261(6): Calculation of Light Deflection due to Gravitation from the ECE Orbital Force.

In the single polarization theory the ECE orbital force is:

$$\underline{F} = -\underline{\nabla}\phi - \frac{\partial \underline{p}}{\partial t} - \omega_0 \underline{p} + \phi \underline{\omega} \quad -(1)$$

By ECE antisymmetry:

$$-\underline{\nabla}\phi - \omega_0 \underline{p} = \frac{d\underline{p}}{dt} - \phi\underline{\omega} \quad -(2)$$

in which it has been assumed that:

$$\frac{d\underline{p}}{dt} = \frac{\partial \underline{p}}{\partial t} \quad -(3)$$

So:

$$\underline{F} = 2\left(\frac{d\underline{p}}{dt} - \phi\underline{\omega}\right) \quad -(4)$$

$$= -2\left(\underline{\nabla}\phi + \omega_0 \underline{p}\right)$$

This result has been derived by assuming:

$$\phi^a_\mu = \phi^{(0)} q^a_\mu \quad -(5)$$

2) The factor 2 in eq. (4) can be eliminated by assuming for convenience that:

$$\phi^a_\mu = \frac{\phi^{(0)}}{2} q^a_\mu \quad -(6)$$

So:

$$\underline{F} = \frac{d\underline{p}}{dt} - \phi\,\underline{\omega} = -\underline{\nabla}\phi - \omega_0 \underline{p}. \quad -(7)$$

As

$$\omega^a_{\mu b} \rightarrow 0 \quad -(8)$$

eq. (7) gives the Newtonian result:

$$\underline{F} = \frac{d\underline{p}}{dt} = -\underline{\nabla}\phi \quad -(9)$$

which is the equivalence principle:

$$\underline{F} = m\underline{g} = -mMG\frac{\underline{r}}{r^3} \quad -(10)$$

where

$$\phi = -\frac{mMG}{r}. \quad -(11)$$

In order to calculate light deflection due to gravitation use the experimental

3) fact that a solar system and all planar orbits are represented by a precessing conical section:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad -(12)$$

for small x.

As in previous UFT papers such as UFT 215 and UFT 216 eq. (12) can be used in the equation:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{m r^2}{L} F(r) \quad -(13)$$

(Marion and Thornton eq. (7.21)) which is valid for all planar orbits in which total angular momentum is conserved. Eqs. (12) and (13) give:

$$F(r) = -\frac{k x^2}{r} - \frac{k(1-x^2)\alpha}{r^3} \quad -(14)$$

where k is a constant. If:

$$\underline{p} = p_r \underline{e}_r \quad -(15)$$

$$\underline{\omega} = \omega_r \underline{e}_r \quad -(16)$$

4) Then:

$$F = \frac{dp_r}{dt} - \phi\,\omega_r = -\frac{kx^2}{r^2} - \frac{k(1-x^2)\alpha}{r^3} \quad -(17)$$

For small deviation from a Newtonian orbit:

$$\frac{dp_r}{dt} = -\frac{kx^2}{r^2}, \quad -(18)$$

$$x \sim 1 \quad -(19)$$

to an excellent approximation. Eq. (18) is the equivalence principle. So:

$$\phi\,\omega_r = \frac{k(1-x^2)\alpha}{r^3} \quad -(20)$$

to an excellent approximation where:

$$\phi = -\frac{k}{r} \quad -(21)$$

and

$$k = mMG \quad -(22)$$

So:

$$\boxed{\omega_r = -(1-x^2)\frac{\alpha}{r^2}} \quad -(23)$$

where

$$\alpha = \frac{b^2}{a} \quad -(24)$$

5) Here α is the half right latitude, a and b are the semi major and minor axes, and ϵ is the eccentricity. In light deflection by the sun the orbit is a hyperbola. The total deflection for the hyperbola is:

$$\Delta\phi = 2\sin^{-1}\frac{1}{\epsilon} \quad -(25)$$

As shown in UFT 216, for small angles of deflection at closest approach, R_0:

$$\sin\phi \sim \phi = \frac{1}{\epsilon} = \left[\frac{m^2\alpha R_0}{x^2L^2}\left(v^2 - \frac{L^2}{m^2}\left(\frac{x^2-1}{R_0^2}\right)\right) - 1\right]^{-1} \quad -(26)$$

where v is the velocity of a mass m orbiting a mass M. In the Newtonian limit this equation reduces to:

$$\sin\phi \sim \phi = \frac{1}{\epsilon} = \left(\frac{R_0v^2}{MG} - 1\right)^{-1} \quad -(27)$$

and for a photon of mass m:

$$v \longrightarrow c \quad -(28)$$

so

$$2\phi = \frac{2MG}{R_0c^2} \quad -(29)$$

This is the so called Newtonian

result for light deflection due to gravitation.

By experimental observation the value of light deflection by any mass M is:

$$2\phi = \frac{4MG}{R_0 c^2} \quad -(30)$$

The experimental result is explained by a choice of x in eq. (26) and therefore by a choice of spin connection ω_r from eq. (23). If eq. (28) is assumed the correction needed to produce eq. (30) is:

$$\frac{m^2 \alpha R_0 c^2}{L^2} = \frac{R_0 c^2}{mG}$$

$$\rightarrow \frac{R_0 c^2}{mG} - \frac{m^2 \alpha R_0}{x^2 m^2}\left(\frac{x^2-1}{R_0^2}\right)$$

$$= \frac{R_0 c^2}{MG} - \frac{\alpha}{R_0}\left(\frac{x^2-1}{x^2}\right)$$

$$= \frac{R_0 c^2}{mG} + \frac{\alpha}{R_0}\left(\frac{1-x^2}{x^2}\right) \quad -(30)$$

7) Now use the result:

$$\alpha = R_0 (1 + \epsilon) \quad -(31)$$

(Maria and Thonta eq. (7.44)) to find that:

$$2\phi = \frac{2 R_0 c^2}{MG} + 2(1+\epsilon)\left(\frac{1-x^2}{x^2}\right) \quad -(31)$$

Experimentally:

$$(1+\epsilon)\left(\frac{1-x^2}{x^2}\right) = \frac{R_0 c^2}{MG}. \quad -(32)$$

the observed.

The eccentricity ϵ is given by eq. (25): angle of deflection is

$$\frac{1}{\epsilon} = \sin\left(\frac{\Delta\phi}{2}\right) \quad -(33)$$

For small deflections:

$$\frac{1}{\epsilon} \sim \frac{\Delta\phi}{2} \quad -(34)$$

so

$$\left(1 + \frac{2}{\Delta\phi}\right)\left(\frac{1-x^2}{x^2}\right) = \frac{R_0 c^2}{MG} \quad -(35)$$

For small deflections:

$$x \sim 1 \quad -(36)$$

So

$$1 - x^2 = \frac{R_0 c^2}{MG}\left(1 + \frac{\alpha}{\Delta\phi}\right)^{-1} \quad -(37)$$

However:

$$\Delta\phi = \frac{4 R_0 c^2}{MG} \quad -(38)$$

so

$$1 - x^2 = \frac{\Delta\phi}{4}\left(1 + \frac{\alpha}{\Delta\phi}\right)^{-1} \quad -(39)$$

so using eq. (23):

$$\omega_r = -\frac{\Delta\phi}{4}\left(1 + \frac{\alpha}{\Delta\phi}\right)^{-1}\frac{\alpha}{r^2}. \quad -(40)$$

Finally from eq. (31):

$$\alpha = R_0(1 + \epsilon) \quad -(41)$$

$$= R_0\left(1 + \frac{\alpha}{\Delta\phi}\right)$$

Fm eqs. (40) and (41):

$$\boxed{\omega_r = -\frac{\Delta\phi}{4}\frac{R_0}{r^2}} \quad -(42)$$

i.e.

4) at closest approach:

$$r = R_0 \quad -(43)$$

So:

$$\boxed{\omega_r = -\frac{\Delta\phi}{4R_0}} \quad -(44).$$

This is a universal result because it is found experimentally that light deflection due to gravitation is always given by twice the Newtonian value. Both $\Delta\phi$ and R_0 are known experimentally so ω_r can be calculated and tabulated for any astronomical object.

It is named the spin curvature for light deflection due to gravity.