262(4): Antisymmetry Law and Theory of Orbits.

The theory of orbits produces the result that the net force is:

$$\underset{\sim}{F}(r) = m(\ddot{r} - r\dot{\theta}^2)\underline{e}_r$$
$$= -\frac{L^2}{mr^2}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right)\underline{e}_r = -\underline{\nabla}\phi \quad -(1)$$

The outward centrifugal force is present in this analysis and can be best understood by writing eq. (1) as:

$$m\ddot{r} = F(r) + mr\dot{\theta}^2 \quad -(2)$$

The centrifugal force is:

$$\underline{F}_c = mr\dot{\theta}^2\underline{e}_r = mr\omega^2\underline{e}_r$$
$$= \frac{L^2}{mr^3}\underline{e}_r \quad -(3)$$

The origin of the centrifugal force is geometrical, it is due to the spin connection written as the angular velocity $\underline{\omega}$ in units of radians per second. The spin connection is the fundamental origin of the plane polar coordinate rotation of the axes of the plane polar coordinate system. Note carefully that of the Cartesian system.

2) coordinate system is used, the axes are static and there is no centrifugal force, i.e.:

$$\underline{F}(r)\,(\text{Cartesian}) = m\ddot{r}\,\underline{e}_r \quad -(4)$$

The centrifugal force is an every day force and is of course real. It provides direct evidence for the existence of the spin connection. So Eq. (1) is an equation of general relativity and of Cartan geometry.

1) Elliptical Orbit

$$\frac{1}{r} = \frac{1}{\alpha}\left(1 + \epsilon\cos\theta\right) \quad -(5)$$

So:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = -\frac{\epsilon}{\alpha}\cos\theta \quad -(6)$$

where

$$\cos\theta = \frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right) \quad -(7)$$

Here α is the half right latitude and ϵ the eccentricity. Therefore:

$$\begin{aligned} m\ddot{r} &= -\frac{L^2}{mr^2}\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) \\ &= \frac{L^2}{mr^2}\frac{\epsilon}{\alpha}\frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right) \quad -(8) \\ &= \frac{L^2}{mr^3} - \frac{L^2}{\alpha m r^2} \end{aligned}$$

3) This is a sum of the centrifugal force, which is positive and directed outwards, and an attractive, negative valued force:

$$\underline{F} = m\underline{a} = m(\ddot{r} - r\dot{\theta}^2)\underline{e}_r \quad -(9)$$

$$= -\frac{L^2}{\alpha m r^2}\underline{e}_r = -\underline{\nabla}\phi$$

The famous Newtonian result is:

$$\underline{F} = -\frac{L^2}{\alpha m r^2}\underline{e}_r = -\frac{mMG}{r^2}\underline{e}_r \quad -(10)$$

The Newtonian result is obtained with:

$$\alpha = \frac{L^2}{m^2 \underline{M} G} \quad -(11)$$

So:

$$m\ddot{r} = \underset{\substack{\text{centrifugal}\\\text{repulsive}}}{\frac{L^2}{mr^3}} - \underset{\substack{\text{attractive}\\\text{Newtonian}}}{\frac{m\underline{M}G}{r^2}} \quad -(12)$$

Newton obtained the inverse square law from Hooke according to John Aubrey, "Brief Lives", and both obtained:

4)

$$m\ddot{r} = -\frac{mMG}{r^2} \quad -(13)$$

because they used Cartesian coordinates and did not realize the existence of the centrifugal force. Unfortunately, eq. (13) is an incorrect description of planar orbits because it means that the attractive force is not counterbalanced by the centrifugal force, so m would fall into M.

2) Precessing Ellipse

$$\frac{1}{r} = \frac{1}{\alpha}\left(1 + \epsilon\cos(x\theta)\right) \quad -(14)$$

so

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = -\frac{\epsilon x^2}{\alpha}\cos(x\theta) \quad -(15)$$

where

$$x \sim 1 \quad -(16)$$

is the precession factor.

So:

$$m\ddot{r} = \frac{L^2}{mr^2}\,\frac{\epsilon x^2}{\alpha}\cos(x\theta)$$

$$= x^2\left(\frac{L^2}{mr^3} - \frac{L^2}{\alpha m r^2}\right) \quad -(17)$$

and is a balance of centrifugal and attractive

) forces multiplied by x^2.

We have:

$$\underline{F} = m\left(\ddot{r} - r\dot{\theta}^2\right)\underline{e}_r = -\underline{\nabla}\phi$$

$$= \left(\left(x^2 - 1\right)\frac{L^2}{mr^3} - \frac{x^2 L^2}{\alpha m r^2}\right)\underline{e}_r \quad -(18)$$

This is <u>non</u> - <u>Einsteinian</u>, and <u>non</u> - <u>Newtonian</u>, but at the same time is the direct result of fundamental geometry. The Newtonian result is recovered when

$$x = 1 \quad -(19)$$

<u>but</u> the <u>Einsteinian result</u> is <u>never</u> <u>recovered</u>, because the Einstein field equation produces the sum of inverse square and inverse fourth terms.

<u>So this analysis shows that the Einstein theory</u> <u>is incorrect</u>.

) <u>The</u> Hyperbolic Spiral

In the infinite r limit any planar orbit must become a hyperbolic spiral:

$$\theta = -\left(\frac{L}{mv_\infty}\right)\frac{1}{r} \quad -(20)$$

if the orbital linear velocity reaches a constant v_∞.

From eq. (20):

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = 0 \quad -(21)$$

So

$$m\ddot{r} = 0, \quad -(22)$$

and

$$\underset{\sim}{F} = m(\ddot{r} - r\dot{\theta}^2)\underline{e}_r = -\frac{L^2}{mr^3}\underline{e}_r \quad -(23)$$

Therefore:

$$m\ddot{r} = \frac{L^2}{mr^3} - \frac{L^2}{mr^3} = 0. \quad -(24)$$

The outward centrifugal force is exactly counterbalanced by the inward attractive force.

So if

$$v = \frac{dr}{dt} \quad -(25)$$

then

$$\frac{dv}{dt} = 0 \quad -(26)$$

i.e.

$$\frac{dv_\infty}{dt} = 0 \quad -(27)$$

and as $r \to \infty$:

$$v \xrightarrow[r \to \infty]{} v_\infty. \quad -(28)$$

So it is possible to explain the velocity curve of a spiral galaxy from eq. (1). Note carefully

7) that eq. (23) is completely non Newtonian and completely non Einsteinian, yet is produced by the fundamental geometry of the plane polar coordinates.

Antisymmetry Law

Denote the Cartan spin connection by:

$$\Omega^a_{\ \mu b} = \left(\frac{1}{c} \Omega^a_{\ 0b} , - \underline{\Omega}^a_{\ b} \right) \quad -(29)$$

then the antisymmetry law produces:

$$-\frac{\partial \underline{p}}{\partial t} - \Omega_0 \underline{p} = - \underline{\nabla} \phi + \underline{\Omega} \phi \quad -(30)$$

in the one polarization model.

A possible solution is:

$$-\frac{\partial \underline{p}}{\partial t} = - \underline{\nabla} \phi \quad -(31)$$

and

$$-\Omega_0 \underline{p} = \underline{\Omega} \phi \quad -(32)$$

Eq. (31) is the equivalence principle and eq. (32) defines the centrifugal force as:

$$\boxed{-\Omega_0 \underline{p} = \phi \underline{\Omega} = mr\omega^2 \underline{e}_r} \quad -(33)$$

8) so the inward momentum is:

$$\underline{p} = -\frac{mr\omega^2}{\Omega_0}\,\underline{e}_r \quad -(34)$$

corresponding to the inward force:

$$\underset{\sim}{F} = -\underline{\nabla}\,\phi \quad -(35)$$

From eq. (16) of note 262(2) the orbital velocity of any planar orbit is:

$$v^2 = \omega^2\left(r^2 + \left(\frac{dr}{d\theta}\right)^2\right) \quad -(36)$$

so for a circular orbit:

$$\frac{dr}{d\theta} = 0, \quad v = \omega r. \quad -(37)$$

If

$$-\underline{p} = -m\underline{v} \quad -(38)$$

then

$$\boxed{\Omega_0 = \omega} \quad -(39)$$

in a circular orbit. Otherwise:

$$mv = m\omega\left(r^2 + \left(\frac{dr}{d\theta}\right)^2\right)^{1/2} = \frac{mr\omega^2}{\Omega_0} \quad -(40)$$

and

$$\boxed{\Omega_0 = \frac{\omega r}{\left(r^2 + \left(\frac{dr}{d\theta}\right)^2\right)^{1/2}}} \quad -(41)$$

9)

Eq. (41) gives the relation between the scalar part of the Cartan spin connection and any planar orbit $dr/d\theta$.

The vector part of the Cartan spin connection is given by:

$$\phi \, \underline{\Omega} = m r \omega^2 \, \underline{e}_r \quad -(42)$$

so

$$\boxed{\underline{\Omega} = \frac{m r \omega^2}{\phi} \, \underline{e}_r} \quad -(43)$$

where:

$$\underline{\nabla} \phi = \frac{L^2}{m r^2} \left(\frac{d^2}{d\theta^2} \left(\frac{1}{r} \right) + \frac{1}{r} \right) \underline{e}_r \quad -(44)$$

So both Ω_0 and $\underline{\Omega}$ can be found for various planar orbits by computer algebra