262(8): Comparison of Theories of Orbital Precession

For any planar orbit:

$$\underline{F}(r) = -\frac{L^2}{mr^2}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)+\frac{1}{r}\right)\underline{e}_r \quad -(1)$$

Theory 1

If
$$\underline{F}(r) = \left(-\frac{mMG}{r^2} - \frac{3GML^2}{mc^2r^4}\right)\underline{e}_r \quad -(2)$$

then
$$\omega = \omega_0\left(1+\frac{6MG}{\alpha c^2}\right) \quad -(3)$$

where ω_0 is the angular velocity of the Newtonian orbit defined by:
$$\underline{F}(r) = -\frac{mMG}{r^2}\,\underline{e}_r \quad -(4)$$

i.e.
$$\omega_0 = \frac{L}{mr^2}. \quad -(5)$$

and
$$\Delta\theta = \left(\frac{3MG}{ac^2(1-\epsilon^2)}\right)\theta \quad -(6)$$

The force law (2) is due to the Cartan spin connection (3). At the orbital turning point
$$\frac{d^2r}{dt^2} = 0 \quad -(7)$$

2) then

$$r = \alpha - r_0 = \frac{\alpha}{1 + \epsilon\cos\theta} \quad - (8)$$

where

$$r_0 = \frac{3MG}{c^2} \quad - (9)$$

<u>Theory 2</u>

If:

$$\underline{F}(r) = (x^2 - 1)\frac{L^2}{mr^3} - \frac{x^2 L^2}{\alpha m r^2} \quad - (10)$$

then:

$$\Delta\theta = x\theta \quad - (11)$$

and

$$r = \frac{\alpha}{1 + \epsilon\cos(x\theta)} \quad - (12)$$

At the orbital turning point:

$$r = \alpha \quad - (13)$$

So Theory 2 changes the angle from θ to $x\theta$, but does not change the turning point of the Newtonian orbit, while Theory 1 leaves the angle the same but changes the turning point:

$$\alpha \rightarrow \alpha - r_0 \quad - (14)$$

3) If the two theories have the same effect then:

$$x = \frac{3MG}{ac^2(1-\epsilon^2)} \quad -(15)$$

So two different force laws (2) and (10) produce the same observed precession. Therefore the precessional method cannot distinguish between the two force laws, proving that the force law (2) is not unique.

In the Newton theory let:

$$r \rightarrow r + r_0 \quad -(16)$$

so

$$F = -\frac{mMG}{r^2} \rightarrow -\frac{mMG}{(r+r_0)^2}$$

$$= -\frac{mMG}{r^2}\left(1+\frac{r_0}{r}\right)^{-2} = -\frac{mMG}{r^2}\left(1-2\frac{r_0}{r}\right) \quad -(17)$$

if

$$r_0 \ll r. \quad -(18)$$

so

$$F = -\frac{mMG}{r^2} + \frac{2mMGr_0}{r^3} \quad -(18)$$

Now compare eqs. (10) and (18) with

4)

$$x \sim 1, \quad \alpha = \frac{L^2}{m^2 M G} \quad -(19)$$

to find:

$$x^2 = 1 + 2\frac{r_0}{\alpha} \quad -(20)$$

to an excellent approximation If:

$$r_0 \ll \alpha \quad -(21)$$

then:

$$x = \left(1 + 2\frac{r_0}{\alpha}\right)^{1/2} = 1 + \frac{r_0}{\alpha} \quad -(22)$$

So if

$$x = 1 + \frac{r_0}{\alpha} \quad -(23)$$

then

$$r \rightarrow r + r_0 \quad -(24)$$

It follows that:

$$r + r_0 = \frac{\alpha}{1 + \epsilon \cos\theta} \quad -(25)$$

is equivalent to:

$$r = \frac{\alpha}{1 + \epsilon \cos\left(\left(1 + \frac{r_0}{\alpha}\right)\theta\right)} \quad -(26)$$

At the turning point:

$$r + r_0 = \alpha \quad -(27)$$

so

$$r = \alpha - r_0 \quad -(28)$$

5) However, eq. (28) is the result of eq. (2), so the orbits described by eqs. (25) and (26) are equivalent to eq. (2). Therefore the Einstein theory (2) is merely a re-expression of an equation of type (12).
