276(2): The Simplest Expression for Precession in 3D Orbit Theory.

From the previous note, the general expression for x is:

$$x = f(\phi) = \frac{1}{\phi} \tan^{-1}\left(\frac{L}{L_Z} \tan\phi\right) \quad - (1)$$

$$= 1 + \frac{3mG}{\alpha c^2}$$

The effect of x is to change 2π to $2\pi + \Delta\phi$ in one complete orbit, where $\Delta\phi$ is very small. In the solar system it is only a few arc seconds per revolution of 2π radians. The x needed for the angle $\Delta\phi$ is:

$$x = \frac{1}{\Delta\phi} \tan^{-1}\left(\frac{L}{L_Z} \tan(\Delta\phi)\right) \quad - (2)$$

Now consider the Maclaurin expansions:

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots \quad - (3)$$

valid for:

$$|x| < 1 \quad - (4)$$

and

$$\tan x = x + \frac{x^3}{3} + \frac{2}{15} x^5 + \ldots \quad - (5)$$

valid for

$$|x| < \pi/2 \quad - (6)$$

2)

It follows that Eqs. (2) to (6) give:

$$x = \frac{1}{\Delta\phi}\left(\frac{L}{L_Z}\tan(\Delta\phi) - \frac{1}{3}\left(\frac{L}{L_Z}\tan(\Delta\phi)\right)^3 + \dots\right) \quad -(7)$$

where

$$\Delta\phi << 1 \quad -(8)$$

in radians. So:

$$\tan\Delta\phi \sim \Delta\phi << 1. \quad -(9)$$

and

$$\boxed{x = \frac{L}{L_Z}} \quad -(10)$$

This result can be obtained in a different way as shown by co author Horst Eckardt by using a Taylor expansion around $\phi = 2\pi$. This gives:

$$\tan\phi = \phi - 2\pi + \frac{1}{3}(\phi - 2\pi)^3 + \dots \quad -(11)$$

For small ϕ:

$$\tan(\phi + 2\pi) \sim \phi \quad -(12)$$

So:

$$\tan^{-1}\left(\frac{L}{L_Z}\tan\phi\right) = \frac{L}{L_Z}\phi - \frac{1}{3}\frac{L^3\phi^3}{L_Z^3} + \dots$$

(13)

3) giving eq. (10) again, Q.E.D.

This leads to the result:

$$\boxed{\frac{L}{L_z} = 1 + \frac{3MG}{\alpha c^2}} \qquad (14)$$

for all orbit precessions.

Conclusion

All observed orbit precessions are due to three dimensional orbit theory, and the use of the spherical polar coordinates in the kinetic energy on the classical level. This is a major advance in the theory of orbits because it means that all orbits are three dimensional.