1) 261(3): PROOF OF IDENTITIES WITH THE HODGE DUAL

In the four dimension of spacetime the Hodge dual of an antisymmetric tensor is an antisymmetric tensor:

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}. \quad -(1)$$

The torsion and curvature are antisymmetric tensors so their Hodge duals in four dimensions are antisymmetric tensors:

$$\tilde{T}^{a}_{\ \mu\nu} = -\tilde{T}^{a}_{\ \nu\mu} \quad -(2)$$

$$\tilde{R}^{a}_{\ b\mu\nu} = -\tilde{R}^{a}_{\ b\nu\mu} \quad -(3)$$

It follows that:

$$D\wedge\tilde{T} := \tilde{R}\wedge q \quad -(4)$$

which is the Evans identity in four dimensional space time. The Cartan identity in 4-D is:

$$D\wedge T := R\wedge q. \quad -(5)$$

Here $\tilde{T}$ is the Hodge dual of T and $\tilde{R}$ is the Hodge dual of R. In four dimensions, eqs. (4) and (5) are exact identities.

2) So the geometry of ECE theory is:

$$T = D \wedge q \qquad (6)$$

$$R = D \wedge \omega \qquad (7)$$

$$D \wedge T := R \wedge q \qquad (8)$$

$$D \wedge \tilde{T} := \tilde{R} \wedge q \qquad (9)$$

This is entirely standard Cartan geometry. The Cartan Evans identity is an example of the Cartan identity and vice-versa.

<u>Therefore ECE theory is Cartan geometry.</u>

Eqs. (6) and (7) are the Cartan Maurer structure equations, eqs. (8) and (9) are the Cartan example in four dimensions of the identity.

Note carefully that the Evans identity is true in any dimension because T and R can always be written as the Hodge duals of some other tensor. However eqns (8) and (9) are true <u>simultaneously</u> only in four dimensions.

3) The two structure equations (6) and (7) are always produced simultaneously by the commutator method, which shows that the Christoffel connection is anti-symmetric. If torsion is zero, then the curvature is also zero.

Finally the tetrad postulate translates Cartan into Riemann geometry:

$$D_\mu q^a_\nu = 0. \quad - (10)$$