262(7): The Change in Spin Connection due to Orbital Precession

As shown in note 262(6) the experimental result for orbital precession in the solar system is:

$$\Delta\theta = \frac{3MG}{ac^2(1-\epsilon^2)} \quad -(1)$$

where M is the mass of the sun, G is Newton's constant, a the semi major axis, ϵ the eccentricity and c the vacuum speed of light.

The result (1) is produced by a shift in the orbital turning point:

$$\alpha \rightarrow \alpha - r_0 \quad -(2)$$

where

$$r_0 = \frac{3MG}{c^2} \quad -(3)$$

The experimentalists claim that the result (1) is very accurate. Accepting this claim for the sake of argument then the corresponding shift in the angular velocity is:

$$\omega = \frac{L}{m\alpha^2} \rightarrow \frac{L}{m(\alpha - r_0)^2} \quad -(4)$$

i.e.

2)

$$\omega \rightarrow \frac{L}{m}\left(\frac{1}{d-r_0}\right)^2$$

$$= \frac{L}{md^2}\left(1-\frac{r_0}{d}\right)^{-2} \quad -(5)$$

Now note that $\quad r_0 << d \quad -(6)$

so

$$\omega \rightarrow \frac{L}{md^2}\left(1+2\frac{r_0}{d}\right) \quad -(7)$$

to an excellent approximation. So:

$$\boxed{\omega \rightarrow \omega\left(1+\frac{6MG}{dc^2}\right)} \quad -(8).$$

Therefore the experimental result (1) is obtained by the Cartan spin connection (8),

Q.E.D.

The spin connection has been expressed in terms of angular velocity, in units of radians per second. To express it in units of inverse metres divide by c.