230(6): Vacuum Correction and Minimal Prescription.

In note 230(5) the basic equation:

$$p^a p_\mu = \left(\frac{m^2 c^2}{p_0}\right) p_\mu^a \quad -(1)$$

was used with the minimal prescription:

$$p_\mu^a \rightarrow p_\mu^a + e A_\mu^a \quad -(2)$$

to give the equation:

$$p^\mu p_\mu = m^2 c^2 \left(1 - \frac{e A_0}{p_0}\right) \quad -(3)$$

In this note eq. (3) is interpreted as the Einstein energy equation corrected with the vacuum potential A_0. If there is an additional electromagnetic field present then eq. (3) becomes:

$$\pi^\mu \pi_\mu = m^2 c^2 \left(1 - \frac{e A_0}{p_0}\right) \quad -(4)$$

where

$$\pi^\mu = p^\mu - e A^\mu \quad -(5)$$

$$-(6)$$

In the SU(2) basis:

$$p^\mu p_\mu = \frac{E^2}{c^2} - \underline{\sigma} \cdot \underline{p}\, \underline{\sigma} \cdot \underline{p} = m^2 c^2 \left(1 - \frac{e A_0}{p_0}\right)$$

so:

$$E^2 - m^2 c^4 \left(1 - \frac{e A_0}{p_0}\right) = c^2 \underline{\sigma} \cdot \underline{p}\, \underline{\sigma} \cdot \underline{p} \quad -(7)$$

i.e.

2)
$$E - mc^2\left(1 - \frac{eA_0}{p_0}\right)^{1/2} = \frac{c^2\,\underline{\sigma}\cdot\underline{p}\;\underline{\sigma}\cdot\underline{p}}{E + mc^2\left(1-\frac{eA_0}{p_0}\right)^{1/2}} \quad -(8)$$

In the non-relativistic limit:
$$E - mc^2 \quad -(9)$$

so if
$$eA_0 \ll p_0, \quad -(10)$$

$$E - mc^2\left(1-\frac{eA_0}{2p_0}\right) \sim c^2\,\underline{\sigma}\cdot\underline{p}\left(\frac{1}{2mc^2 - \frac{eA_0 mc^2}{2p_0}}\right)\underline{\sigma}\cdot\underline{p} \quad -(11)$$

i.e.
$$E - mc^2\left(1-\frac{eA_0}{2p_0}\right) = \frac{1}{2m}\,\underline{\sigma}\cdot\underline{p}\left(\frac{1}{1-\frac{eA_0}{4p_0}}\right)\underline{\sigma}\cdot\underline{p}$$

$$\doteq \frac{1}{2m}\,\underline{\sigma}\cdot\underline{p}\left(1+\frac{eA_0}{4p_0}\right)\underline{\sigma}\cdot\underline{p} \quad -(12)$$

This equation may be quantized to produce a Schroedinger like equation as in the next note.