260(a). Solution for V

The starting equation is:

$$(V-E)^3 - a(V-E) + b = 0 \quad -(1)$$

where

$$a = \frac{\hbar^2}{2m}\nabla^2 V \quad -(2)$$

$$b = \frac{\hbar^2}{2m}\nabla V \cdot \nabla V \quad -(3)$$

Eq. (1) is a cubic in $V - E$. It can be written as:

$$E^3 - 3VE^2 + (3V^2 + a)E + V^3 - aV + b = 0 \quad -(4)$$

which is a cubic for E.

Using computer algebra the three roots of the cubic (1) can be found, giving the three solutions for $V - E$. Alternatively the three roots of eq. (4) can be found for E. The roots of eq. (4) can be checked by computer algebra, therefore E can be found as a function of V, ∇V and $\nabla^2 V$.

Now use the fact that the total energy E is constant, so:

$$\nabla E = 0 \quad -(5)$$

for each of the three roots of eq. (1).

2) Eq. (5) gives a differential equation in V, an equation that can be solved numerically.

Finally use this V in the Schroedinger equation:

$$-\frac{\hbar^2}{2m}\nabla^2\phi + V\phi = E\phi \quad -(6)$$

and find the wave functions ϕ and energy levels E of the partons of any elementary particle.

<u>Analytical Solution</u> of <u>Eq. (1)</u>

The solution of the cubic:

$$x^3 + px = q \quad -(7)$$

can be found using:

$$x = w - \frac{p}{3w} \quad -(8)$$

Then $\quad w^3 = \frac{1}{2}\left(q \pm \left(q^2 + \frac{4}{27}p^3\right)^{1/2}\right)$