256(4): Double Checking Note

Inhomogeneous Field Equation

This is:

$$D_\mu F^{a\mu\nu} = A^{(0)} R^{a}{}_{\mu}{}^{\mu\nu} \quad -(1)$$

which means that:

$$\partial_\mu F^{a\mu\nu} + \omega^{a}{}_{\mu b} F^{b\mu\nu} = A^{(0)} R^{a}{}_{\mu}{}^{\mu\nu} \quad -(2)$$

The field tensor for each a is defined by:

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_Y & -E_Z \\ E_x & 0 & -cB_Z & cB_Y \\ E_Y & cB_Z & 0 & -cB_X \\ E_Z & -cB_Y & cB_X & 0 \end{bmatrix} \quad -(3)$$

The standard model is:

$$\partial_\mu F^{\mu\nu} = \frac{1}{\epsilon_0 c} J^\nu \quad -(4)$$

where

$$J^\nu = (c\rho, \underline{J}) \quad -(5)$$

is the charge current density, and where ϵ_0 is the vacuum permittivity, in S.I. units.

For $\nu = 0$:

$$\partial_1 F^{10} + \partial_2 F^{20} + \partial_3 F^{30} = \frac{J^0}{c\epsilon_0} \quad -(6)$$

i.e

$$\frac{\partial E_X}{\partial X} + \frac{\partial E_Y}{\partial Y} + \frac{\partial E_Z}{\partial Z} = \frac{\rho}{\epsilon_0} \quad -(7)$$

i.e.

$$\boxed{\underline{\nabla} \cdot \underline{E} = \frac{\rho}{\epsilon_0}} \quad - (8)$$

which is the Coulomb Law.

For $\nu = 1$:

$$\partial_0 F^{01} + \partial_2 F^{21} + \partial_3 F^{31} = \frac{J^1}{\epsilon_0 c} \quad - (9)$$

i.e.

$$-\frac{1}{c}\frac{\partial E_X}{\partial t} + \left(\frac{\partial B_Z}{\partial Y} - \frac{\partial B_Y}{\partial Z}\right) = \frac{J_X}{\epsilon_0 c} \quad - (10)$$

or

$$\left(\underline{\nabla} \times \underline{B}\right)_X - \frac{1}{c^2}\frac{\partial E_X}{\partial t} = \frac{J_X}{\epsilon_0 c^2} = \mu_0 J_X \quad - (11)$$

using:

$$\epsilon_0 \mu_0 = \frac{1}{c^2} \quad - (12)$$

where μ_0 is the vacuum permeability in S.I.

Eq. (11) is the X component of:

$$\boxed{\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J}} \quad - (13)$$

which is the Ampère Maxwell law.

In these derivations:

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \underline{\nabla}\right) \quad - (14)$$

$$\omega^a_{\mu b} = \left(\omega^a_{0b}, -\underline{\omega}^a_b\right) \quad - (15)$$

3) It follows that:

$$\partial_\mu F^{a\mu\nu} \rightarrow \begin{array}{l} \underline{\nabla} \cdot \underline{E}^a \\ \underline{\nabla} \times \underline{B}^a - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} \end{array} \quad -(16)$$

$$\omega^a_{\mu b} F^{b\mu\nu} \rightarrow \begin{array}{l} -\underline{\omega}^a{}_b \cdot \underline{E}^b \\ -\omega^a_{0b} \underline{E}^b - \underline{\omega}^a{}_b \times \underline{B}^b \end{array} \quad -(17)$$

So:

$$\boxed{\underline{\nabla} \cdot \underline{E}^a = \underline{\omega}^a{}_b \cdot \underline{E}^b - c\underline{A}^b \cdot \underline{R}^a{}_b(\text{orb})} \quad -(18)$$

The charge density is therefore:

$$\rho^a = \epsilon_0 \left(\underline{\omega}^a{}_b \cdot \underline{E}^b - c\underline{A}^b \cdot \underline{R}^a{}_b(\text{orb}) \right) \quad -(19)$$

which is UFT255, eq. (66), QED.

The Ampère Maxwell law is generalized to ECE as follows:

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2} \frac{\partial \underline{E}^a}{\partial t} - \frac{\omega^a_0 \underline{E}^b}{c} - \underline{\omega}^a{}_b \times \underline{B}^b \quad -(20)$$

$$= A^{(0)} R^a{}_\mu{}^{\mu\nu}, \quad \nu = 1, 2, 3$$

$$A^{(0)} \ldots^b R^a{}_b{}^{\mu\nu}$$

4) The curvature tensor is defined in analogy with eq.(3) ~~as~~: for each a and b as:

$$R^{\mu\nu} = \begin{bmatrix} 0 & -R_X(\text{orb}) & -R_Y(\text{orb}) & -R_Z(\text{orb}) \\ R_X(\text{orb}) & 0 & -R_Z(\text{spin}) & R_Y(\text{spin}) \\ R_Y(\text{orb}) & R_Z(\text{spin}) & 0 & -R_X(\text{spin}) \\ R_Z(\text{orb}) & -R_Y(\text{spin}) & R_X(\text{spin}) & 0 \end{bmatrix} \quad -(21)$$

For $\nu = 0$:

$$R^a{}_\mu{}^{\mu 0} = q^b{}_1 R^a{}_b{}^{10} + q^b{}_2 R^a{}_b{}^{20} + q^b{}_3 R^a{}_b{}^{30}$$
$$= -\underline{q}^b \cdot \underline{R}^a{}_b(\text{orb}) \quad -(22)$$

using

$$q^b{}_\mu = (q^b{}_0, -\underline{q}^b) \quad -(23)$$

For $\nu = 1$:

$$R^a{}_\mu{}^{\mu 1} = q^b{}_\mu R^a{}_b{}^{\mu 1}$$
$$= q^b{}_0 R^a{}_b{}^{01} + q^b{}_2 R^a{}_b{}^{21} + q^b{}_3 R^a{}_b{}^{31}$$
$$= -q^b{}_0 R^a{}_{bX}(\text{orb}) - q^b{}_Y R^a{}_{bZ}(\text{spin}) + q^b{}_Z R^a{}_{bY}(\text{spin})$$
$$= -q^b{}_0 R^a{}_{bX}(\text{orb}) - \left(\underline{q}^b \times \underline{R}^a{}_b(\text{spin})\right)_X$$

This is the X component of $\quad -(24)$

$$-q^b{}_0 \underline{R}^a{}_b(\text{orb}) - \underline{q}^b \times \underline{R}^a{}_b(\text{spin})$$

5) Therefore the Ampère Maxwell law is generalized to:

$$\boxed{\begin{aligned}\underline{\nabla}\cdot\underline{B}^a - \frac{1}{c^2}\frac{\partial \underline{E}^a}{\partial t} &= \underline{\omega}^a{}_b \times \underline{B}^b + \frac{\underline{\omega}_0}{c}\,\underline{E}^b \\ &\quad - A^b{}_0\,\underline{R}^a{}_b(\text{orb}) - \underline{A}^b \times \underline{R}^a{}_b(\text{spin}) \\ &= \mu_0 \underline{J}^a\end{aligned}}$$

$$-(25)$$

which is eq. (81) of 4FT255 QED
