266(10): Results for Graphing

The precessing ellipse of the Sommerfeld theory is given by:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad -(1)$$

where:

$$\alpha = r\left(1 - \frac{1}{x^2}\left(1 - \frac{1}{\gamma}\right)\right)^{-1} - (2)$$

$$\epsilon^2 = 1 + \frac{2E\alpha}{k} \quad -(3)$$

Here:

$$\gamma = \left(1 - \left(\frac{\alpha_f}{n}\right)^2\right)^{-1/2} \quad -(4)$$

where α_f is the fine structure constant:

$$\alpha_f = \frac{e^2}{4\pi\hbar c\epsilon_0} \quad -(5)$$

and n is the principal quantum number:

$$n = 1, 2, 3, \ldots \quad -(6)$$

In eq. (2):

$$r = \frac{4\pi\epsilon_0 n^2\hbar^2}{me^2} \quad -(7)$$

In Eq. (3): $\quad -(8)$

$$E = mc^2\left(\left(1 - \left(\frac{\alpha_f}{n}\right)^2\right)^{-1/2} - 1 - \left(\frac{\alpha_f}{n}\right)^2\right)$$

2) and

$$k = \frac{e^2}{4\pi\epsilon_0} \quad \text{—} (9)$$

Therefore

$$x^2 = \left(1 - \frac{1}{\gamma}\right)\left(1 - \frac{r}{\alpha}\right)^{-1} \quad \text{—} (10).$$

is very close to unity and ϵ is very close to zero.

<u>Suggested Procedure</u>

1) Choose x in a suitable range, for example:

$$1 < x < 1.00001 \quad \text{—} (11).$$

and evaluate α from eq. (2)

2) Evaluate ϵ from eq. (3).

3) Plot r as a function of θ.