272(1): Transition to Planar Ellipses

The three dimensional orbit is:

$$r = \frac{\alpha}{1+\epsilon\cos\beta} \quad -(1)$$

where

$$\tan\beta = \frac{L_Z}{L}\tan\phi \quad -(2)$$

$$= \frac{\sin\beta}{\cos\beta} = \frac{(1-\cos^2\beta)^{1/2}}{\cos\beta} \quad -(3)$$

so

$$\cos\beta = \left(1+\left(\frac{L_Z}{L}\right)^2\tan^2\phi\right)^{-1/2} \quad -(4)$$

and the orbit is:

$$r = \frac{\alpha}{1+\epsilon\left(1+\left(\frac{L_Z}{L}\right)^2\tan^2\phi\right)^{-1/2}} \quad -(5)$$

This function approaches the ellipse:

$$r = \frac{\alpha}{1+\epsilon\cos\phi} \quad -(6)$$

as

$$L \rightarrow L_Z. \quad -(7)$$

Therefore plot eq. (5) on top of eq. (6) to see the effect of three dimensionality.

Also:

$$r = \frac{\alpha}{1+\epsilon\left(1-\frac{L^2}{(L^2-L_Z^2)}\cos^2\theta\right)^{1/2}} \quad -(8)$$

2) Because:

$$\sin\beta = -\frac{L}{(L^2-L_Z^2)^{1/2}}\cos\theta, \quad -(9)$$

$$\sin^2\beta = \frac{L^2}{L^2-L_Z^2}\cos^2\theta \quad -(10)$$

$$= 1-\cos^2\beta$$

So

$$\cos\beta = \left(1-\frac{L^2}{L^2-L_Z^2}\cos^2\theta\right)^{1/2} \quad -(11)$$

In the planar orbit there is no dependence of r on θ. From eqs (4) and (11)

$$\left(1+\left(\frac{L_Z}{L}\right)^2\tan^2\phi\right)^{-1/2} = \left(1-\frac{L^2}{L^2-L_Z^2}\cos^2\theta\right)^{1/2} \quad -(12)$$

so that graphs of ϕ versus θ and θ versus ϕ can be constructed. From eq. (12):

$$\frac{L^2-L_Z^2}{1+\left(\frac{L_Z}{L}\right)^2\tan^2\phi} = L^2-L_Z^2-L^2\cos^2\theta \quad -(13)$$

and as $L \rightarrow L_Z$, $\cos\theta \rightarrow 0 \quad -(14)$

then $0 = 0 \quad -(15)$