275(2): Cartesian Representations of the Beta Ellipse and Three Dimensional Orbits.

Consider the beta ellipse:

$$r = \frac{\alpha}{1 + \epsilon \cos \beta} \quad -(1)$$

which gives various conic sections as follows.

) Ellipse

In this case:

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1 \quad -(2)$$

and

$$Z^2 = \left(1 - \frac{L_Z}{L}\right) Y^2 \quad -(3)$$

with

$$Z = -\left(1 - \frac{L_Z}{L}\right) Y. \quad -(4)$$

Here:

$$X = a\epsilon + r\cos\beta \quad -(5)$$

$$Y = r\sin\beta \quad -(6)$$

$$Z = r\cos\theta \quad -(7)$$

so

$$\cos\theta = -\left(1 - \left(\frac{L_Z}{L}\right)\right)\sin\beta \quad -(8)$$

The ellipticity is:

$$\epsilon^2 = 1 - \frac{b^2}{a^2} \quad -(9)$$

and the half right latitude is:

$$d = a(1-\epsilon^2). \quad -(10)$$

Here a and b are the major and minor semi axes of the seta ellipse.

2) <u>Hyperbola</u>

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad -(11)$$

$$z^2 = \left(1 - \frac{L_z}{L}\right) y^2 \quad -(12)$$

$$z = -\left(1 - \frac{L_z}{L}\right) y. \quad -(13).$$

where:

$$x = -a\epsilon + r\cos\beta \quad -(14)$$

$$y = r\sin\beta \quad -(15)$$

$$z = r\cos\theta \quad -(16)$$

The ellipticity is:

$$\epsilon^2 = 1 + \frac{b^2}{a^2} \quad -(17)$$

and the half right latitude is:

$$d = a(\epsilon^2 - 1) \quad -(18)$$

where

$$\epsilon > 1 \quad -(19)$$

3) 3) The Parabola

Here
$$y^2 = 4aX \quad -(20)$$
$$z = -\left(1 - \frac{L_z}{L}\right) y \quad -(21)$$
$$z^2 = \left(1 - \frac{L_z}{L}\right) y^2 \quad -(22)$$

and
$$\epsilon = 1 \quad -(23)$$

4) The Circle

Here:
$$x^2 + y^2 = r^2 \quad -(24)$$

where
$$r = a = b \quad -(25)$$
$$z = -\left(1 - \frac{L_z}{L}\right) Y \quad -(26)$$
$$z^2 = \left(1 - \frac{L_z}{L}\right)^2 y^2 \quad -(27)$$

and
$$\epsilon = 0 \quad -(28)$$
$$d = r \quad -(29)$$
$$X = r\cos\beta \quad -(30)$$
$$Y = r\sin\beta \quad -(31)$$
$$Z = r\cos\theta \quad -(32)$$

4)

The polar representation of eq. (20) is

$$r = \frac{-2a}{1+\cos\beta} \quad -(33)$$

where:

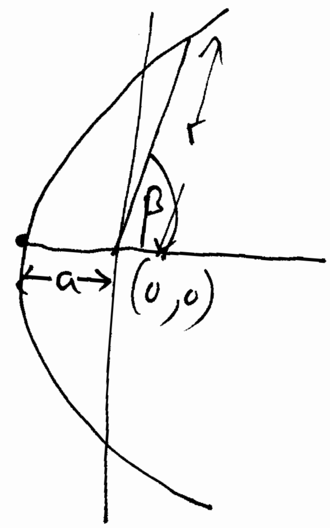

Fig (1)

$$(X', Y') = (X-a, Y) \quad -(34)$$

$$r^2 = X'^2 + Y'^2 \quad -(35)$$

$$\beta = \tan^{-1}\frac{Y'}{X'} \quad -(36)$$

$$\cos\beta = \frac{X-a}{((X-a)^2+Y^2)^{1/2}} \quad -(37)$$

$$r^2 = (X-a)^2 + Y^2 \quad -(38)$$

From these equations various 3D orbits can be constructed as in the next note.