267(1): Comparison of the Topological Structure of ECE Theory and Barrett's Equations

Gauss Law of Magnetism

1) ECE Theory

$$\underline{\nabla} \cdot \underline{B} = \underline{\omega} \cdot \underline{B} - \underline{A} \cdot \underline{R}(\text{spin}) \quad -(1)$$

2) Barrett's Theory

$$\underline{\nabla} \cdot \underline{B} = -iq\left(\underline{B} \cdot \underline{A} - \underline{A} \cdot \underline{B}\right) \quad -(2)$$

Faraday Law of Induction

1) ECE Theory

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = c\left(A_0 \underline{R}(\text{spin}) - \omega_0 \underline{B}\right) + \underline{\omega} \times \underline{E} - c\,\underline{A} \times \underline{R}(\text{orb}) \quad -(3)$$

2) Barrett's Theory

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = -iq\left(A_0 \underline{B} - \underline{B} A_0\right) + iq\left(\underline{A} \times \underline{E} - \underline{E} \times \underline{A}\right) \quad -(4)$$

Coulomb Law

1) ECE Theory

$$\underline{\nabla} \cdot \underline{E} = \underline{\omega} \cdot \underline{E} - c\,\underline{A} \cdot \underline{R}(\text{orb}) \quad -(5)$$

2) Barrett's Theory

$$\underline{\nabla} \cdot \underline{E} = J_0 - iq\left(\underline{A} \cdot \underline{E} - \underline{E} \cdot \underline{A}\right) \quad -(6)$$

2)

Ampère Maxwell Law

1) ECE Theory

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \frac{\omega_0}{c}\underline{E} - A_0 \underline{R}\,(\text{orb}) + \underline{\omega} \times \underline{B} - \underline{A} \times \underline{R}\,(\text{spin}) \quad -(7)$$

2) Barrett's Theory

$$\underline{\nabla} \times \underline{B} - \frac{\partial \underline{E}}{\partial t} = \underline{J} + iq\,(A_0 \underline{E} - \underline{E} A_0) + iq\,(\underline{A} \times \underline{B} - \underline{B} \times \underline{A}) \quad -(8)$$

The overall structure of Barrett's theory is:

$$\partial_\mu \tilde{F}^{\mu\nu} - iq A_\mu \tilde{F}^{\mu\nu} = -iq \tilde{F}^{\mu\nu} A_\mu \quad -(9)$$

and

$$\partial_\mu F^{\mu\nu} - iq A_\mu F^{\mu\nu} = J^\nu - iq F^{\mu\nu} A_\mu \quad -(10)$$

so the homogeneous field equations are

$$\partial_\mu \tilde{F}^{\mu\nu} = -iq\left(\tilde{F}^{\mu\nu} A_\mu - A_\mu \tilde{F}^{\mu\nu}\right) \quad -(11)$$

and the inhomogeneous field equations are:

$$\partial_\mu F^{\mu\nu} = J^\nu - iq\left(F^{\mu\nu} A_\mu - A_\mu F^{\mu\nu}\right) \quad -(12)$$

so the spin connection is:

$$\omega_\mu = -ig A_\mu \quad (13)$$

The terms on the right hand side of eqs. (11) and (12) are tensor cross products of field and potential. The simplified ECE equations are:

$$\partial_\mu \tilde{F}^{\mu\nu} + \omega_\mu \tilde{F}^{\mu\nu} = A_\mu \tilde{R}^{\mu\nu} \quad (14)$$

and

$$\partial_\mu F^{\mu\nu} + \omega_\mu F^{\mu\nu} = A_\mu R^{\mu\nu} \quad (15)$$

Eq. (14) is transformed into eq. (9) using:

$$\omega_\mu \tilde{F}^{\mu\nu} \rightarrow -ig A_\mu \tilde{F}^{\mu\nu} \quad (16)$$

and

$$A_\mu \tilde{R}^{\mu\nu} \rightarrow -ig \tilde{F}^{\mu\nu} A_\mu \quad (17)$$

Eq. (15) is transformed into eq. (10) using:

$$\omega_\mu F^{\mu\nu} \rightarrow -ig A_\mu F^{\mu\nu} \quad (18)$$

and

$$A_\mu R^{\mu\nu} \rightarrow J^\nu - ig F^{\mu\nu} A_\mu \quad (19)$$

The topology of the two sets of equations is the same: SU(2), homomorphic with O(3). Barrett's theory is an example of Cartan geometry.