72(5): Equations for More Direct Interpretations

In the notation of previous work the complete set of equations for three dimensional orbits include the following:

$$\ddot{r} = r\left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta\right) - \frac{MG}{r^2} \quad -(1)$$

$$r\ddot{\theta} - r\dot{\phi}^2 \sin\theta\cos\theta + 2\dot{r}\dot{\theta} = 0 \quad -(2)$$

$$r\ddot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta + 2\dot{r}\dot{\phi}\sin\theta = 0 \quad -(3)$$

$$\dot{\phi} = \frac{L_z}{mr^2\sin^2\theta} \quad -(4)$$

$$\dot{\theta} = \frac{1}{mr^2}\left(L^2 - \frac{L_z^2}{\sin^2\theta}\right)^{1/2} \quad -(5)$$

$$\sin^2\theta = \left(\frac{L_z}{L}\right)^2 + \left(1 - \left(\frac{L_z}{L}\right)^2\right)\left(\frac{\cos^2\phi}{\cos^2\phi + \left(\frac{L_z}{L}\right)^2\sin^2\phi}\right) \quad -(6)$$

$$\cos^2\theta = 1 - \sin^2\theta \quad -(7)$$

$$r = \frac{\alpha}{1 + \epsilon\cos\beta} \quad -(8)$$

$$\cos\beta = \frac{\cos\phi}{\left(\cos^2\phi + \left(\frac{L_z}{L}\right)^2\sin^2\phi\right)^{1/2}} \quad -(9)$$

2)
$$\dot{r} = \left(\frac{2}{m} \left(E - \frac{L^2}{2mr^2} + \frac{k}{r} \right) \right)^{1/2} \quad -(10)$$

1) Using these equations $\ddot{r}$ can be expressed in terms of ϕ in 3-D, and the result graphed.

In two dimensional theory:
$$r = \frac{\alpha}{1 + \epsilon \cos \phi} \quad -(11)$$

and
$$\ddot{r} = r \dot{\phi}^2 - \frac{MG}{r} \quad -(12)$$

where
$$\dot{\phi} = \frac{L}{mr^2} \quad -(13)$$

So $\ddot{r}$ can also be expressed in terms of ϕ in two dimensional theory, and the two types of curve compared directly.

2) The function $\ddot{\phi}$ can also be expressed in terms of ϕ without involving the tangent function. In previous work this function was shown to produce orbital like structure by use of graphics. So:

3)

$$\ddot{\phi} = -2\left(\dot{\theta}\dot{\phi}\frac{\cos\theta}{\sin\theta} + \frac{\dot{r}\dot{\phi}}{r}\right) \quad -(14)$$

This can be worked out over the complete range of ϕ to give orbital like structure over the complete range of ϕ. The $\cos\theta$ and $\sin\theta$ functions are defined by eqs. (6) and (7).

In two dimensions:

$$\ddot{\phi} = -2\frac{\dot{r}}{r}\dot{\phi} \quad -(15)$$

where $\dot{\phi}$ is expressed in terms of $\sin^2\theta$ and r in eqs. (4), (6), (8) and (9), so:

$$\ddot{\phi} = f(\phi) \quad -(16)$$

and

$$\ddot{r} = f(\phi) \quad -(17)$$

in two dimensions and three dimensions.
