266(6) : Bohr Theory of the atom.

The hamiltonian is:

$$H = E = \frac{1}{2} m \left(\left(\frac{dr}{dt} \right)^2 + r^2 \left(\frac{d\theta}{dt} \right)^2 \right) - \frac{k}{r} \quad -(1)$$

where

$$k = \frac{e^2}{4\pi\epsilon_0} \quad -(2)$$

In general this hamiltonian produces the ellipse:

$$r = \frac{\alpha}{1 + \epsilon \cos\theta} \quad -(3)$$

where

$$\alpha = \frac{L^2}{mk} \quad -(4)$$

and

$$\epsilon = \left(1 - \frac{2EL^2}{mk^2} \right)^{1/2} \quad -(5)$$

The force law is:

$$m\frac{d^2 r}{dt^2} = -\frac{k}{r^2} + \frac{L^2}{mr^3} \quad -(6)$$

Bohr used the quantization condition:

$$L = n\hbar \quad -(7)$$

where

$$n = 0, 1, 2, 3, \ldots \quad -(8)$$

and the turning point condition:

2)

$$\frac{d^2 r}{dt^2} = 0 \quad -(9)$$

so

$$\frac{k}{r^2} = \frac{L^2}{mr^3} \quad -(10)$$

i.e.

$$r = \alpha = \frac{L^2}{mk} = \frac{n^2 \hbar^2}{mk} \quad -(11)$$

when

$$n = 1 \quad -(12)$$

Eq. (11) defines the Bohr radius:

$$\boxed{r = \frac{\hbar^2}{mk} = \frac{4\pi\epsilon_0 \hbar^2}{me^2}} \quad -(13)$$

The condition (9) reduces the ellipse to a circle:

$$\epsilon = 0 \quad -(14)$$

so from Eq. (5):

$$\frac{2EL^2}{mk^2} = 1 \quad -(15)$$

i.e.

$$E = \frac{mk^2}{2L^2} = \frac{mk^2}{2\hbar^2 n^2} \quad -(16)$$

i.e the energy levels of the atom are:

$$\boxed{E = \frac{me^4}{8\pi^2 \epsilon_0^2 \hbar^2 n^2}} \quad -(17)$$

These are the energy levels of H from the Schroedinger equation.