230(5): Development of the General Fermion equation.

The general fermion equation can be written as:

$$p^a p_\mu = m^2 c^2 q^a_\mu \quad -(1)$$

where

$$p^a_\mu = p_0 q^a_\mu \quad -(2)$$

is the momentum tetrad. So:

$$p^a p_\mu = \left(\frac{m^2 c^2}{p_0}\right) p^a_\mu \quad -(3)$$

The ECE minimal prescription is:

$$p^a_\mu \rightarrow p^a_\mu - eA^a_\mu \quad -(4)$$

so in the presence of electromagnetic radiation:

$$p^a p_\mu = \left(\frac{m^2 c^2}{p_0}\right)\left(p^a_\mu - eA^a_\mu\right) \quad -(5)$$

i.e.

$$p^a_\mu - eA^a_\mu = \left(\frac{p_0}{m^2 c^2}\right) p^a p_\mu \quad -(6)$$

$$= \left(\frac{p_0}{m^2 c^2}\right) q^a_\mu p^\mu p_\mu \quad -(7)$$

Therefore:

$$q^\mu_a\left(p^a_\mu - eA^a_\mu\right) = \left(\frac{p_0}{m^2 c^2}\right) p^\mu p_\mu \quad -(8)$$

2)

In this equation:

$$q^{\mu}_{a} p^{a}_{\mu} = p_0 \; ; \; e q^{\mu}_{a} A^{a}_{\mu} = e A_0 \quad -(9)$$

So

$$\boxed{p^{\mu} p_{\mu} = m^2 c^2 \left(1 - \frac{e A_0}{p_0}\right)} \quad -(10)$$

In this equation A_0 may be regarded as the magnitude of a vacuum potential.

Eq. (10) may be written as:

$$E^2 - c^2 p^2 = m^2 c^4 \left(1 - \frac{e A_0}{p_0}\right) \quad -(11)$$

i.e.

$$E - mc^2 \left(1 - \frac{e A_0}{p_0}\right)^{1/2} = \frac{c^2 p^2}{E + mc^2 \left(1 - \frac{e A_0}{p_0}\right)^{1/2}} \quad -(12)$$

This equation may now be approximated and reduced to a Schroedinger type equation.

In the non relativistic limit on the right hand side:

$$E \rightarrow mc^2 \quad -(13)$$

For a weak perturbation:

$$e A_0 << p_0 \quad -(14)$$

3) then:

$$E - mc^2\left(1-\frac{eA_0}{p_0}\right)^{1/2} \sim \frac{p^2}{2m}, \quad -(15)$$

and

$$\left(1-\frac{eA_0}{p_0}\right)^{1/2} \sim 1 - \frac{eA_0}{2p_0}, \quad -(16)$$

so

$$E - mc^2 + \frac{emc^2A_0}{2p_0} \sim \frac{p^2}{2m} \quad -(17)$$

Denote

$$E_1 = E - mc^2 \quad -(18)$$

then

$$E_1 + \frac{emc^2A_0}{2p_0} \sim \frac{p^2}{2m} \quad -(19)$$

This quantizes to:

$$\boxed{-\frac{\hbar^2\nabla^2}{2m}\phi = \left(E_1 + \frac{emc^2A_0}{2p_0}\right)\phi} \quad -(20)$$

In the presence of a potential V:

$$\left(-\frac{\hbar^2\nabla^2}{2m} + V\right)\phi = \left(E_1 + \frac{emc^2A_0}{2p_0}\right)\phi \quad -(21)$$

i.e.

$$-\frac{\hbar^2\nabla^2}{2m}\phi = \left(E_1 - V + \frac{emc^2A_0}{2p_0}\right)\phi \quad -(22)$$

4)

So:

$$\nabla^2 \phi = -\frac{2m}{\hbar^2}\left(E_1 - V + \frac{emc^2 A_0}{2p_0}\right)\phi \quad -(23)$$

This equation may now be used in tunnelling theory.