256(2): Final Vector Equations of the ECE Theory and New Fundamental Equations for the Spin Connection

The complete field equations of the Engineering Model are as follows:

Homogeneous

$$\underline{\nabla} \cdot \underline{B}^a - \underline{\omega}^a{}_b \cdot \underline{B}^b = - \underline{A}^b \cdot \underline{R}^a{}_b (\text{spin}) \quad -(1)$$

$$\frac{\partial \underline{B}^a}{\partial t} + \underline{\nabla} \times \underline{E}^a + c\omega_0 \underline{B}^a - \underline{\omega}^a{}_b \times \underline{E}^b = A^b_0 \underline{R}^a{}_b (\text{spin}) - \underline{A}^b \times \underline{R}^a{}_b (\text{orb}) \quad -(2)$$

Inhomogeneous

$$\underline{\nabla} \cdot \underline{E}^a - \underline{\omega}^a{}_b \cdot \underline{E}^b = - c \underline{A}^b \cdot \underline{R}^a{}_b (\text{orb}) \quad -(3)$$

$$\underline{\nabla} \cdot \underline{B}^a - \frac{1}{c^2} \frac{\partial \underline{E}^a}{\partial t} - \underline{\omega}^a{}_b \times \underline{B}^b - \frac{\omega_0}{c} \underline{E}^b = \left[A^b_0 \underline{R}^a{}_b (\text{orb}) - \underline{A}^b \times \underline{R}^a{}_b (\text{spin}) \quad -(4)$$

The homogeneous equations can be written as:

$$\underline{\nabla} \cdot \underline{B}^a = 0 \quad -(5)$$

$$\frac{\partial \underline{B}^a}{\partial t} + \underline{\nabla} \times \underline{E}^a = \underline{0} \quad -(6)$$

if:

2)

$$\underline{A}^b \cdot \underline{R}^a{}_b(\text{spin}) = \underline{\omega}^a{}_b \cdot \underline{B}^b \quad -(7)$$

and:

$$\underline{\omega}^a{}_b \times \underline{E}^b - c\omega_0 \underline{B}^a = A_0^b \underline{R}^a{}_b(\text{spin}) - \underline{A}^b \times \underline{R}^a{}_b(\text{orb}) \quad -(8)$$

The inhomogeneous equation can be written as:

$$\underline{\nabla} \cdot \underline{E}^a = \rho^a / \epsilon_0 \quad -(9)$$

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2} \frac{\partial \underline{E}^a}{\partial t} = \mu_0 \underline{J}^a \quad -(10)$$

if

$$\rho^a = \epsilon_0 \left(c \underline{A}^b \cdot \underline{R}^a{}_b(\text{orb}) - \underline{\omega}^a{}_b \cdot \underline{E}^b \right) \quad -(11)$$

$$\underline{J}^a = \frac{1}{\mu_0} \left(A_0^b \underline{R}^a{}_b(\text{orb}) - \underline{A}^b \times \underline{R}^a{}_b(\text{spin}) + \underline{\omega}^a{}_b \times \underline{B}^b + \frac{\omega_0}{c} \underline{E}^b \right) \quad -(12)$$

The spin part of the torsion is:

$$\underset{\sim}{\underline{T}}^a = \underline{\nabla} \times \underline{q}^a - \underline{\omega}^a{}_b \times \underline{q}^b \quad -(13)$$

In the absence of a magnetic monopole the spatial part of the Cartan identity reduces to:

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{q}^b = 0 \quad -(14)$$

Therefore:

$$\boxed{\underline{\nabla} \cdot \underline{\nabla} \times \underline{q}^a = \underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{q}^b = 0 \quad -(15)}$$

3) A possible solution is:

$$\boxed{\underline{\nabla} \times \underline{q}^{a} = -i\, \underline{\omega}^{a}{}_{b} \times \underline{q}^{b}} \quad -(16)$$

For the vacuum plane waves in the complex circular basis:

$$\underline{q}^{(1)} = \frac{1}{\sqrt{2}} \left(\underline{i} - i \underline{j} \right) e^{i\phi} \quad -(17)$$

$$\underline{q}^{(2)} = \frac{1}{\sqrt{2}} \left(\underline{i} + i \underline{j} \right) e^{-i\phi} \quad -(18)$$

$$\underline{q}^{(3)} = \underline{k} \quad -(19)$$

with

$$\phi = \omega t - \kappa Z \quad -(20)$$

These equations produce:

$$\begin{aligned} \underline{q}^{(1)} \times \underline{q}^{(2)} &= i \underline{q}^{(3)*} \\ \underline{q}^{(3)} \times \underline{q}^{(1)} &= i \underline{q}^{(2)*} \\ \underline{q}^{(2)} \times \underline{q}^{(3)} &= i \underline{q}^{(1)*} \end{aligned} \quad -(21)$$

and

$$\underline{\nabla} \times \underline{q}^{(1)} = \kappa \underline{q}^{(1)} \quad -(22)$$

$$\underline{\nabla} \times \underline{q}^{(2)} = \kappa \underline{q}^{(2)} \quad -(23)$$

$$\underline{\nabla} \times \underline{q}^{(3)} = \underline{0} \quad -(24)$$

In general:

4)

$$\underline{\omega}^{(1)}{}_{(1)} \times \underline{q}^{(1)} + \underline{\omega}^{(1)}{}_{(2)} \times \underline{q}^{(2)} + \underline{\omega}^{(1)}{}_{(3)} \times \underline{q}^{(3)} = \kappa \underline{q}^{(1)} \quad -(25)$$

$$\underline{\omega}^{(2)}{}_{(1)} \times \underline{q}^{(1)} + \underline{\omega}^{(2)}{}_{(2)} \times \underline{q}^{(2)} + \underline{\omega}^{(2)}{}_{(3)} \times \underline{q}^{(3)} = \kappa \underline{q}^{(2)} \quad -(26)$$

$$\underline{\omega}^{(3)}{}_{(1)} \times \underline{q}^{(1)} + \underline{\omega}^{(3)}{}_{(2)} \times \underline{q}^{(2)} + \underline{\omega}^{(3)}{}_{(3)} \times \underline{q}^{(3)} = \underline{0} \quad -(27)$$

The Simplified Engineering Model

Denote:

$$\underline{q} = \underline{q}^{(1)} = \underline{e}^{(1)} e^{i\phi} \quad -(28)$$
$$= \frac{1}{\sqrt{2}} (\underline{i} - i\underline{j}) e^{i\phi}$$

This is a complex vector in the Cartesian basis and the index (1) is eliminated. It follows that eq. (16) simplifies to:

$$\underline{\nabla} \times \underline{q} = \kappa \underline{q} = -i \underline{\omega} \times \underline{q} \quad -(29)$$

in which:

$$\underline{q} = \underline{q}^{a} = \underline{q}^{b} \quad -(30)$$

The solution of eq. (29) is:

$$\boxed{\underline{\omega} = \kappa \underline{k}} \quad -(31)$$

The spin connection is the momentum of the photon accompanying the wave within $\hbar$:

$$\underline{p} = \hbar \underline{\kappa} = \hbar \underline{\omega} \quad -(31a)$$

5) With a spin connection of type (31) the engineering model simplifies considerably. The ECE hypothesis gives:

$$\underline{E} = c A^{(0)} \underset{\sim}{T}(\text{orb}) \quad -(32)$$

$$\underline{B} = A^{(0)} \underset{\sim}{T}(\text{spin}) \quad -(33)$$

The first Cartan structure equation gives:

$$\underline{E} = -\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t} - c\omega_0 \underline{A} + \phi \underline{\omega} \quad -(34)$$

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \quad -(35)$$

The Cartan and Evans identities give:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(36)$$

$$\frac{\partial \underline{B}}{\partial t} + \underline{\nabla} \times \underline{E} = \underline{0} \quad -(37)$$

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(38)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} \quad -(39)$$

The spacelike part of the Cartan identity gives

$$\underline{\nabla} \cdot \underline{\omega} \times \underline{A} = \underline{\omega} \cdot \underline{\nabla} \times \underline{A} - \underline{A} \cdot \underline{\nabla} \times \underline{\omega} \quad -(40)$$

The absence of magnetic charge [density] gives:

$$\underline{\omega} \cdot \underline{B} = \underline{A} \cdot \underline{R}(\text{spin}) \quad -(41)$$

6) The absence of magnetic current density gives:

$$\underline{\omega} \times \underline{E} - c\omega_0 \underline{B} = c\left(\phi \underline{R}(\text{spin}) + \underline{A} \times \underline{R}(\text{orb})\right) \quad -(42)$$

The electric charge density is:

$$\rho = \epsilon_0 \left(c\underline{A} \cdot \underline{R} - \underline{\omega} \cdot \underline{E} \right) \quad -(43)$$

The electric current density is:

$$\underline{J} = \frac{1}{\mu_0} \left(c\,\phi \underline{R}(\text{orb}) - \underline{A} \times \underline{R}(\text{spin}) + \underline{\omega} \times \underline{B} + \frac{\omega_0}{c} \underline{E} \right) \quad -(44)$$

The orbital curvature is

$$\underline{R}(\text{orb}) = -\underline{\nabla}\omega_0 - \frac{1}{c}\frac{\partial \underline{\omega}}{\partial t} - \omega_0 \underline{\omega} + \omega_0 \underline{\omega}$$
$$= -\underline{\nabla}\omega_0 - \frac{1}{c}\frac{\partial \underline{\omega}}{\partial t} \quad -(45)$$

The spin curvature is:

$$\underline{R}(\text{spin}) = \underline{\nabla} \times \underline{\omega} \quad -(46)$$

Using eq. (31), for vacuum plane waves:

$$\underline{R}(\text{orb}) = \underline{R}(\text{spin}) = \underline{0} \quad -(47)$$

The charge density is:

$$\rho = +\epsilon_0 \underline{\omega} \cdot \underline{E} = 0 \quad -(48)$$

The current density reduces to:

$$\underline{J} = \frac{1}{\mu_0}\left(\underline{\omega}\times\underline{B} + \frac{\omega_0}{c}\underline{E}\right) \quad -(49)$$

$$= \underline{0}$$

if:

$$\boxed{\omega_0 = \kappa = \frac{\omega}{c}} \quad -(50)$$

The scalar part of the conversion is the photon energy within $\hbar c$:

$$\boxed{E = \hbar\omega = \hbar c\,\omega_0} \quad -(51)$$

Here we have used:

$$\underline{E} = \frac{E^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)e^{i\phi} \quad -(52)$$

$$\underline{B} = \frac{B^{(0)}}{\sqrt{2}}\left(i\underline{i} + \underline{j}\right)e^{i\phi} \quad -(53)$$

$$E^{(0)} = cB^{(0)} \quad -(54)$$

Self consistently, there are no charge and current densities in the vacuum (QED).

The spin connection four vector is:

$$\omega^\mu = \left(\frac{\omega}{c}, \underline{\kappa}\right) \quad -(55)$$

$$= \kappa^\mu$$

8) where the angular frequency ω should not be confused with the connection.

The spinning and translating frame itself generates the photon energy and momentum along the direction of translation. This is a remarkable result of general relativity.

The free field is described by:

$$\boxed{\begin{aligned} \underline{\omega} \cdot \underline{B} &= 0 \\ \underline{\omega} \cdot \underline{E} &= 0 \\ \omega_0 \underline{B} - \frac{1}{c}\, \underline{\omega} \times \underline{E} &= \underline{0} \\ \omega_0 \underline{E} + c\, \underline{\omega} \times \underline{B} &= \underline{0} \end{aligned}} \quad -(56)$$

and by:

$$\boxed{\begin{aligned} \underline{\nabla} \cdot \underline{B} &= 0 \\ \underline{\nabla} \cdot \underline{E} &= 0 \\ \frac{\partial \underline{B}}{\partial t} + \underline{\nabla} \times \underline{E} &= \underline{0} \\ \underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} &= \underline{0} \end{aligned}} \quad -(57)$$

Eq. (56) is completely new to physics and solves the problem of why the wave is transverse to the photon.

9) The structure of eq. (49) suggests the Lorentz force law:

$$\underline{F} = e\left(\underline{E} + \underline{v} \times \underline{B}\right) \quad - (56)$$

In the absence of a magnetic field and far from material matter:

$$\rho \rightarrow + \epsilon_0 \underline{\omega} \cdot \underline{E} \quad - (57)$$

$$\underline{J} \rightarrow \frac{\omega_0}{c} \underline{E} \quad - (58)$$
