266(1): The Fundamental Origin of Perihelion Precession

The fundamental origin of precession is:

$$d\theta' = d\theta + \omega dt \quad -(1)$$

when used in any metric. In this context the de Sitter and Thomas precessions become the same thing. With the definition

$$v_\theta = \omega r \quad -(2)$$

eq. (1) gives:

$$r^2 (d\theta')^2 = r^2 d\theta^2 + 3 v_\theta^2 dt^2 \quad -(3)$$

The equivalence principle gives:

$$\frac{1}{2} m v_\theta^2 = \frac{mMG}{r} \quad -(4)$$

so:

$$r^2 (d\theta')^2 = r^2 d\theta^2 + \left(\frac{6MG}{rc^2}\right) c^2 dt^2 \quad -(5)$$

The effect of eq. (1) is therefore to change the original infinitesimal line element:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\theta^2 \quad -(6)$$

to

$$c^2 d\tau^2 = \left(1 - \frac{6MG}{c^2 r}\right) c^2 dt^2 - dr^2 - r^2 d\theta^2 \quad -(7)$$

The orbital velocity is:

$$v^2 = v_r^2 + v_\theta^2 = \left(\frac{dr}{dt}\right)^2 + r^2 \left(\frac{d\theta}{dt}\right)^2 \quad -(8)$$

So:

$$\left(\frac{d\tau}{dt}\right)^2 = \left(1-\frac{6mG}{c^2 r}\right) - \frac{v^2}{c^2}. \quad -(9)$$

In the absence of precession:

$$\left(\frac{d\tau}{dt}\right)^2 = 1 - \frac{v^2}{c^2}. \quad -(10)$$

So

$$\frac{dt}{d\tau} = \left(\left(1-\frac{6mG}{c^2 r}\right) - \frac{v^2}{c^2}\right)^{-1/2} \quad -(11)$$

$$\sim 1 + \frac{3mG}{c^2 r} + \frac{1}{2}\frac{v^2}{c^2}$$

If

$$\frac{mG}{c^2} << r \text{ and } v << c. \quad -(12)$$

the extra effect of precession is therefore:

$$\Delta\left(\frac{dt}{d\tau}\right) = 1 + \frac{3mG}{c^2 r} \quad -(13)$$

in the approximations (12).

At the turning point:

$$v_r = \frac{dr}{dt} = 0 \quad -(14)$$

and

$$v = v_\theta = r\frac{d\theta}{dt} = \omega r, \quad -(15)$$

which is eq. (2).

At the turning point

$$r = \alpha \quad - (16)$$

both for the conical section:

$$r = \frac{\alpha}{1 + \epsilon \cos(\theta)} \quad - (17)$$

and the processing conical section:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad - (18)$$

so the precession of the perihelion is defined by:

$$\Delta\left(\frac{dt}{d\tau}\right) = 1 + \frac{3MG}{c^2 \alpha} \quad - (19)$$

which is exactly the experimental result.

Note carefully that the Thomas velocity v_θ always defines a turning point, because v_r is zero.

The turning point is defined by:

$$m\frac{d^2 r}{dt^2} = F + \frac{L^2}{mr^3} = 0 \quad - (20)$$

i.e.

$$\frac{d}{dt}\left(\frac{dr}{dt}\right) = 0, \quad \frac{dr}{dt} = 0. \quad - (21)$$

For the ellipse (17):

4)

$$F = -\frac{mMG}{r^2} \quad -(22)$$

and

$$\alpha = \frac{L^2}{m^2 MG} \quad -(23)$$

so eq. (20) gives $r = \alpha. \quad -(24)$

This point, the half right latitude, is sketched in Fig (1):

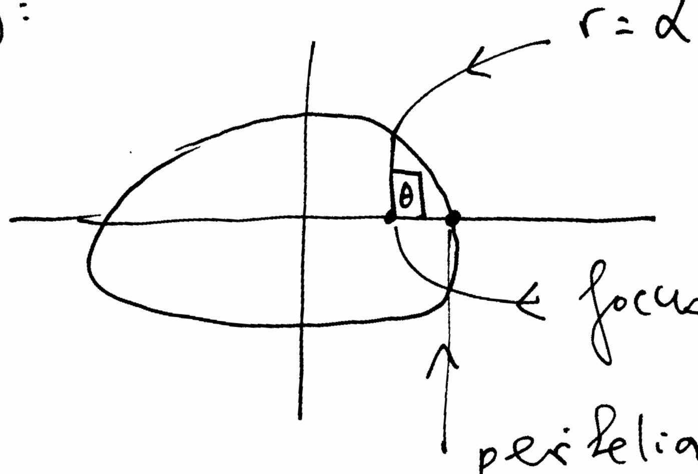

Fig (1)

At the point $r = \alpha$ the extra precession due to eq. (1) is given by eq. (19), the experimental result.

It follows that the extra precession at all points of the ellipse is given by Eq. (19).

The perihelion is defined by the distance of closest approach:

$$r = \frac{\alpha}{1+\epsilon} \quad -(25)$$

5) The rotation of the frame at the velocity v_θ is equivalent to the circular function:

$$r = \alpha \quad -(26)$$

which is the conical section with:

$$\epsilon = 0. \quad -(27)$$

The ellipse is defined as:

$$r = \frac{\alpha}{1+\epsilon\cos\theta}. \quad -(28)$$

Eqs (26) and (28) are the same when:

$$\cos\theta = 0, \quad \epsilon \neq 0, \quad -(29)$$
$$0 < \epsilon < 1$$

This is the point with: $\theta = \pi / 2 \quad -(30)$

marked in Fig (1).

At this point the velocity of a mass m on the ellipse and on the circle is the same, i.e. v_θ. This velocity is imparted to the ellipse, and it rotates or precesses. All points on the ellipse rotate at the same velocity v_θ, including the perihelion and aphelion.

In ECE the angular velocity ω in Eq. (1) is a spin connection of spacetime. This is the fundamental cause of the precession of an ellipse. The perihelion is chosen as a point of measurement.