256(3) : Double Checking Note

Homogeneous Field Equation

This is : $$D_\mu \tilde{F}^{a\mu\nu} = \tilde{R}^{a}{}_{\mu}{}^{\mu\nu} \quad -(1)$$

which means that :

$$\partial_\mu \tilde{F}^{a\mu\nu} + \omega^{a}_{\mu b}\tilde{F}^{b\mu\nu} = \tilde{R}^{a}{}_{\mu}{}^{\mu\nu} \quad -(2)$$

where

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \underline{\nabla}\right) \quad -(3)$$

$$\omega^{a}_{\mu b} = \left(\omega^{a}_{0 b}, -\underline{\omega}^{a}{}_{b}\right) \quad -(4)$$

Note that there is a sign change between (3) and (4). This comes from contravariant - covariant definitions in relativity.

The field tensor for each a is defined by:

$$\tilde{F}^{\mu\nu} = \begin{bmatrix} 0 & -cB_x & -cB_y & -cB_z \\ cB_x & 0 & E_z & -E_y \\ cB_y & -E_z & 0 & E_x \\ cB_z & E_y & -E_x & 0 \end{bmatrix} \quad -(5)$$

The standard model is :

$$\partial_\mu \tilde{F}^{\mu\nu} = 0 \quad -(6)$$

For $\nu = 0$:

$$\tilde{F}^{10} = cB_x, \; \tilde{F}^{20} = cB_y, \; \tilde{F}^{30} = cB_z$$

So :

a) $$\partial_1 \tilde{F}^{10} + \partial_2 \tilde{F}^{20} + \partial_3 \tilde{F}^{30} = 0 \quad -(7)$$

where summation over μ indices has been used. Eq. (7) means:

$$\frac{\partial B_X}{\partial X} + \frac{\partial B_Y}{\partial Y} + \frac{\partial B_Z}{\partial Z} = 0 \quad -(8)$$

i.e.

$$\boxed{\underline{\nabla} \cdot \underline{B} = 0} \quad -(9)$$

For $\nu = 1$:

$$\partial_0 \tilde{F}^{01} + \partial_2 \tilde{F}^{21} + \partial_3 \tilde{F}^{31} = 0 \quad -(10)$$

i.e.

$$-\frac{\partial B_X}{\partial t} - \frac{\partial E_Z}{\partial Y} + \frac{\partial E_Y}{\partial Z} = 0 \quad -(10)$$

Note that:

$$\underline{\nabla} \times \underline{E} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial/\partial X & \partial/\partial Y & \partial/\partial Z \\ E_X & E_Y & E_Z \end{vmatrix} \quad -(11)$$

$$= \left(\frac{\partial E_Z}{\partial Y} - \frac{\partial E_Y}{\partial Z} \right) \underline{i} - \left(\frac{\partial E_Z}{\partial X} - \frac{\partial E_X}{\partial Z} \right) \underline{j} + \left(\frac{\partial E_Y}{\partial X} - \frac{\partial E_X}{\partial Y} \right) \underline{k}$$

So eq. (10) is:

$$-\frac{\partial B_X}{\partial t} - \left(\underline{\nabla} \times \underline{E} \right)_X = 0 \quad -(12)$$

Note carefully that the standard physics definition (5) produces a minus sign in eq. (13). This is never pointed out in textbooks.

3) Similarly the $\nu = 2$ and $\nu = 3$ components produce:

$$-\frac{\partial B_Y}{\partial t} - (\underline{\nabla} \times \underline{E})_Y = 0 \quad -(13)$$

and

$$-\frac{\partial B_Z}{\partial t} - (\underline{\nabla} \times \underline{E})_Z = 0 \quad -(14)$$

In standard physics eqs. (12) to (14) are written as:

$$\boxed{\frac{\partial \underline{B}}{\partial t} + \underline{\nabla} \times \underline{E} = \underline{0}} \quad -(15)$$

and the minus sign is omitted.

Eq. (9) is the Gauss law of magnetism and eq. (15) is the Faraday law of induction, both for standard physics.

In ECE physics there is an additional term $\omega^a_{\ \mu b} \tilde{F}^{b\mu\nu}$ on the left hand side of eq. (2). The sign change in eq. (4) means that:

$$\boxed{\begin{aligned} \partial_\mu \tilde{F}^{a\mu\nu} &\rightarrow \underline{\nabla} \cdot \underline{B}^a \\ &\quad -\frac{\partial \underline{B}^a}{\partial t} - \underline{\nabla} \times \underline{E}^a \\ \omega^a_{\ \mu b} \tilde{F}^{b\mu\nu} &\rightarrow -\underline{\omega}^a_{\ b} \cdot \underline{B}^b \\ &\quad -c\,\omega_0 \underline{B}^a + \underline{\omega}^a_{\ b} \times \underline{B}^b \end{aligned}} \quad -(16)$$

4) Therefore:

$$\underline{\nabla} \cdot \underline{B}^a - \underline{\omega}^a{}_b \cdot \underline{B}^b = \tilde{R}^a{}_\mu{}^{\mu 0} \qquad -(17)$$
$$\nu = 0$$

and:

$$-\frac{\partial \underline{B}^a}{\partial t} - \underline{\nabla} \times \underline{E}^a - c\omega_0 \underline{B}^a + \underline{\omega}^a{}_b \times \underline{B}^b$$
$$= \tilde{R}^a{}_\mu{}^{\mu\nu}, \quad \nu = 1, 2, 3 \qquad -(18)$$

The right hand side is developed with:

$$\tilde{R}^a{}_\mu{}^{\mu\nu} = q^b{}_\mu \tilde{R}^a{}_b{}^{\mu\nu} \qquad -(19)$$

with summation over b indices and μ indices.

In direct analogy with the electromagnetic field tensor (5), and for each a and b, the curvature tensor in eq. (19) is defined as:

$$\tilde{R}^{\mu\nu} = \begin{bmatrix} 0 & -R_X(\text{spin}) & -R_Y(\text{spin}) & -R_Z(\text{spin}) \\ & & R_Z(\text{orb}) & -R_Y(\text{orb}) \\ R_X(\text{spin}) & 0 & & \\ & & 0 & R_X(\text{orb}) \\ R_Y(\text{spin}) & -R_Z(\text{orb}) & & 0 \\ R_Z(\text{spin}) & R_Y(\text{orb}) & -R_X(\text{orb}) & \end{bmatrix} \qquad -(20)$$

The labels "spin" and "orbital" are introduced from the well known description of the electromagnetic field

) tensor as being made up of electric (orbital) components and magnetic (spin) components. We can now proceed to translate eq. (19) into vector notation.

For

$$\nu = 0 \quad -(21)$$

$$\tilde{R}^{a}{}_{\mu}{}^{\mu 0} = q^{b}_{\mu} \tilde{R}^{a}{}_{b}{}^{\mu 0}$$

$$= q^{b}_{1} \tilde{R}^{a}{}_{b}{}^{10} + q^{b}_{2} \tilde{R}^{a}{}_{b}{}^{20} + q^{b}_{3} \tilde{R}^{a}{}_{b}{}^{30}$$

$$-(22)$$

Note that:

$$q^{b}_{\mu} = \left(q^{b}_{0}, -\underline{q}^{b}\right) \quad -(23)$$

is covariant four vector notation. So:

$$\tilde{R}^{a}{}_{\mu}{}^{\mu 0} = -\underline{q}^{b} \cdot \underline{R}^{a}{}_{b}(\text{spin}) \quad -(24)$$

Therefore:

$$\boxed{\underline{\nabla} \cdot \underline{B}^{a} = \underline{\omega}^{a}{}_{b} \cdot \underline{B}^{b} - \underline{A}^{b} \cdot \underline{R}^{a}{}_{b}(\text{spin})}$$

$$-(25)$$

For:

$$\nu = 1 \quad -(26)$$

$$\tilde{R}^{a}{}_{\mu}{}^{\mu 1} = q^{b}_{0} \tilde{R}^{a}{}_{b}{}^{01} + q^{b}_{2} \tilde{R}^{a}{}_{b}{}^{21} + q^{b}_{3} \tilde{R}^{a}{}_{b}{}^{31}$$

$$= -q^{b}_{0} R_{x}(\text{spin}) + q^{b}_{y} R^{a}{}_{bz}(\text{orb}) - q^{b}_{z} R^{a}{}_{yb}(\text{orb})$$

6)

So

$$\tilde{R}^{a}{}_{\mu}{}^{\mu\nu} \rightarrow -q^{b}{}_{0} \underline{R}^{a}{}_{b}(\text{spin}) + \underline{q}^{b} \times \underline{R}^{a}{}_{b}(\text{orb})$$

$-(28)$

Therefore:

$$\boxed{\frac{\partial \underline{B}^{a}}{\partial t} + \underline{\nabla} \times \underline{E}^{a} + c\omega_{0} \underline{B}^{a} - \underline{\omega}^{a}{}_{b} \times \underline{E}^{a} = A^{b}{}_{0} \underline{R}^{a}{}_{b}(\text{spin}) - \underline{A}^{b} \times \underline{R}^{a}{}_{b}(\text{orb})}$$

$-(29)$
