266(4): Non-Relativistic Quantization in x Theory

Consider the non relativistic Hamiltonian:

$$H = \frac{1}{2} m v^2 + U(r) \quad - (1)$$

In plane polar coordinates:

$$v^2 = \dot{r}^2 + r^2 \dot{\theta}^2 \quad - (2)$$

and the force law is:

$$F(r) = -\frac{L^2}{mr^2}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right)$$

$$= \frac{m d^2 r}{dt^2} - \frac{L^2}{mr^3} \quad - (3)$$

Therefore the 1689 Leibniz equation is:

$$\frac{m d^2 r}{dt^2} = F(r) + \frac{L^2}{mr^3} = -\frac{L^2}{mr^2}\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) - (4)$$

For an elliptical orbit:

$$r = \frac{\alpha}{1 + \epsilon \cos(\theta)} \quad - (5)$$

and

$$F(r) = -\frac{mMG}{r^2} \quad - (6)$$

The elliptical orbit however shows no additional structure or quantization.

2) Quantization appears with the precessing elliptical orbit:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad - (7)$$

From eqs. (4) and (7):

$$m\frac{d^2 r}{dt^2} = x^2\left(\frac{L^2}{mr^3} - \frac{L^2}{mr^2\alpha}\right) \quad - (8)$$

$$= x^2\left(-\frac{mMG}{r^2} + \frac{L^2}{mr^3}\right)$$

where

$$\alpha = \frac{L^2}{m^2 MG} \quad - (9)$$

Note carefully that the right hand side of eq. (8) is x^2 times the right hand side of the 1689 Leibniz equation for an elliptical orbit.

The factor x for all planetary precessions is

$$x = 1 + \frac{3MG}{c^2\alpha} \quad - (10)$$

From eqs (3) and (8):

$$F(r) = m\frac{d^2 r}{dt^2} - \frac{L^2}{mr^3} \quad - (11)$$

So the force law needed for quantization is:

$$F(r) = -x^2 \frac{mMG}{r^2} + (x^2 - 1)\frac{L^2}{mr^3} \quad -(12)$$

The potential needed for quantization is:

$$U(r) = -\int F(r)\,dr = -x^2 \frac{mMG}{r} + (x^2-1)\frac{L^2}{2mr^2} \quad -(13)$$

Therefore the Hamiltonian for quantization is:

$$\boxed{H = \frac{1}{2}mv^2 - x^2\frac{mMG}{r} + (x^2-1)\frac{L^2}{2mr^2} \quad -(14).}$$

Q.E.D.

Note carefully that quantization can be investigated by simply graphing Eq. (7). Closed orbits are quantized orbits.

Atoms and Molecules

A precisely analogous procedure can be used in the simplest case of a hydrogen atom, in which the hamiltonian is:

$$H = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\epsilon_0 r} \quad -(15)$$

Therefore:

$$nMG \longrightarrow \frac{e^2}{4\pi\epsilon_0} \quad -(16)$$

Quantization appears through the precessing ellipse:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(17)$$

and the hamiltonian:

$$H = \frac{1}{2}mv^2 - \frac{x^2 e^2}{4\pi\epsilon_0 r} + \frac{(x^2-1)L^2}{2mr^2} \quad -(18)$$

where m is the electron mass, $-e$ is the electron charge, and ϵ_0 the vacuum permittivity.

If it is assumed that gravitation and electromagnetism are equivalent, as indicated in eq. (16), then for electromagnetism:

$$x = 1 + \frac{3e^2}{4\pi\epsilon_0 mc^2\alpha} \quad -(19)$$
$$= 1 + \frac{3nG}{c^2\alpha}$$

Here α is the right semi latitude of the orbital.

The fine structure constant is:

5)

$$\alpha_f = \frac{e^2}{4\pi\hbar c\epsilon_0} = 0.007297351 \quad -(20)$$

so

$$\boxed{x = 1 + 3\frac{\hbar}{mc}\frac{\alpha_f}{\alpha}} \quad -(21)$$

In this model Planck quantization occurs through

$$x = 1 + \hbar\left(\frac{3\alpha_f}{mc\alpha}\right) \quad -(22)$$

The conventional route to quantization is the well known Schroedinger equation, which is obtained from eq. (15) with:

$$H = \frac{p^2}{2m} - \frac{e^2}{4\pi\epsilon_0 r} \quad -(23)$$

and

$$\underline{p}\phi = -i\hbar\underline{\nabla}\phi \quad -(24)$$

so

$$-\frac{\hbar^2}{2m}\nabla^2\phi - \frac{e^2}{4\pi\epsilon_0 r}\phi = E\phi \quad -(25)$$

i.e.

$$\hat{H}\phi = E\phi \quad -(26)$$

It is very much easier to ~~graph eq.~~

(17) than solve Eq. (25).

In order to introduce the l quantum number it is possible to use:

$$L^2 \phi = \hbar^2 l(l+1)\phi \quad -(27)$$

in eq. (18). The conventional approach to the Schroedinger quantization uses:

$$H = \frac{1}{2} m v^2 - \frac{e^2}{4\pi\epsilon_0 r} \quad -(28)$$

$$= \frac{1}{2} m \left(\frac{d^2 r}{dt^2} + r^2 \left(\frac{d\theta}{dt} \right)^2 \right) - \frac{e^2}{4\pi\epsilon_0 r}$$

where

$$\frac{d\theta}{dt} = \frac{L}{m r^2} \quad -(29)$$

So:

$$H = \frac{1}{2} m \frac{d^2 r}{dt^2} + \frac{L^2}{2 m r^2} - \frac{e^2}{4\pi\epsilon_0 r} \quad -(30)$$

with the potential:

$$U = \frac{L^2}{2 m r^2} - \frac{e^2}{4\pi\epsilon_0 r} \quad -(31)$$

$$= \frac{\hbar^2 l(l+1)}{2 m r^2} - \frac{e^2}{4\pi\epsilon_0 r}$$

Eq. (27) is actually a classical result of the spherical harmonics.

) In the new route to quantization the principal quantum numbers are defined by:

$$n = x = 1 + \frac{3\hbar}{mc}\frac{\alpha_f}{\alpha} \quad -(32)$$

where the Compton wavelength of an electron is:

$$\lambda_c = \frac{h}{mc} = 2.426309 \times 10^{-12}\, m \quad -(33)$$

and the fine structure constant is:

$$\alpha_f = \frac{e^2}{4\pi\hbar c\epsilon_0} = 0.007297351. \quad -(34)$$

Note that

$$\hbar = h / (2\pi) \quad -(35)$$

If

$$x = 1 + \frac{r_0}{\alpha} \quad -(36)$$

then integer x occurs at:

$$\alpha = \frac{r_0}{n-1} \quad -(37)$$

where

$$r_0 = \frac{3\hbar}{mc}\alpha_f = \frac{3}{2\pi}\lambda_c\alpha_f \quad -(38)$$

So

$$\boxed{\alpha = \frac{3}{2\pi(n-1)}\lambda_c\alpha_f} \quad -(39)$$

For the quantum state:

$$n = 1 \quad - (40)$$

then

$$\alpha \rightarrow \infty \text{ and } x \rightarrow 1 \quad - (41)$$

and an ellipse is obtained. In the traditional quantization this is:

$$n = 1, \; \ell = 0 \, (1s) \quad - (42)$$

From eqs. (36) and (37):

$$x = 1 + n - 1 = n \quad - (43)$$

For

$$x = 2 \quad - (44)$$

Eq. (17) develops structure, and in the traditional quantization:

$$\left.\begin{array}{l} n = 2, \; \ell = 0 \, (2s) \\ n = 2, \; \ell = 1 \, (2p) \end{array}\right\} \quad - (45)$$

The S orbitals in the new quantization can be understood by using the circular limit of eq. (17):

$$\alpha = r \quad - (46)$$

so that the ellipse becomes a circle. So eq. (37) is interpreted as:

$$\boxed{\alpha = \frac{r_0}{n-1}, \quad n \neq 1} \quad - (47)$$

This is a plausible theory, but it is

) make systematic to regard the classical theory as a limit of the Bohr Sommerfeld quantization, which leads to:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(48)$$

where

$$\alpha = \frac{1}{K} = \frac{L^2}{mkZe^2}\left(1+\frac{H}{mc^2}\right)^{-1} \quad -(49)$$

and

$$k = \frac{1}{4\pi\epsilon_0} . \quad -(50)$$

For the H atom:

$$Z = 1 . \quad -(51)$$

Here

$$x^2 = 1 - \left(\frac{e^2}{4\pi\epsilon_0 c L}\right)^2 \quad -(52)$$

The principal quantum number is defined by:

$$n\hbar = \oint m\frac{dr}{dt}\,dr = \oint p_r\,dr$$

$$= L\oint\left(\frac{1}{r}\frac{dr}{d\theta}\right)^2 d\theta \quad -(53)$$

and the angular momentum quantum number by:

$$\oint L\,d\theta = 2\pi L = \hbar l \quad -(54)$$

The fine structure constant is:

10)

$$\alpha_f = \frac{e^2}{4\pi\hbar c \epsilon_0} \quad -(55)$$

So

$$\boxed{x^2 = 1 - \left(\frac{\hbar \alpha_f}{L}\right)^2} \quad -(56)$$

So for x, eq. (54) must be used:

$$L = \hbar \ell \quad -(57)$$

and

$$x^2 = 1 - \left(\frac{\alpha_f}{\ell}\right)^2 \quad -(58)$$

<u>Summary</u>

In the classical limit of the Sommerfeld Hamiltonian

$$T = (\gamma^2 - 1) m c^2 \rightarrow \frac{1}{2} m v^2 \quad -(59)$$

and

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)}, \quad -(60)$$

where

$$x^2 = 1 - \left(\frac{\alpha_f}{\ell}\right)^2 \quad -(61)$$

and

$$\alpha = \frac{L^2}{4\pi \epsilon_0 m e^2} \left(1 + \frac{H}{mc^2}\right)^{-1} \quad -(62)$$

The eccentricity ϵ may be varied