260(8): Development of the Schroedinger Equation under the Beltrami Constraint for Momentum

The Beltrami constraint for momentum produces:

$$\left(\nabla^2 + \kappa^2\right)\phi = 0 \quad -(1)$$

and
$$\kappa^2 \nabla^2 \kappa^2 = \underline{\nabla}\kappa^2 \cdot \underline{\nabla}\kappa^2 + \kappa^6 \quad -(2)$$

where
$$\kappa^2 = \frac{2m}{\hbar^2}\left(V - E\right), \quad -(3)$$

and where E is a constant.

So:
$$\underline{\nabla}\kappa^2 = \frac{2m}{\hbar^2}\underline{\nabla}V \quad -(4)$$

and
$$\nabla^2\kappa^2 = \frac{2m}{\hbar^2}\nabla^2 V \quad -(5)$$

Therefore:
$$\frac{2m}{\hbar^2}\left(V-E\right)\frac{2m}{\hbar^2}\left(\nabla^2 V\right) = \frac{4m^2}{\hbar^4}\left(\underline{\nabla}V\right)^2 + \left(\frac{2m}{\hbar^2}\right)^3\left(V-E\right)^3 \quad -(6)$$

so:
$$\frac{4m^2}{\hbar^4}\left[\left(V-E\right)\nabla^2 V - \underline{\nabla}V \cdot \underline{\nabla}V\right] = \left(\frac{2m}{\hbar^2}\right)^3\left(V-E\right)^3 \quad -(7)$$

i.e.
$$\left(V-E\right)^3 = \frac{\hbar^2}{2m}\left[\left(V-E\right)\nabla^2 V - \underline{\nabla}V \cdot \underline{\nabla}V\right] \quad -(8)$$

a)

This is a cubic equation in $V-E$:

$$\left(V-E\right)^3 - \frac{\hbar^2}{2m}\nabla^2 V\left(V-E\right) + \frac{\hbar^2}{2m}\left(\underline{\nabla} V \cdot \underline{\nabla} V\right) = 0 \quad -(9)$$

i.e.

$$x^3 - ax + b = 0 \quad -(10)$$

where:

$$x = V - E \quad -(11)$$

$$a = \frac{\hbar^2}{2m}\nabla^2 V \quad -(12)$$

$$b = \frac{\hbar^2}{2m}\underline{\nabla} V \cdot \underline{\nabla} V \quad -(13)$$

and

$$\left(\nabla^2 + \kappa^2\right)\phi = 0 \quad -(14)$$

where

$$\kappa^2 = \frac{2m}{\hbar^2}x \quad -(15)$$

Therefore the equation (14) can be expressed entirely in terms of V, $\underline{\nabla} V$ and $\nabla^2 V$, and solved numerically for the wave function ϕ. The expectation values of α are:

$$\langle \alpha \rangle = \int \phi^* \alpha \phi \, d\tau \quad -(16)$$

3) where α is any quantity.

If the electron, proton or neutron is made up of partons, then the probability density of the parton being in an infinitesimal volume element:

$$d\tau = r^2 dr \sin\theta \, d\theta \, d\phi \quad -(17)$$

is

$$P_1 = \phi \phi^* d\tau. \quad -(18)$$

The probability of finding the parton in the spherical shell of thickness dr and radius r is:

$$P_2 = \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi \, \phi \phi^* r^2 dr \quad -(19)$$

which is the distribution function of the parton.

<u>Conclusion</u>

The parton structure of elementary particles can be found by finding ϕ and calculating the probability density and distribution function