276(6): Thomas Precession in Three Dimensions, Thomas Effect in a Pendulum and de Sitter Precession.

Consider the three dimensional metric:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 \quad -(1)$$

The 3D precession in general is:

$$\phi \rightarrow \phi + \omega t, \quad -(2)$$

$$d\phi \rightarrow d\phi + \omega dt, \quad -(3)$$

So:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 (d\phi + \omega dt)^2 \quad -(4)$$

where:

$$(d\phi + \omega dt)^2 = d\phi^2 + 2\omega d\phi dt + \omega^2 dt^2 \quad -(5)$$

So:

$$ds^2 = c^2 d\tau^2 = (c^2 - r^2\omega^2) dt^2 - dr^2 - r^2 d\phi^2 - 2\omega r^2 d\phi dt. \quad -(6)$$

Now define:

$$v_\theta = |\underline{\omega} \times \underline{r}| = \omega r \quad -(7)$$

then:

$$ds^2 = \left(1 - \frac{v_\theta^2}{c^2}\right) c^2 dt^2 - dr^2 - r^2 d\phi^2 - 2\omega r^2 d\phi dt \quad -(8)$$

Now define the angular velocity:

$$\omega = \frac{d\phi}{dt} = \frac{v_\theta}{r} \quad -(9)$$

2) and it follows that:

$$ds^2 = c^2 d\tau^2 = \left(1 - 3\frac{v_\theta^2}{c^2}\right) c^2 dt^2 - v^2 dt^2$$

— (10)

As in previous notes the precession of orbits follows from this result.

The above theory can be developed in a different way to give (UFT 110):

$$ds^2 = \left(1 - \frac{v_\theta^2}{c^2}\right)\left(c^2 dt^2 - 2r^2 \Omega \, d\phi \, dt\right) - dr^2 - r^2 d\phi^2$$

— (11)

where the relativistic angular velocity is defined as:

$$\Omega = \omega \left(1 - \frac{v_\theta^2}{c^2}\right)^{-1} \quad — (12)$$

The relativistic time interval is:

$$dt' = \left(1 - \frac{v_\theta^2}{c^2}\right)^{1/2} dt \quad — (13)$$

In a rotation of 2π radians:

$$\omega t = 2\pi \quad — (14)$$

and

$$\alpha = \Omega t' - \omega t = 2\pi \left(\left(1 - \frac{v_\theta^2}{c^2}\right)^{1/2} - 1\right)$$

— (15)

) This result is observed in the Foucault pendulum as described in UFT 110. The relativistic time of eq. (13) is observed in spin orbit coupling in atoms and molecules.

The result:

$$dt' = \left(1 - \frac{3v_\theta^2}{c^2}\right)^{1/2} dt \qquad -(16)$$

$$= \left(1 - \frac{3mG}{c^2 r}\right)^{1/2} dt$$

is conventionally obtained by rotating the so called Schwarzschild metric using three dimensional theory with:

$$\phi \rightarrow \phi + \omega t \qquad -(17)$$

Conventionally:

$$dt' = \left(1 - \frac{mG}{c^2 r}\right)^{1/2} dt \qquad -(18)$$

is attributed to the gravitational red shift,

and

$$dt' = \left(1 - \frac{2mG}{c^2 r}\right)^{1/2} dt \qquad -(19)$$

is attributed to the geodetic effect or de Sitter precession.

However, the result (16) can be

4) Started from the metric (1). By using:

$$\beta \rightarrow \beta + \left(\frac{\omega}{\sqrt{2}}\right) dt \quad - (16)$$

So:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 \left(d\beta + \frac{\omega}{\sqrt{2}} dt\right)^2$$

$$- (17)$$

So:

$$\left(d\beta + \frac{\omega}{\sqrt{2}} dt\right)^2 = d\beta^2 + \omega \, d\beta \, dt + \frac{\omega^2}{2} dt^2 \quad - (18)$$

and:

$$ds^2 = c^2 d\tau^2 = \left(c^2 - \frac{r^2 \omega^2}{2}\right) dt^2 - dr^2 - r^2 d\beta^2$$

$$- \omega r^2 d\beta \, dt \quad - (19)$$

Now use:

$$V_\theta = |\underline{\omega} \times \underline{r}| = \omega r \quad - (20)$$

and

$$\omega = \frac{d\beta}{dt} = \frac{V_\theta}{r} \quad - (21)$$

so

$$\omega r^2 d\beta \, dt = \omega^2 r^2 dt^2 \quad - (22)$$

It follows that:

5)
$$ds^2 = \left(c^2 - \frac{3}{2} r^2 \omega^2\right) dt^2 - dr^2 - r^2 d\phi^2 \quad -(23)$$

$$= \left(1 - \frac{3}{2}\frac{v_\theta^2}{c^2}\right) c^2 dt^2 - dr^2 - r^2 d\phi^2.$$

Using the equivalence principle as in UFT 265:

$$ds^2 = \left(1 - \frac{3MG}{c^2 r}\right) c^2 dt^2 - dr^2 - r^2 d\phi^2 \quad -(24)$$

and
$$\Delta t' = \left(1 - \frac{3MG}{c^2 r}\right)^{1/2} dt \quad -(25).$$

which is eq. (16) QED.

Therefore the gravitational red shift and the de Sitter precession can be obtained from the Minkowski metric (1) with the frame rotation (16).