

235(5): Orbital Linear Velocity as Spi Connection.

The velocity is defined by:

$$\underline{v} = \frac{d}{dt} (r \underline{e}_r) = \frac{dr}{dt} \underline{e}_r + r \frac{d\underline{e}_r}{dt} \quad - (1)$$

$$= \frac{dr}{dt} \underline{e}_r + \omega r \underline{e}_\theta$$

$$= \frac{dr}{dt} \underline{e}_r + \underline{\omega} \times \underline{r}$$

The Newtonian or inertial velocity is:

$$\underline{v}_N = \frac{dr}{dt} \underline{e}_r \quad - (2)$$

and the orbital linear velocity is:

$$\underline{v}_o = \underline{\omega} \times \underline{r} \quad - (3)$$

which does not exist is an inertial frame. This shows that applying Newtonian dynamics to an orbit is self contradictory. For an elliptical

orbit:

$$\underline{v} = \omega \frac{dr}{d\theta} \underline{e}_r + \underline{\omega} \times \underline{r} \quad - (4)$$

where

$$\frac{dr}{d\theta} = \frac{\epsilon r^2}{d} \sin \theta, \quad - (5)$$

$$r = \frac{d}{1 + \epsilon \cos \theta} \quad - (6)$$

$$2) \text{ So } \sin^2 \theta = 1 - \frac{1}{\epsilon^2} \left(\frac{d}{r} - 1 \right)^2 \quad - (7)$$

$$\text{and } \left(\frac{dr}{dt} \right)^2 = \left(\frac{\epsilon}{d} \right)^2 r^4 \left(1 - \frac{1}{\epsilon^2} \left(\frac{d}{r} - 1 \right)^2 \right)$$

$$= \left(\frac{r}{d} \right)^2 \left(\epsilon^2 r^2 - (d-r)^2 \right) \quad - (8)$$

$$\text{So } \frac{dr}{dt} = \frac{r}{d} \left(\left(r(\epsilon+1) - d \right) \left(r(\epsilon-1) + d \right) \right)^{1/2} \quad - (9)$$

$$\text{and } \boxed{\underline{v} = \frac{\omega r}{d} \left(\left(r(\epsilon+1) - d \right) \left(r(\epsilon-1) + d \right) \right)^{1/2} \underline{e}_r + \underline{\omega} \times \underline{r}} \quad - (10)$$

consisting of a centrally directed Newtonian or vertical component and a tangential orbital linear velocity $\underline{\omega} \times \underline{r}$ generated by the spin connection. In a circular orbit:

$$\epsilon = 0, \quad d = r \quad - (11)$$

$$\text{so } \underline{v} = \underline{\omega} \times \underline{r} \quad - (12)$$

and is due entirely to the frame movement and the movement of spacetime.