262(6) : Analysis of Planar Orbital Theory by Analysis of the Turning Point of Orbits.

The turning point of orbits is defined by :

$$m \frac{d^2 r}{dt^2} = F(r) + \frac{L^2}{mr^3} \qquad - (1)$$

$$= 0$$

where $F(r)$ is the force defined by :

$$F(r) = -\frac{L^2}{mr^2} \left(\frac{d^2}{d\theta^2} \left(\frac{1}{r} \right) + \frac{1}{r} \right) \qquad - (2)$$

along the $\underline{e}_r$ axis

1) Newtonian Theory

In this case :

$$F(r) = -\frac{mMG}{r^2} \qquad - (3)$$

so the turning point is defined by :

$$r = d \qquad - (4)$$

and

$$\cos \theta = 0 \qquad - (5)$$

where

$$d = r_{min} (1 + \epsilon) \qquad - (6)$$

where r_{min} is the perihelion, and where :

$$r = \frac{d}{1 + \epsilon \cos \theta} \qquad - (7)$$

'Here α is the half right latitude and ϵ is the eccentricity.

) Precessing Ellipse

Here
$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad -(8)$$

and as in previous notes the turning point is defined by:

$$m\ddot{r} = x^2 \left(\frac{L^2}{mr^3} - \frac{L^2}{\alpha m r^2} \right) - (9)$$

i.e
$$r = \alpha \quad - (10)$$

and
$$\cos(x\theta) = 0, \quad - (11)$$
$$x\theta = \pi/2 \quad - (12)$$

so the advance in the angle due to the precession factor x is:

$$\Delta\theta = \frac{\pi}{2}(1 - x) - (13)$$

with
$$x \sim 1 \quad - (14)$$

Using eq. (6) this can be measured through the precession of the perihelion.

Einstein Theory

In this case, the incorrect Einstein

field equation gives:

$$F(r) = -\frac{mMG}{r^2} - \frac{3GML^2}{mc^2 r^4} \qquad -(15)$$

so at the turning point:

$$-\frac{mMG}{r^2} - \frac{3GML^2}{mc^2 r^4} + \frac{L^2}{mr^3} = 0 \qquad -(16)$$

i.e.

$$r^2 - \alpha r + r_0 \alpha = 0 \qquad -(17)$$

where

$$r_0 = \frac{3MG}{c^2} \qquad -(18)$$

The solution of eqv. (17) is:

$$r = \frac{1}{2}\left(\alpha \pm \left(\alpha^2 - 4r_0\alpha\right)^{1/2}\right) \qquad -(19)$$

$$= \frac{1}{2}\left(\alpha \pm \alpha\left(1 - \frac{4r_0}{\alpha}\right)^{1/2}\right)$$

To an excellent approximation:

$$4r_0 \ll \alpha \qquad -(20)$$

So

$$r \sim \frac{1}{2}\left(\alpha \pm \alpha\left(1 - \frac{2r_0}{\alpha}\right)\right) \qquad -(21)$$

i.e.

$$r = \alpha - r_0 \qquad -(22)$$

or

$$r = r_0 \qquad -(23)$$

4) Assuming that eq. (23) is the physical result then at the turning point:

$$r = d - r_0 = \frac{d}{1 + \epsilon \cos\theta} \quad - (24)$$

The turning point is changed by:

$$r_0 = \frac{3mG}{c^2} \quad - (25)$$

The change in angle can so calculated from:

$$d = (d - r_0)(1 + \epsilon \cos\theta), \quad - (26)$$

So

$$1 + \epsilon \cos\theta = \frac{d}{d - r_0} = \left(1 - \frac{r_0}{d}\right)^{-1} \quad \ldots$$

$$- (27)$$

Using $r_0 \ll d$:

$$1 + \epsilon \cos\theta \sim 1 + \frac{r_0}{d} \quad - (28)$$

So

$$\boxed{\cos\theta = \frac{r_0}{\epsilon d}} \quad - (29)$$

So

$$\theta = \cos^{-1}\frac{r_0}{\epsilon d} \quad - (30)$$

$$\sim \frac{\pi}{2} - \frac{r_0}{\epsilon d} + \ldots$$

') using the Maclaurin series. The Newtonian result is:

$$\theta = \frac{\pi}{2} \quad - (31)$$

So

$$\boxed{\Delta\theta = \frac{3mG}{\epsilon c^2 d}} \quad - (32)$$

In eq. (32)

$$\epsilon d = a(1-\epsilon^2) - (33)$$

So we obtain:

$$\Delta\theta = \frac{3MG}{ac^2(1-\epsilon^2)} \quad - (34)$$

where a is the semi major axis.

Discussion

Eq. (34) checks that the turning point method works in the solar system, because eq. (34) is the same as the result obtained by Maria and Thornton, but in a much more complicated way. Despite the fact that eq. (34) is the usual textbook result, it is obtained by assuming that the orbit is an ellipse, i.e.

6) The half right latitude is changed but the angle θ in the equation of the ellipse remains the same.

It is known that the Einstein field equation is geometrically incorrect, but the result $(34)^V$ is accurate in the solar system.

The correct method of calculating (34) is to use the force law of ECE theory:

$$F(r) = -\frac{\partial \phi}{\partial r} + \Omega \phi \quad - (35)$$

where Ω is a spin connection of Cartan. This replaces the incorrectly symmetric Christoffel connection used by Einstein. In order to obtain the experimental result (34):

$$-\frac{\partial \phi}{\partial r} = -\frac{mMG}{r^2} \quad - (36)$$

and

$$\Omega \phi = -\frac{3GML^2}{mc^2 r^4} \quad - (37)$$

so:

$$\boxed{\Omega = \frac{3L^2}{m^2 c^2 r^3}} \quad - (38)$$

in units of inverse metres.