249(1): Translation of Particle Collision Theory to the Theory of Low Energy Nuclear Reactions

The general particle collision theory is:

$$E_1 + E_2 = E_3 + E_4 \quad -(1)$$

and

$$\underline{p}_1 + \underline{p}_2 = \underline{p}_3 + \underline{p}_4 \quad -(2)$$

The process of low energy nuclear reaction can be described as in UFT 226 ff. by assuming that a particle of energy E_1 collides with a potential barrier V. The latter models the nuclear nuclear repulsion.

In this model:

$$E_2 = -V \quad -(3)$$

$$\underline{p}_2 = \underline{0} \quad -(4)$$

so

$$E_1 - V = E_3 + E_4 \quad -(5)$$

$$\underline{p}_1 = \underline{p}_3 + \underline{p}_4 \quad -(6)$$

The kinetic energy E_1 of the incoming particle is described by the Einstein energy equation:

$$E_1^2 = c^2 p_1^2 + m^2 c^4 \quad -(7)$$

where m is the mass of the incoming particle. Eq. (7) can be written as:

$$E = E_1 - mc^2 = \frac{c^2 p_1^2}{E_1 + mc^2} \quad -(8)$$

In the low energy approximation:

2) The energy $E_1 - V$ is a relativistic kinetic energy that can be written as:

$$T = E_1 - V = (\gamma - 1) mc^2 \quad -(9)$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(10)$$

If

$$v \ll c \quad -(11)$$

then

$$T \sim \frac{1}{2} mv^2 = \frac{p_1^2}{2m} \quad -(12)$$

so

$$E_1 - V = \frac{p_1^2}{2m} \quad -(13)$$

$$= E_3 + E_4$$

in this approximation.

Eq. (13) quantizes to the Schroedinger equation

$$\hat{H} \phi = E_1 \phi \quad -(14)$$

where

$$\hat{H} = -\frac{\hbar^2 \nabla^2}{2m} + V \quad -(15)$$

Eq. (14) can be written as:

$$-\frac{\hbar^2}{2m} \phi + (V - E_1) \phi = 0$$

$$-(16)$$

3) Therefore the particle collision equations (1) and (2) have been translated into eq. (16).

If the potential barrier is very high then:

$$V > E_1 \quad -(17)$$

and as in 4FT226 ff. the Schroedinger equation gives the transmission coefficient:

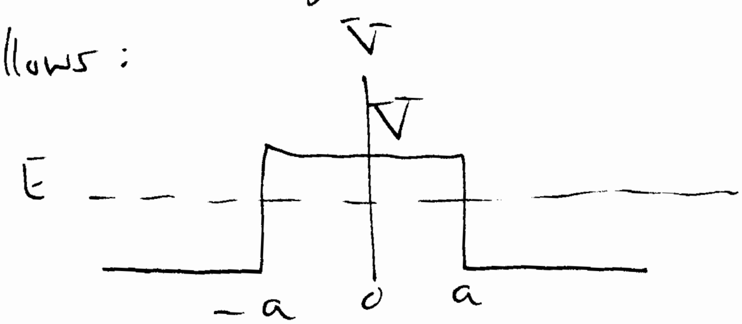
$$T \sim 16 \exp(-4\kappa a) \left(\frac{k\kappa}{k^2 + \kappa^2} \right)^2 \quad -(18)$$

where

$$k = \frac{1}{\hbar} (2mE_1)^{1/2} \quad -(19)$$

$$\kappa = \frac{1}{\hbar} (2m(V - E_1))^{1/2} \quad -(20)$$

for a model V as follows:



This means that the nuclear nuclear repulsion can be eliminated by quantum tunnelling.