230(4): Development of the Schroedinger Postulate on the ECE Level

The Schroedinger postulate is:

$$p^\mu = i\hbar \partial^\mu \, ; \quad p_\mu = i\hbar \partial_\mu \quad -(1)$$

In ECE theory:

$$p^a = q^a_\mu \, p^\mu \quad -(2)$$

where q^a_μ is the Cartan tetrad. Multiply both sides of eq. (2) by p_μ:

$$p^a p_\mu = q^a_\mu \, p^\mu p_\mu = q^a_\mu \, m^2 c^2 \quad -(3)$$

where

$$p^\mu p_\mu = m^2 c^2 \quad -(4)$$

is the Einstein energy equation. So:

$$q^a_\mu = \frac{1}{m^2 c^2} \, p^a p_\mu \quad -(5)$$

Now define the momentum tetrad:

$$p^a_\mu = p_0 \, q^a_\mu \quad -(6)$$

so

$$\boxed{p^a_\mu = \left(\frac{p_0}{m^2 c^2} \right) p^a p_\mu} \quad -(7)$$

In the presence of electromagnetic radiation:

$$p_\mu^a \rightarrow p_\mu^a + eA_\mu^a \quad -(8)$$

which is the ECE minimal prescription.

It is seen that there is a more general Einstein energy equation in ECE theory:

$$\boxed{p^a p_\mu = m^2 c^2 q_\mu^a} \quad -(9)$$

and therefore a more general Klein Gordon equation and Dirac equation.

Illustration with Circular Polar Basis

Three dimensional space can be represented by the circular polar basis:

$$\underline{e}^{(1)} = \frac{1}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right) \quad -(10)$$

$$\underline{e}^{(2)} = \frac{1}{\sqrt{2}}\left(\underline{i} + i\underline{j}\right) \quad -(11)$$

$$\underline{e}^{(3)} = \underline{k} \quad -(12)$$

Here:

$$\underline{e}^{(1)} \times \underline{e}^{(2)} = i\,\underline{e}^{(3)*} \quad -(13)$$

et cyclicum

and

$$\underline{i} \times \underline{j} = \underline{k} \quad -(14)$$

et cyclicum

3) So:

$$p^a = \left(p^{(0)}, p^{(1)}, p^{(2)}, p^{(3)}\right) \quad -(15)$$

$$p^\mu = \left(p^0, p^1, p^2, p^3\right) \quad -(16)$$

where:

$$p^{(0)} = p^0 = \frac{E}{c} \quad -(17)$$

$$p^{(1)} = \frac{1}{\sqrt{2}}\left(p_x - i p_y\right) = \frac{1}{\sqrt{2}}\left(p^1 - i p^2\right) \quad -(18)$$

$$p^{(2)} = \frac{1}{\sqrt{2}}\left(p_x + i p_y\right) = \frac{1}{\sqrt{2}}\left(p^1 + i p^2\right) \quad -(19)$$

$$p^{(3)} = p^3 \quad -(20)$$

Therefore:

$$\begin{bmatrix} p^{(0)} \\ p^{(1)} \\ p^{(2)} \\ p^{(3)} \end{bmatrix} = \begin{bmatrix} q^{(0)}_0 & q^{(0)}_1 & q^{(0)}_2 & q^{(0)}_3 \\ q^{(1)}_0 & q^{(1)}_1 & q^{(1)}_2 & q^{(1)}_3 \\ q^{(2)}_0 & q^{(2)}_1 & q^{(2)}_2 & q^{(2)}_3 \\ q^{(3)}_0 & q^{(3)}_1 & q^{(3)}_2 & q^{(3)}_3 \end{bmatrix} \begin{bmatrix} p^0 \\ p^1 \\ p^2 \\ p^3 \end{bmatrix} \quad -(21)$$

Therefore:

$$q^a_\mu = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -i/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & i/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad -(22)$$

These are relations of a higher topology than the

4) usual Cartesian representation. This topology contains more information and generalizes well known relations of quantum mechanics.

For example:

$$\underline{p}^{(1)} = \frac{1}{\sqrt{2}} \left(p_x \underline{i} - p_y \underline{j} \right) \quad - (23)$$

$$\underline{p}^{(2)} = \frac{1}{\sqrt{2}} \left(p_x \underline{i} + p_y \underline{j} \right). \quad - (24)$$

The Schroedinger postulates are:

$$p_x = -i\hbar \frac{\partial}{\partial x}, \quad p_y = -i\hbar \frac{\partial}{\partial y}, \quad - (25)$$

so:

$$p_x^{(1)} = \frac{1}{\sqrt{2}} p_x = -\frac{i\hbar}{\sqrt{2}} \frac{\partial}{\partial x} \quad - (26)$$

$$p_x^{(2)} = \frac{1}{\sqrt{2}} p_x = -\frac{i\hbar}{\sqrt{2}} \frac{\partial}{\partial x} \quad - (27)$$

$$p_y^{(1)} = -\frac{1}{\sqrt{2}} p_y = \frac{i\hbar}{\sqrt{2}} \frac{\partial}{\partial y} \quad - (28)$$

$$p_y^{(2)} = \frac{1}{\sqrt{2}} p_y = -\frac{i\hbar}{\sqrt{2}} \frac{\partial}{\partial y} \quad - (29)$$

$$p^{(1)} = -\frac{i\hbar}{\sqrt{2}} \left(\frac{\partial}{\partial x} - i \frac{\partial}{\partial y} \right) \quad - (30)$$

$$p^{(2)} = -\frac{i\hbar}{\sqrt{2}} \left(\frac{\partial}{\partial x} + i \frac{\partial}{\partial y} \right) \quad - (31)$$

The d'Alembertian operator may be generalized

5) using results such as:

$$p^{(1)} p_1 = \frac{\hbar^2}{\sqrt{2}}\left(\frac{\partial^2}{\partial x^2} - i\frac{\partial}{\partial Y}\frac{\partial}{\partial X}\right) \quad -(32)$$

$$p^{(1)} p_2 = \frac{\hbar^2}{\sqrt{2}}\left(\frac{\partial^2}{\partial x^2} - i\frac{\partial^2}{\partial Y^2}\right) \quad -(33)$$

$$p^{(2)} p_1 = \frac{\hbar^2}{\sqrt{2}}\left(\frac{\partial^2}{\partial x^2} + i\frac{\partial}{\partial Y}\frac{\partial}{\partial X}\right) \quad -(34)$$

$$p^{(2)} p_2 = \frac{\hbar^2}{\sqrt{2}}\left(\frac{\partial^2}{\partial x^2} + i\frac{\partial^2}{\partial Y^2}\right) \quad -(35)$$

therefore eq. (9) may be quantized to give new types of Klein Gordon equation. For example:

1) If $a = (1)$, $\mu = 1$:

$$\frac{\hbar^2}{\sqrt{2}}\left(\frac{\partial^2}{\partial x^2} - i\frac{\partial}{\partial Y}\frac{\partial}{\partial X}\right)\phi = m^2c^2 q_1^{(1)}\phi \quad -(36)$$

$$= \frac{1}{\sqrt{2}} m^2 c^2 \phi \quad -(37)$$

i.e.

$$\boxed{\left(\frac{\partial^2}{\partial x^2} - i\frac{\partial}{\partial Y}\frac{\partial}{\partial X}\right)\phi = \left(\frac{mc}{\hbar}\right)^2 \phi} \quad -(38)$$

The Laplacian may be defined by

6) relations such as:

$$p_x^2 = \sqrt{2}\left(p^{(1)} + p^{(2)}\right) p_x \quad -(39)$$

$$p_y^2 = -i\sqrt{2}\left(p^{(2)} - p^{(1)}\right) p_y \quad -(40)$$

$$p_z^2 = p^{(3)} p_z \quad -(41)$$

and $$p_x^2 + p_y^2 + p_z^2 = -\hbar^2 \nabla^2 . \quad -(42)$$

In general:

$$p^a p_\mu = \frac{E^2}{c^2} - \underline{p}^a \cdot \underline{p} \quad -(43)$$

so $$\boxed{\left(E^2 - c^2 \underline{p}^a \cdot \underline{p}\right)\phi = m^2 c^{4} q^a_\mu \phi} \quad -(44)$$

which is a generalized Klein Gordon and generalized fermion equation.

The acceleration tetrad is defined by:

$$a^a_{\mu\nu} = \partial_\mu p^a_\nu - \partial_\nu p^a_\mu + \omega^a_{\mu b} p^b_\nu - \omega^a_{\nu b} p^b_\mu \quad -(45)$$

and may also be quantized.