256(7): Precise Definition of "Single Polarization" ECE Theory.

The entire ECE theory is based directly on Cartan geometry, which in turn is based on the definition of the tetrad:

$$V^a = q^a_\mu V^\mu \quad -(1)$$

The tetrad is the matrix linking any vector $V^a$ and $V^\mu$ in two different representations or bases. Restricting attention to three dimensional space and to unit vectors, then:

$$\begin{bmatrix} \underline{e}^{(1)} \\ \underline{e}^{(2)} \end{bmatrix} = \begin{bmatrix} q^{(1)}_X & q^{(1)}_Y \\ q^{(2)}_X & q^{(2)}_Y \end{bmatrix} \begin{bmatrix} \underline{i} \\ \underline{j} \end{bmatrix} \quad -(2)$$

for the transverse components of a vacuum plane wave.

In this case:

$$\underline{e}^{(1)} = \frac{1}{\sqrt{2}} (\underline{i} - i\underline{j}) e^{i\phi} \quad -(3)$$

$$\underline{e}^{(2)} = \frac{1}{\sqrt{2}} (\underline{i} + i\underline{j}) e^{-i\phi} \quad -(4)$$

where $\phi$ is the phase:

$$\phi = \omega t - \kappa Z. \quad -(5)$$

Here $\omega$ is the angular frequency at time $t$ and $\kappa$ is the wave vector component at point $Z$. The frame of reference defined in this way is rotating and translating with respect to the static frame. This is the most important point in ECE theory.

2) Therefore there exist the tetrad vectors:

$$\underline{v}^{(1)} = v_x^{(1)} \underline{i} + v_y^{(1)} \underline{j} \quad -(6)$$

$$\underline{v}^{(2)} = v_x^{(2)} \underline{i} + v_y^{(2)} \underline{j} \quad -(7)$$

$$\underline{v}^{(3)} = v_z^{(3)} \underline{k} \quad -(8)$$

where

$$v_x^{(1)} = \frac{1}{\sqrt{2}} e^{i\phi}, \quad v_y^{(1)} = -\frac{i}{\sqrt{2}} e^{i\phi}$$

$$v_x^{(2)} = \frac{1}{\sqrt{2}} e^{-i\phi}, \quad v_y^{(2)} = \frac{i}{\sqrt{2}} e^{-i\phi}$$

$$v_z^{(3)} = 1. \quad -(9)$$

The rotating and translating frame is defined by the phase factors $e^{i\phi}$ and $e^{-i\phi}$, so these ... This is what makes it necessary to define the connection. ECE theory, a theory of general relativity. It has more information than Maxwell Heaviside theory, in which the concept of connection does not exist. Recent work has shown that the Cartan identity gives a simple way of calculating the connection from the tetrad. Note 286(2) showed that

$$\boxed{\underline{\nabla} \times \underline{v}^a = -i \underline{\omega}^a{}_b \times \underline{v}^b} \quad -(10)$$

3) So:

$$\underline{\nabla} \times \underline{q}^{(1)} = \kappa \underline{q}^{(1)} = -i\underline{\omega}^{(1)}{}_b \times \underline{q}^b \quad -(11)$$

The single polarization model means that:

$$\underline{q}^{(1)} = \underline{q} \quad -(12)$$

i.e only the vector $\underline{q}^{(1)}$ is being considered. This is sufficient to define the vacuum plane wave:

$$\underline{q} = \frac{1}{\sqrt{2}}(\underline{i} - i\underline{j})e^{i\phi} \quad -(13)$$

Single polarization means:

$$a = b. \quad -(14)$$

Denoting:

$$\underline{\omega} = \underline{\omega}^{(1)}{}_{(1)} \quad -(15)$$

then it follows that:

$$\underline{\nabla} \times \underline{q} = \kappa \underline{q} = -i\underline{\omega} \times \underline{q} \quad -(16)$$
$$= -i\kappa \underline{k} \times \underline{q}.$$

Therefore:

$$\underline{k} \times \underline{q} = i\underline{q} \quad -(17)$$

where:

$$\underline{k} = \underline{q}^{(3)} \quad -(18)$$
$$\underline{q} = \underline{e}^{(1)} e^{i\phi} \quad -(19)$$

So eq. (17) is:

$$\underline{q}^{(3)} \times \underline{q}^{(1)} = i\underline{q}^{(1)}. \quad -(20)$$

Using

$$\underline{q}^{(1)} = \underline{q}^{(2)*} \quad -(21)$$

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eq. (20) can be written as:

$$\underline{q}^{(3)} \times \underline{q}^{(1)} = i\underline{q}^{(2)*} \quad -(22)$$

Eq. (22) implies:

$$\underline{q}^{(2)} \times \underline{q}^{(3)} = i\underline{q}^{(1)*} \quad -(23)$$

$$\underline{q}^{(1)} \times \underline{q}^{(2)} = i\underline{q}^{(3)*} \quad -(24)$$

Eqs. (22) to (24) define the complex circular basis. The (1), (2) and (3) superscripts can always be removed, to give:

$$\boxed{\begin{aligned} \underline{k} \times \underline{q} &= i\underline{q} \\ \underline{q}^* \times \underline{k} &= i\underline{q}^* \\ \underline{q} \times \underline{q}^* &= i\underline{k} \end{aligned}} \quad \begin{aligned} &-(25) \\ &-(26) \\ &-(27) \end{aligned}$$

Eqs. (22) to (24) are written in the (1), (2), (3) basis, eqs. (25) to (27) are written in the $\underline{i}$, $\underline{j}$, $\underline{k}$ basis, the Cartesian basis. They are two different ways of representing the same cyclic symmetry of space

Double Check on Eqs. (25) to (27)

Eq. (25) can be written as:

$$\underline{k} \times \underline{q} = \frac{e^{i\phi}}{\sqrt{2}} \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0 & 1 \\ 1 & -i & 0 \end{vmatrix}$$

$$= \frac{e^{i\phi}}{\sqrt{2}} \left( i\underline{i} + \underline{j} \right) \longrightarrow (28)$$

$$= \frac{i e^{i\phi}}{\sqrt{2}} \left( \underline{i} - i\underline{j} \right)$$

$$= i\underline{q} \qquad \text{QED}$$

Eq. (26) can be written as:

$$\underline{q}^* \times \underline{k} = \frac{e^{-i\phi}}{\sqrt{2}} \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & i & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad - (29)$$

$$= \frac{e^{-i\phi}}{\sqrt{2}} \left( i\underline{i} - \underline{j} \right)$$

$$= \frac{i e^{-i\phi}}{\sqrt{2}} \left( \underline{i} + i\underline{j} \right)$$

$$= i\underline{q}^* \qquad \text{QED}$$

Eq. (27) can be written as:

$$\underline{q} \times \underline{q}^* = \frac{1}{2} \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -i & 0 \\ 1 & i & 0 \end{vmatrix}$$

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$$= i\underline{k} = i\underline{q}^{(3)} = i\underline{q}^{(3)*} \quad -(30)$$

QED.

Eq. (27) is known as the conjugate product

$$\underline{q} \times \underline{q}^* = \underline{q}^{(1)} \times \underline{q}^{(2)}. \quad -(31)$$

It defines the $\underline{B}^{(3)}$ field and inverse Faraday effect as follows:

$$\underline{B}^{(3)} = \underline{B}^{(3)*} = -iB^{(0)}\underline{q}^{(1)} \times \underline{q}^{(2)}$$

$$= -iB^{(0)}\underline{q} \times \underline{q}^* \quad -(32)$$

$$= B^{(0)}\underline{k}$$

The spin connection is defined as:

$$\underline{\omega} = -i\omega_0 \underline{q}^{(1)} \times \underline{q}^{(2)} \quad -(33)$$

$$= -i\omega_0 \underline{q} \times \underline{q}^*$$

$$= \omega_0 \underline{k}$$

It follows ~~that~~:

$$\boxed{\underline{B}^{(3)} = B^{(0)}\frac{\underline{\omega}}{\omega_0}} \quad -(34)$$

The $\underline{B}^{(3)}$ field is defined by the spin

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convention. This means that general relativity is needed to define the $\underline{B}^{(3)}$ field, which is observed in the inverse Faraday effect as the conjugate product of circularly polarized radiation. The old physics did not use the spin connection and did not define the $\underline{B}^{(3)}$ because the old theory was special relativity. The spin connection defines the energy / momentum of the photon as follows:

$$p^\mu = \left( \frac{E}{c}, \underline{p} \right) = \hbar \left( \frac{\omega}{c}, \underline{\kappa} \right) \quad -(35)$$

$$= \hbar \left( \omega_0, \underline{\omega} \right)$$

where $\omega$ the angular frequency should not be confused with the spin connection four vector:

$$\omega^\mu = \left( \omega_0, \underline{\omega} \right). \quad -(36)$$

Therefore the photon energy momentum is defined by the $\underline{B}^{(3)}$ field as follows:

$$\boxed{p^\mu = \hbar \omega_0 \left( 1, \frac{\underline{B}^{(3)}}{B^{(0)}} \right)} \quad -(37)$$