2\theta - (15)$$

and

$$\omega = \omega_1 + \omega_2 - (16)$$

The change of frequency due to refraction is evaluated by using:

$$\omega_2 = \omega - \omega_1 - (17)$$

So:

$$n^2\omega_1^2 = \omega^2 + (\omega - \omega_1)^2 - 2\omega_1(\omega - \omega_1)\cos 2\theta$$

$$- (18)$$

4) so ω_1 can be expressed in term of ω and $\cos 2\theta$.

The change of frequency due to reflection is evaluated by using:

$$\omega_1 = \omega - \omega_2 \quad -(19)$$

So:

$$n^2(\omega-\omega_2)^2 = \omega^2 + \omega_2^2 - 2(\omega-\omega_2)\omega_2 \cos 2\theta \quad -(20)$$

so ω_2 can be expressed in term of ω and $\cos 2\theta$.

From eq. (18):

$$n^2\omega_1^2 = \omega^2 + \omega^2 - 2\omega\omega_1 + \omega_1^2 - 2\omega\omega_1 \cos 2\theta + 2\omega_1^2 \cos 2\theta \quad -(20)$$

So:

$$\omega_1^2(n^2 - 2\cos 2\theta) = 2\omega^2 + \omega_1^2 - 2\omega\omega_1(1+\cos 2\theta) \quad -(21)$$

and

$$\omega_1^2(n^2 - 1 - 2\cos 2\theta) = 2\omega^2 - 2\omega\omega_1(1+\cos 2\theta) \quad -(22)$$

i.e

$$A\omega_1^2 + B\omega_1 - C = 0 \quad -(23)$$

where:

$$A = n^2 - 1 - 2\cos 2\theta \quad -(24)$$

$$B = 2\omega(1 + \cos 2\theta) \quad -(25)$$

$$C = 2\omega^2 \quad -(26)$$

so

$$\omega_1 = \frac{1}{2A}\left(-B \pm \left(B^2 + 4AC\right)^{1/2}\right) \quad -(27)$$

In the conventional theory of reflection and refraction it is assumed that the frequency does not change. However this violates the law of conservation of energy, eq. (3).

Computer algebra can be used to check the result (27) and plot ω_1 against ω for a given n and θ. Similarly it can be used to plot ω_2 against ω from eq. (20).
