278(2): Development of the Current Density for the Evans / Morris Effects

As in UFT 256, the ECE current density is:

$$\underline{J}^a = \frac{1}{\mu_0}\left(\underline{\omega}^a{}_b \times \underline{B}^b + \frac{\omega_0}{c}\underline{E}^b - \left(\underline{A}^b \times \underline{R}^a{}_b(\text{spin}) + A_0^b \underline{R}^a{}_b(\text{orb})\right)\right) \quad -(1)$$

where μ_0 is the vacuum permeability, $\underline{B}^b$ the magnetic flux density, ω_0 and $\underline{\omega}^a{}_b$ are the scalar and vector parts of the spin connection, $\underline{A}^b$ the vector potential and $\underline{R}^a{}_b(\text{orb})$ and $\underline{R}^a{}_b(\text{spin})$ are the orbital and spin parts of the curvature.

Eq. (1) is part of the inhomogeneous ECE equation:

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2}\frac{\partial \underline{E}^a}{\partial t} = \mu_0 \underline{J}^a \quad -(2)$$

In the simplified ECE theory:

$$\underline{J} = \frac{1}{\mu_0}\left(\underline{\omega} \times \underline{B} + \frac{\omega_0}{c}\underline{E} - \left(\underline{A} \times \underline{R}(\text{spin}) + A_0 \underline{R}(\ )\right)\right) \quad -(3)$$

From eq. (5) of note 278(1):

$$\underline{J} = \underline{\nabla} \times \underline{M} + \frac{\partial \underline{P}}{\partial t} \quad -(4)$$

$$= \underline{0}$$

2) Therefore the current for the Evans Morris effects is:

$$\underline{\nabla} \times \underline{M} + \frac{\partial \underline{P}}{\partial t} = \frac{1}{\mu_0}\left(\underline{\omega} \times \underline{B} + \frac{\omega_0}{c}\underline{E} - \left(\underline{A} \times \underline{R}(\text{spin}) + A_0 \underline{R}(\text{orb})\right)\right) \quad -(5)$$

$$= \underline{0}$$

This equation leads to the shift:

$$\omega \rightarrow \frac{\omega}{\epsilon_r^{1/2}} \quad -(6)$$

which is a particular solution of:

$$\frac{\omega}{\kappa} \rightarrow \frac{1}{(\mu_r \epsilon_r)^{1/2}}\left(\frac{\omega}{\kappa}\right). \quad -(7)$$

The phase velocity is defined by

$$v_p = \frac{\omega}{\kappa} \quad -(8)$$

so

$$v_p \rightarrow \left(\frac{1}{(\mu_r \epsilon_r)^{1/2}}\right) v_p \quad -(9)$$

and the refractive index is:

$$n^2 = \mu_r \epsilon_r \quad -(10)$$

The condition for eqs (6) to (10) is:

$$\underline{\omega} \times \underline{B} + \frac{\omega_0}{c} \underline{E} = \underline{A} \times \underline{R}(\text{spin}) + A_0 \underline{R}(\text{orb}) \quad -(11)$$

which is the same as the free space condition except for the fact that the phase velocity is v_p and not c.

The phase of the plane wave is changed as follows:

$$\omega_0 \left(t - \frac{Z}{c} \right) \rightarrow \omega \left(t - \frac{Z}{v_p} \right) \quad -(12)$$

where

$$v_p = \frac{c}{(\mu_r \epsilon_r)^{1/2}} \quad -(13)$$

The incoming angular frequency is ω_0 and the shifted angular frequency is ω. So:

$$\boxed{\frac{\omega_0}{\omega} = \frac{t - (\mu_r \epsilon_r)^{1/2} \frac{Z}{c}}{t - \frac{Z}{c}}} \quad -(14)$$

If the time origin is:

$$t = 0 \quad -(15)$$

4) then:

$$\boxed{\frac{\omega_0}{\omega} = (\mu_r \epsilon_r)^{1/2}} \quad - (16)$$

In general:

$$\mu_r = \mu' + i\mu'' \quad - (17)$$

and

$$\epsilon_r = \epsilon' + i\epsilon'' \quad - (18)$$

where ϵ' is the dielectric permittivity and ϵ'' the dielectric loss.

The condition for eq. (16) is a condition on the spin connection (11)