230(10): Particle Travelling in the Z direction:

In this case:

$$\Box \phi = -\left(\frac{m^2c^2}{\hbar^2 p_0}\right)\left(\hat{p}^{(0)}_0 + \hat{p}^{(3)}_3\right)\phi \quad -(1)$$

which implies that

$$\left(\hat{p}^{(0)}_0 + \hat{p}^{(3)}_3\right)\phi = p_0 \phi. \quad -(2)$$

From the general result:

$$p^a_\mu = \frac{p_0}{m^2c^2}\left(\underline{p}^a \cdot \underline{p}_\mu\right) \quad -(3)$$

it follows that:

$$\hat{p}^{(0)}_0 \phi = \left(\frac{p_0}{m^2c^2}\right)\left(\underline{\hat{p}}^{(0)} \cdot \underline{\hat{p}}_0\right)\phi \quad -(4)$$

and

$$\hat{p}^{(3)}_3 \phi = \left(\frac{p_0}{m^2c^2}\right)\left(\underline{\hat{p}}^{(3)} \cdot \underline{\hat{p}}_3\right)\phi \quad -(5)$$

where

$$\underline{\hat{p}}^{(0)} \cdot \underline{\hat{p}}_0 = \hat{p}^0 \hat{p}_0 \quad -(6)$$

and

$$\underline{\hat{p}}^{(3)} \cdot \underline{\hat{p}}_3 = \hat{p}^3 \hat{p}_3 \quad -(7)$$

so

$$-\hbar^2 \Box \phi = \left(\hat{p}^0\hat{p}_0 + \hat{p}^3\hat{p}_3\right)\phi. \quad -(8)$$

However:

$$\left(\Box + \left(\frac{mc}{\hbar}\right)^2\right)\phi = 0 \quad -(9)$$

so

$$\left(\hat{p}^0\hat{p}_0 + \hat{p}^3\hat{p}_3\right)\phi = m^2c^2\phi \quad -(10)$$

2)

i.e.

$$\boxed{\left(\underline{\hat{p}}^{(0)} \cdot \underline{\hat{p}}_0 + \underline{\hat{p}}^{(3)} \cdot \underline{\hat{p}}_3\right)\phi = m^2c^2\phi} \quad -(11)$$

Therefore $\underline{p}^{(0)} \cdot \underline{p}_0 + \underline{p}^{(3)} \cdot \underline{p}_3 = \frac{E^2}{c^2} - p^2 \quad -(12)$

$$= \frac{m^2c^2}{p_0}\left(p_0^{(0)} + p_0^{(1)}\right)$$

However:

$$\frac{E^2}{c^2} - p^2 = m^2c^2 \quad -(13)$$

so in this case:

$$p_0^{(0)} + p_0^{(1)} = p_0, \quad -(14)$$

which is consistent with eq. (2).

For the purposes of low energy nuclear reaction it is convenient to express eq. (10) as:

$$\left(p_0^2 - p_Z^2\right)\phi = m^2c^2\phi \quad -(15)$$

i.e. $\left(p_0^2 - m^2c^2\right)\phi = p_Z^2\phi \quad -(16)$

This can be reduced to a Schroedinger equation using the usual method:

3)
$$(p_0 - mc)\phi = \left(\frac{p_z^2}{p_0 + mc}\right)\phi \quad -(17)$$

in which:
$$p_z^2 = -\hbar^2 \nabla^2 \quad -(18)$$

is regarded as an operator.

<u>Conclusions.</u>

1) A rigorously self consistent ECE theory has been developed using the classical result:
$$p_\mu^a = \left(\frac{p_0}{m^2c^2}\right) \underline{p}^a \cdot \underline{p}_\mu \quad -(19)$$

which quantizes to:
$$\hat{p}_\mu^a \phi = \left(\frac{p_0}{m^2c^2}\right)\left(\underline{\hat{p}}^a \cdot \underline{\hat{p}}_\mu\right)\phi \quad -(20)$$

2) In the simple case of the complex circular Cartesian basis, and for basis superimposed on the Z axis: motion is
$$\left(\hat{p}_0^{(0)} + \hat{p}_3^{(3)}\right)\phi = p_0 \phi \quad -(21)$$

and
$$\left(\underline{\hat{p}}^{(0)} \cdot \underline{\hat{p}}_0 + \underline{\hat{p}}^{(3)} \cdot \underline{\hat{p}}_3\right)\phi = m^2c^2\phi \quad -(22)$$

which reduce to the Schroedinger equation.