230(2): Photon Absorption as an Increase in Mass.

Consider the Einstein energy equation:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(1)$$

and quantize it as:

$$(E^2 - m^2 c^4)\phi = c^2 p^2 \phi \quad -(2)$$

i.e

$$(E - mc^2)\phi = \left(\frac{c^2 p^2}{E + mc^2}\right)\phi \quad -(3)$$

Make the approximation:

$$E = \gamma mc^2 \rightarrow mc^2 \quad -(4)$$

in the low velocity limit. Then:

$$(E - mc^2)\phi \sim \frac{p^2}{2m}\phi \quad -(5)$$

Use the Schroedinger postulate:

$$\underline{p} = -i\hbar \underline{\nabla} \quad -(6)$$

then

$$-\frac{\hbar^2 \nabla^2}{2m}\phi = (E - mc^2)\phi \quad -(7)$$

Now add the potential energy V to find that:

2)

$$\left(-\frac{\hbar^2\nabla^2}{2m} + V\right)\phi = (E - mc^2)\phi \quad -(8)$$

So:

$$\boxed{\nabla^2\phi = -\frac{2m}{\hbar^2}\left(E - mc^2 - V\right)\phi} \quad -(9)$$

Consider the reduced mass of the fused entity in a low energy nuclear reaction:

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \quad -(10)$$

Then:

$$\nabla^2\phi = -\frac{2\mu}{\hbar^2}\left(E - \mu c^2 - V\right)\phi \quad -(11)$$

where

$$E^2 = c^2 p^2 + \mu^2 c^4 \quad -(12)$$

Apply the postulates of Einstein and de Broglie:

$$E = \hbar\omega, \quad \underline{p} = \hbar\underline{\kappa} \quad -(13)$$

where ω and $\underline{\kappa}$ are the angular frequency and wavenumber of the fused entity. From eqs. (12) and (13)

$$\mu = \frac{\hbar}{c^2}\left(\omega^2 - \kappa^2 c^2\right). \quad -(14)$$

3) The transmission coefficient of quantum tunnelling of m_1 into m_2 is:

$$T = \frac{4}{\left(2\theta + \frac{1}{2\theta}\right)^2} \quad -(15)$$

where:

$$\theta = \exp\left(\left(\frac{2\mu}{\hbar}\right)^{1/2}\int_a^b \left(V - E + mc^2\right)^{1/2} dr\right) \quad -(16)$$

If the fused entity is at rest:

$$E \doteq mc^2, \quad -(17)$$

$$\mu \doteq \hbar\omega_0 / c^2 \quad -(18)$$

where ω_0 is the rest angular frequency of the fused entity. In this case:

$$\theta \doteq \exp\left(\left(\frac{2\omega_0}{\hbar}\right)^{1/2}\cdot\frac{1}{c}\int_a^b V^{1/2} dr\right) \quad -(19)$$

In this approximation the absorption of a photon occurs simply by:

$$\boxed{\omega_0 \rightarrow \omega_0 + \omega} \quad -(20)$$

and the effect on the transmission coefficient is worked out from eqs. (15), (19) and (20).