254(2): The Hodge Dual Identities and the Gauss Law of Magnetism.

The Hodge dual identities of Cartan geometry are the original Cartan identity:

$$D_\mu \tilde{T}^{a\mu\nu} := \tilde{R}_\mu{}^{a\mu\nu} \quad -(1)$$

and the Evans identity:

$$D_\mu T^{a\mu\nu} := R_\mu{}^{a\mu\nu} \quad -(2)$$

Here the tilde denotes Hodge dual. Eq. (1) is the same as:

$$D_\mu T^a{}_{\nu\rho} + D_\rho T^a{}_{\mu\nu} + D_\nu T^a{}_{\rho\mu} := R^a{}_{\mu\nu\rho} + R^a{}_{\rho\mu\nu} + R^a{}_{\nu\rho\mu} \quad -(3)$$

which is the antisymmetrized tensor product of a one form and two form. Eq. (2) is the same as:

$$D_\mu \tilde{T}^a{}_{\nu\rho} + D_\rho \tilde{T}^a{}_{\mu\nu} + D_\nu \tilde{T}^a{}_{\rho\mu} := \tilde{R}^a{}_{\mu\nu\rho} + \tilde{R}^a{}_{\rho\mu\nu} + \tilde{R}^a{}_{\nu\rho\mu} \quad -(4)$$

These equations are true because in four dimensions the Hodge dual of a two form is also a two form.

The ECE field equations of electrodynamics and gravitation are found from eqs (1) and (2), meaning that field equations of physics are identities.

2)

From eqs (1) and (2) the homogeneous and inhomogeneous field equations are respectively:

$$\partial_\mu \tilde{T}^{a\mu\nu} = 0 \quad -(5)$$

and

$$\partial_\mu T^{a\mu\nu} = j^{a\nu} \quad -(6)$$

where:

$$j^{a\nu} = R_\mu{}^{a\mu\nu} - \omega^a{}_{\mu b} T^{b\mu\nu} \quad -(7)$$
$$\neq 0$$

and

$$\tilde{j}^{a\nu} = \tilde{R}_\mu{}^{a\mu\nu} - \omega^a{}_{\mu b} \tilde{T}^{b\mu\nu} \quad -(8)$$
$$= 0$$

experimentally in electrodynamics, and probably also in gravitation.

Eq. (8) means that:

$$\omega^a{}_{\mu b} T^b{}_{\nu\rho} + \omega^a{}_{\rho b} T^b{}_{\mu\nu} + \omega^a{}_{\nu b} T^b{}_{\rho\mu} \quad -(9)$$
$$= R^a{}_{b\mu\nu} q^b{}_\rho + R^a{}_{b\rho\mu} q^b{}_\nu + R^a{}_{b\nu\rho} q^b{}_\mu$$

For space indices, i.e. for spin torsion or magnetism,

$$\underline{\omega}^a{}_b \cdot (\underline{\nabla} \times \underline{q}^b + \underline{\omega}^b{}_c \times \underline{q}^c) = \underline{q}^b \cdot \underline{R}^a{}_b$$
$$= \underline{q}^b \cdot (\underline{\nabla} \times \underline{\omega}^a{}_b + \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b)$$
$$-(10)$$

3) Now use:

$$\underline{\omega}^a{}_b \cdot \underline{\omega}^b{}_c \times \underline{q}^c = q^b \cdot \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b \quad -(11)$$

Fm eqs. (10) and (11):

$$\underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{q}^b = \underline{q}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b \quad -(12)$$

i.e.

$$\underline{\nabla} \cdot \underline{q}^b \times \underline{\omega}^a{}_b = 0 \quad -(13)$$

or:

$$\boxed{\underline{\nabla} \cdot \underline{B}^a = 0} \quad -(14)$$

which is exactly what is implied by a zero homogeneous current, QED.

As in the ECE engineering model the complete field equations of the ECE theory are:

$$\underline{\nabla} \cdot \underline{B}^a = 0, \quad -(15)$$

$$\underline{\nabla} \times \underline{E}^a + \frac{\partial \underline{B}^a}{\partial t} = \underline{0} \quad -(16)$$

from eq. (5), and:

$$\underline{\nabla} \cdot \underline{E}^a = \rho^a / \epsilon_0 \quad -(17)$$

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2} \frac{\partial \underline{E}^a}{\partial t} = \mu_0 \underline{J}^a \quad -(18)$$

from eq. (6).

4) The notation is explained fully in the ECE engineering model. It is seen that the spacelike eq. (15) is given by the spacelike Cartan identity, as in note 254 (1). This note has shown that it is given self consistently by the eq. (9) of the vanishing homogeneous current.

The electric and magnetic fields are given by the first Cartan structure equation:

$$\underline{E}^a = -\underline{\nabla}\phi^a - \frac{\partial \underline{A}^a}{\partial t} - \omega_0{}^a{}_b \underline{A}^b + \underline{\omega}^a{}_b \phi^b \quad -(19)$$

and

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \quad -(20)$$

The new insight of note 254 to date is that:

$$\underline{\nabla} \cdot \underline{B}^a = \underline{\nabla} \cdot \underline{\nabla} \times \underline{A}^a - \underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{A}^b$$

$$= 0 \quad -(21)$$

because

$$\underline{\nabla} \cdot \underline{\nabla} \times \underline{A}^a = 0 \quad -(22)$$

and

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{A}^b = 0 \quad -(23)$$

In contrast to eq. (21) there exists an electric charge or electric monopole, so:

5)

$$\underline{\nabla} \cdot \underline{E}^a = \rho^a / \epsilon_0 \quad -(24)$$

The antisymmetry constraints are:

$$\underline{\nabla} \phi^a - \frac{\partial \underline{A}^a}{\partial t} - \omega^a_{0b} \underline{A}^b - \underline{\omega}^a{}_b \phi^b = 0 \quad -(25)$$

and the simplified Lindstrom constraint:

$$\underline{\nabla} \times \underline{A}^a + \underline{\omega}^a{}_b \times \underline{A}^b = 0 \quad -(26)$$

which implies self consistently that:

$$\underline{\nabla} \cdot \underline{\nabla} \times \underline{A}^a = \underline{\nabla} \cdot \underline{A}^b \times \underline{\omega}^a{}_b \quad -(27)$$
$$= 0 .$$

The Aharonov Bohm effects are described by the existence of potentials in the absence of fields, so:

$$-\underline{\nabla} \phi^a - \frac{\partial \underline{A}^a}{\partial t} - \omega^a_{0b} \underline{A}^b + \underline{\omega}^a{}_b \phi^b = 0 \quad -(28)$$

and

$$\underline{\nabla} \times \underline{A}^a = \underline{\omega}^a{}_b \times \underline{A}^b \quad -(29)$$

From eq. (25) and (28):

$$\left(-\frac{\partial \underline{A}^a}{\partial t} - \omega^a_{0b} \underline{A}^b \right)_{vac} = \underline{0} \quad -(30)$$

From eqs. (26) and (29):

$$\left(\underline{\omega}^a{}_b \times \underline{A}^b \right)_{vac} = 0 \quad -(31)$$

$$\left(\underline{\nabla} \times \underline{A}^a \right)_{vac} = \underline{0} \quad -(32)$$